www.iop.org/journals/physed

Rotation in secondary school: teaching the effects of frictional force

Paulo Simeão Carvalho and Adriano Sampaio e Sousa

Departamento de Física, Faculdade de Ciências da Universidade do Porto,
Rua do Campo Alegre, 687, 4169-007 Porto, Portugal

Abstract
Frictional force is a source of misconceptions among students, as teachers know from daily experience. This is confirmed by many studies carried out by investigators from all over the world. Surprisingly (or perhaps not), we have found some of these misconceptions among physics school teachers and senior students of physics education courses participating in a workshop in Portugal. In this article we discuss conceptual problems involving frictional force in rotating bodies and suggest teaching strategies based on problem solving, in order to ensure meaningful learning of this difficult topic.

Introduction

Classical mechanics, and particularly dynamics, is one of the most important subjects in physics courses. Therefore, the amount of time spent on this subject in secondary school is very significant when compared with other physics topics. Nevertheless, secondary school students continue to show serious problems when facing basic situations, especially those involving frictional forces, and misconceptions on this subject are frequently found (Driver *et al* 1985, Salazar *et al* 1990), many of them induced by common sense. To our surprise, this picture was also found among university students, in a physics education course given by the authors; according to the literature (Caldas and Saltiel 1999a, 1999b, Caldas *et al* 2001, Viennot 2003), we are sure this is neither exclusive to our university nor to our country. Some strategies to overcome this situation can also be found in the literature (Viennot 2003, Mungan 2001, Arons 1997, Puri 1996, Sousa and Pina 1997); most of them are based on the discussion of specific problems. However, these approaches do not make systematic use of strategies that confront students with physical situations where they can discuss relevant features of the frictional force, like its origin, its direction and eventual effects on translation and rotation, and the comparative motion of sliding blocks and rolling objects.

We have promoted a workshop during a National Conference (Carvalho and Sousa 2002), in which participants (15 school physics teachers and senior students of physics education courses) were questioned about basic concepts such as the direction of frictional force, normal forces at an interface and Newton's third law, in the context of situations involving rolling objects. In this paper we present and discuss the results of this workshop, showing that many teachers exhibit the same difficulties about rolling objects as students do. With this in mind, relevant problems (with solutions) and teaching strategies are then suggested, with the aim that they can be carefully explored with undergraduate students or physics school teachers, so that they can discuss

0031-9120/05/030257+09$30.00

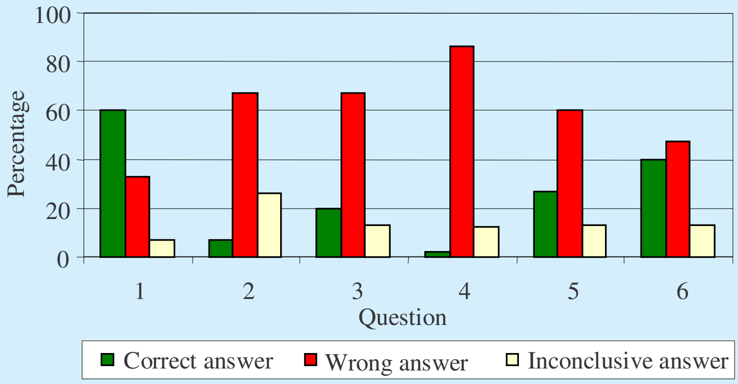

Figure 1. Graphical analysis of the results.

them later, on a conceptual basis, with secondary school students.

Results from the workshop

We asked the participants to comment on six sentences, for about half an hour, without telling them whether they were right or wrong. These sentences were:

1. Frictional forces are always opposite to the direction of the motion.
2. Frictional forces are always opposite to the direction of the velocity at the instantaneous point(s) of contact.
3. Kinetic friction and rolling friction are equivalent designations for friction: the first one is used for sliding objects, while the second one is used for rolling objects.
4. A rigid cylinder rolling without sliding on a rigid horizontal surface is always subjected to a frictional force.
5. A body standing on a horizontal surface is always subjected to a normal force from the surface, whose line of action is the same as that of the gravitational force acting on it (weight).
6. The free body diagram of a solid block freely ascending an incline is the same as that of a solid cylinder freely ascending the same incline.

The first three sentences refer to the basic concepts of friction, frictional force and its direction. With sentence 4, we explicitly introduce the problem of frictional force on rolling objects. Sentence 5 aims at the location of the normal force exerted by the surface on the body, which is especially important in situations concerning sliding objects on surfaces with friction. Finally, with sentence 6, we wanted to confront participants with sliding and rolling motions, by comparing the free body diagrams of sliding objects and rolling objects on an incline.

We classified the answers into three categories: correct answer, wrong answer and inconclusive answer (the last including non-existent or evasive answers). The results are shown graphically in figure 1.

Discussion

The results show that the majority of participants of the workshop seem to know that the frictional force does not always have the same direction as the motion. This fact could be related to some common examples like a person walking, or a passenger standing in a bus as it accelerates.

However, the results concerning sentence 5 clearly show that undergraduate students and teachers do not perceive the contribution of the torques of frictional and normal forces to body motion (sliding and/or rolling). As a result, for all the other sentences where the frictional force in rolling objects must be taken into account, there is a general lack of understanding and a variety of misconceptions about free body diagrams in such contexts.

After a review of Portuguese and international physics textbooks aimed at secondary school level (and some at university level), we have realized that in many examples and even experiments where the authors make use of rolling objects, the mathematical treatment of the body's motion is reduced to that of a particle (for example, a sphere rolling down a 'frictionless' incline, which is physically impossible). Furthermore, objects with wheels are frequently used as examples for describing sliding motion, therefore inducing misconceptions in students' minds.

Moreover, when teachers try to give students a real description of phenomena, sometimes they consider some forces to be applied at the centre of the body (typically the weight and the normal force) but the frictional force is systematically represented at the bottom, between the body and the surface, which is a nonsense because the free body diagram will not be physically coherent. What should be done, then?

Teachers must realize that the main thing about rolling objects is that a rotation of the body takes place. This means that the physical and mathematical description of rolling objects implies not only the fundamental equation for translation,

$$\sum_i \boldsymbol{F}_i = m\boldsymbol{a} \qquad (1)$$

but also the fundamental equation for rotation,

$$\sum_i \boldsymbol{M}_{\boldsymbol{F}_i} = I\boldsymbol{\alpha}. \qquad (2)$$

Before students can understand this, they must be first introduced to the basic concepts of *angular speed* and *angular acceleration*, *moment of inertia of a body* (inertia to rotation) and eventually *torque*, concepts that are taught only in the final years of secondary school and in optional courses.

The difficulties shown by students in learning these subjects justify the use of examples and counter-examples in the classroom. They must be fully discussed with students, so that they can have first a good conceptual understanding and subsequently a mathematical understanding of them. In addition to all the problems already suggested in textbooks, which are, of course, valuable contributions, we suggest here some relevant and stimulating problems, which aim directly at the gaps pointed out in the introductory study of frictional forces.

I. Rolling without slipping on a horizontal surface

Problem 1

Consider a rigid coin rolling freely, without slipping, on the top of a metallic table. What is the direction of the frictional force acting on the coin?

Solution

The coin can be seen as having two distinct but combined motions: translation of its centre of mass (CM) with constant speed v, and rotation about the CM with angular speed ω. However, as it rolls without slipping, v and ω are related to each other by the expression $v = \omega R$, where R is the radius of the coin (students should be asked to describe why, and if possible to demonstrate it). The weight and the normal force acting on the coin are vertical collinear vectors, pointing in opposite directions, so the net torque about the CM is null. As the CM is not accelerated, then the coin is not subjected to any frictional force.

What students think about this...

There is a common idea that whenever there is motion, there must be a frictional force. The absence of a frictional force is also difficult to understand because people know that objects do not move forever; what they don't know is that this fact is due to rolling friction that comes from the deformation of objects or surfaces (Doménech *et al* 1987), and here we are dealing with ideal (non-deformable) objects.

Teaching approaches

This is a basic problem that, as far as we know, is rarely discussed in the classroom. For its analysis, it is of the utmost importance to ask for a free body diagram, because it will conceptually confront students with the non-existence of any horizontal forces acting on the coin during its motion. Additionally, we suggest a mathematical treatment of the problem, in order to analyse the consequences of a frictional force at the point of contact between the coin and the table: the equations of motion derived will be physically incompatible for both translation and rotation, unless this force does not exist. Finally, we suggest that a similar problem is discussed in a different context: 'If you do this experiment in outer space

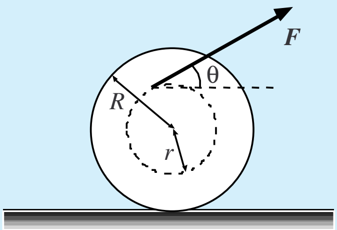

Figure 2. Motion of a spool in a horizontal plane.

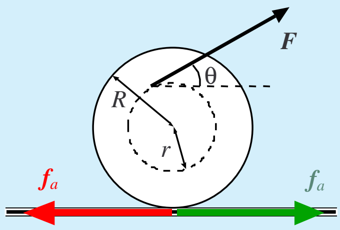

Figure 3. Frictional forces acting on the spool.

and suddenly remove the table, what will happen to the rolling coin?'

For advanced students, we can go further in this discussion and let them evaluate what would happen if, instead of a rigid coin, we had considered a rubber ball or cylinder (this will allow the discussion of a new frictional force, that resulting from rolling friction). As an exploratory activity, let them analyse why a car slows down and stops in a horizontal plane, when the engine cuts out.

Problem 2

Place a spool (two wheels of radius R connected by a cylinder of radius r) of mass m on top of a table. If you pull it as shown in figure 2, can you predict the direction of the frictional force acting on the spool?

Solution

This problem has already been discussed by many authors (Shaw 1979, Caldas and Saltiel 1999a, Pinto and Fiolhais 2001, Mungan 2001, Hewitt 2002, Bartoš and Musilová 2004), although it has not been used as a teaching strategy for secondary school students. $\boldsymbol{F}$ has a non-zero component along the direction of motion (horizontal axis); its torque about the CM reinforces the translation motion. It is impossible to say in which direction the frictional force vector, $\boldsymbol{f}_a$, will point because this can only be achieved by considering equations (1) and (2).

Calculating the acceleration of the CM and the frictional force acting on the spool we get

$$a = \frac{FR(r + R\cos\theta)}{mR^2 + I} > 0$$
$$f_a = \frac{F(mRr - I\cos\theta)}{mR^2 + I}. \qquad (3)$$

where I is the moment of inertia of the spool. Equations (3) show that the spool will always accelerate in the same direction as the horizontal component of $\boldsymbol{F}$. However, three situations are possible, as shown in figure 3:

(i)	$r > \frac{I}{mR}\cos\theta$	$\boldsymbol{f}_a$ is in the direction of motion
(ii)	$r = \frac{I}{mR}\cos\theta$	$\boldsymbol{f}_a = 0$
(iii)	$r < \frac{I}{mR}\cos\theta$	$\boldsymbol{f}_a$ is opposite to the direction of motion

The main issue is that, within the same situation and although the direction of motion is maintained, the frictional force can point in different directions when the cylinder is pulled, depending on several parameters.

What students think about this…

The misconceptions here are basically the same as in the previous problem. However, students usually think that frictional forces always oppose the motion. After some clarifying examples (e.g., a person standing on a moving bus, one block sliding with another block at rest on top of it), students can realize that sometimes the frictional force has the same direction as the motion; what they don't usually realize is that the frictional force can change its direction relative to motion within the same situation, as we heard from our own students!

Teaching approaches

In this example, students are confronted with a more complex situation: in problem 1 there is motion but no frictional force; now there is motion and the frictional force has either the same direction as the motion, or the opposite direction or simply does not exist. Teachers can therefore reinforce the idea that we can never state that the frictional force always points in one direction, because it depends on several parameters.

We suggest starting with a conceptual approach, by drawing the free-body diagrams representing the forces acting on both the spool and the surface. For simplicity, consider the force $\boldsymbol{F}$ to be horizontal (i.e., make $\theta = 0$). The problem should be discussed considering the effects of forces $\boldsymbol{F}$ and $\boldsymbol{f}_a$ upon translation and rotation and both possibilities for the direction of $\boldsymbol{f}_a$. Particular attention must be given to the cause of the frictional force: it exists because the spool exerts a horizontal force on the surface; as the spool rolls without slipping, then by Newton's third law the surface must exert a force opposite to it but of equal magnitude, $\boldsymbol{f}_a$, on the spool. At this point, students should be aware that if the frictional force was eventually not there, there would be a relative motion between the spool and the surface at the contact point, and therefore the frictional force always points in the opposite direction to that eventual relative motion (Caldas *et al* 2001). This conceptual interpretation of frictional forces is general and applies to either sliding or rolling objects, so this problem is very fruitful for a general discussion about the subject. When the problem is discussed in the classroom and the mathematical approach is derived, teachers can use an experimental model to demonstrate or simply let the students try by themselves; we suggest that a small carpet is placed under the spool, over a surface of very low friction: the motion of the carpet will reveal the direction of the frictional force acting upon it. Our experience with students proves that a better understanding is achieved when they confront physical and mathematical reasoning with practical experience, as mentioned by other investigators (Caldas *et al* 2001, Sousa and Pina 1999, Robinson 1979, Zacharia 2003).

A complete mathematical description can be given by considering $\theta \neq 0$, in order to obtain the general situations described above. This is not a simple problem and should only be discussed with undergraduate students or teachers. We suggest that students do not decompose $\boldsymbol{F}$ when calculating its torque, although they have to do it for translation.

II. Rolling without slipping on an inclined surface

Problem

Place a cylinder (mass M, radius r) on an incline. Let it roll freely along the surface, either ascending (with a small impulse) or descending the ramp. In which situation will the acceleration of the cylinder be higher: when it goes up or when it goes down?

Solution

When a block moves freely on a ramp and there is friction between the block and the surface, the frictional force points in opposite directions according to whether it goes up or down. As a consequence, the acceleration of the block takes different values in ascending and descending motions of the block. But this is not the case with cylinders!

As we can see in the free-body diagram of figure 4, the only force with a non-zero torque about the CM of the cylinder (point O) is the frictional force. This force is the *only one* responsible for the rotation of the cylinder. The gravitational force $\boldsymbol{W}$ and the normal reaction force $\boldsymbol{R}_\mathrm{n}$ do not produce any torque about the CM.

The angular acceleration (resulting from the torque) must always be compatible with the

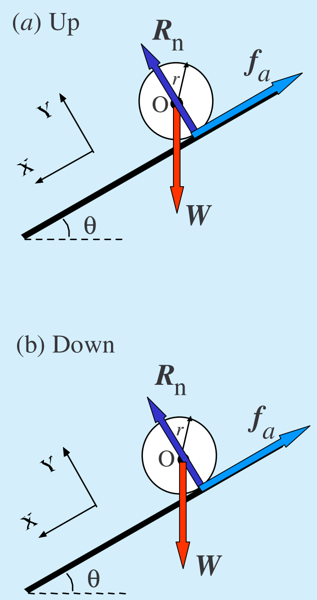

Figure 4. Free-body diagrams for a cylinder going up (*a*) and down (*b*) an incline.

acceleration of the CM, so $\boldsymbol{f}_a$ will point upwards. Therefore, the free-body diagram of forces acting on the cylinder is exactly *the same* when (a) ascending and (b) descending the ramp, and so is the linear acceleration of the cylinder's CM. This is a counter-intuitive example that can easily be explored experimentally with students in the laboratory.

What students think about this...

Objects are frequently treated as particles and this means rotation is not considered. Our experience with undergraduate students tells us that, even after having discussed the two previous problems in the classroom, with the incline as a context, they still persist in using the mathematical equations for a particle to describe the motion of a rolling object. This reasoning is not so obvious for a descending object, but seems to be very clear for the ascending motion, when they draw the frictional force pointing downwards.

Teaching approaches

This example is rather simple but shows the importance of a separate study of sliding and rolling objects, so teachers should be aware of this. The problem can be approached by first drawing the forces acting on the cylinder, in the free body diagram. Attention should be paid to the forces producing a torque: only the frictional force does so. This is a very important detail, because in descending motion this torque accelerates the cylinder (and most students agree that $\boldsymbol{f}_a$ must point upwards); however, for ascending motion, the rotation of the cylinder must decrease (i.e., it must have a 'negative' angular acceleration), which can only be achieved if $\boldsymbol{f}_a$ also points upwards!

It is most important that teachers discuss this matter carefully with their own students, eventually performing a simple but very enlightening experiment: take a ramp with a very gradual slope (about 5°) and conveniently place two photogates separated by, let's say, 30 cm. Take a small cylinder (2 cm diameter or less) and let it roll down the ramp from a fixed point at the top. Measure the time the cylinder takes to pass each photogate: dividing the diameter of the cylinder by this time, you have approximately the speed of the cylinder at the time it reaches each photogate. Determine the change in speed. Now repeat the experiment, launching the cylinder from *the same point* and measuring the time it takes from one photogate to the other. If you divide the change in speed by this time interval, you will have the average linear acceleration of the cylinder. Now it is time to measure the acceleration in ascending motion: take another ramp with a slightly greater slope (let's say 15°) and put it close to the bottom of the previous ramp; without moving the photogates from their position, place the cylinder high on the second ramp and let it roll, so that it crosses the two photogates when ascending the first ramp. Repeat the calculations in the previous experiment, i.e., calculate the speed when it crosses the photogates and then, in a second experiment, the average acceleration in ascending. Students will find it amazing that the accelerations on ascent and descent are about the same, confirming the physical interpretation. Such an experiment can also be done *before* the mathematical description of motion, supporting the conceptual discussion.

Comparing this situation with blocks ascending and descending the same ramp is also highly recommended. Details of this classical experiment can be found in most textbooks.

III. Bicycles and cars

Problem

When a bicycle's wheels roll without slipping on a level road, what are the frictional forces acting on the wheels? Are these forces in favour of or against motion?

Solution

Although bicycles and cars are part of our daily life and there are several discussions in the literature about forces acting on them (Caldas and Saltiel 1999a, Sousa and Pina 1999, Viennot 2003), no mathematical description has been given in detail.

To simplify the problem, let us consider the bicycle as two cylinders (the wheels) of masses m_1 and m_2, connected by a rigid bar of mass M (which also includes the rider and the bicycle's accessories). Figure 5 is a free-body diagram of just the bicycle's wheels. The drive on the rear wheel (wheel 1) is a consequence of the motion of the rider's feet and is physically represented as a powering torque, $\boldsymbol{M}_C$, acting on this wheel.

As a consequence, both wheels exert compressive forces on the rigid bar, so that the corresponding reaction forces on the wheels, $\boldsymbol{T}_1$ and $\boldsymbol{T}_2$, must be considered.

Assuming the bicycle is accelerating to the right, the fundamental equation for translation, $\sum_i \boldsymbol{F}_i = m\boldsymbol{a}$, implies that there must be at least one force acting on the rear wheel pointing in this direction; this can only be the frictional force and students should be aware of the importance of this fact: if this force did not exist, the wheel could not be accelerated in the direction of motion!

For the front wheel, none of the forces $\boldsymbol{W}_2$, $\boldsymbol{R}_2$ or $\boldsymbol{T}_2$ contributes to the torque about the wheel's centre, but the frictional force $\boldsymbol{f}_{a2}$ does. The use of the fundamental condition $\sum_i \boldsymbol{M}_{\boldsymbol{F}_i} = I\boldsymbol{\alpha}$, satisfying the relation $a = \alpha r$, shows that $\boldsymbol{f}_{a2}$ must point in the opposite direction to the motion. Therefore, one important conclusion is that the frictional forces on the bicycle's wheels point in opposite directions!

The linear acceleration of the bicycle,

$$a = \frac{f_{a1} - f_{a2}}{m_1 + m_2 + M}, \tag{4}$$

shows that $f_{a1} - f_{a2} > 0$, i.e., the frictional force acting on the rear wheel is *stronger* than that acting on the front wheel.

Considering the moment of inertia I to be the same for both wheels, and taking $m = m_1 + m_2 + M$ to be the total mass of the bicycle, the full relations for a, f_{a1} and f_{a2} can be obtained analytically:

$$a = \frac{M_C R}{2I + mR^2}$$

$$f_{a1} = \frac{(I + mR^2)M_C}{(2I + mR^2)R} \tag{5}$$

$$f_{a2} = \frac{I M_C}{(2I + mR^2)R}.$$

We must emphasize that neither f_{a1} nor f_{a2} represents a maximal value of the frictional force. This mistake is often made by students and should be confronted in the classroom.

If we want to study the case where both f_{a1} and f_{a2} have maximal values, we can easily start from equations (5) and reach the mathematical relation

$$\frac{R_1}{R_2} = 1 + \frac{mR^2}{I}. \tag{6}$$

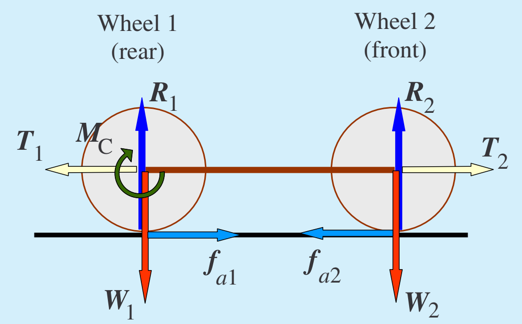

Figure 5. Free-body diagram concerning the wheels of a bicycle.

This expression shows that if we want both wheels to be simultaneously submitted to a maximal effort, the centre of mass of the bicycle must be located on the rear part! Analogous reasoning can also be done for cars (with rear-wheel drive)!

What students think about this...

Students have many misconceptions about how a car or a bicycle works. Firstly, they think it moves because there is a kind of driving force produced by the engine (or the legs), pulling the vehicle in the direction of motion. This idea is sometimes induced by textbooks by representing this force acting upon the vehicle (see, for example, Problem 7.3 in Alonso and Finn 1973). Secondly, friction is always seen as a 'bad' thing for motion, so students believe both f_{a1} and f_{a2} point in the opposite direction to the motion, without realizing that it is the frictional force that accelerates the vehicle. Usually it is far beyond their imagination that the frictional forces on the rear and front wheels can act in opposite directions.

Teaching approaches

Bicycles and cars are often used in textbooks to illustrate Newton's second law $\boldsymbol{F} = m\boldsymbol{a}$, assuming that the acceleration of the vehicle results from the forces acting, generated by the engine or exerted on the pedals. Frictional forces are usually considered as opposing the motion, which is not true in general! Therefore, this subject should be made clear to students. Facing this problem, students must work simultaneously with physical relations describing the motion and apply their reasoning to a challenging situation that combines rotation and translation.

Before a mathematical description can be given, a conceptual discussion about how the forces represented in the free-body diagram can influence the motion of the bicycle is absolutely essential. In fact, when this problem is discussed in the literature (Caldas *et al* 2001, Viennot 2003, Wehrbein 2004, Bloomfield 1997), the authors usually start from a force diagram. Nevertheless, they consider the vehicle as a whole and so students may have some difficulty in understanding how the front wheel will be accelerated if the frictional force is apparently the only horizontal force. For this reason, we suggest that the free body diagram of the two wheels should be represented as in figure 5, showing the forces exerted by the rigid bar—it must be emphasized that they are a consequence of Newton's third law, resulting from compressive forces exerted by the wheels upon the rigid bar.

In any case, teachers must decide, according to their students' understanding and their mathematical and cognitive development, how deep they can go in exploring this problem.

For advanced students, we suggest, as a challenging homework problem, considering a front wheel powered vehicle (for example, a car) and finding the direction of the frictional forces upon the wheels. It is a very enlightening problem, because these forces, as before, have different directions, although opposite to those for a rear wheel drive vehicle.

Conclusions

Traditionally, students tend to simplify problems concerning rigid bodies, by using the same reasoning they do with material particles. This leads frequently to misconceptions concerning the direction of the frictional force and the representation of forces in free body diagrams, which are an obstacle to solving problems involving rotation. Some of these misconceptions also prevail in teachers, as our workshop has revealed.

One of the most important reasons for this is that students and teachers always look for a unique and general answer to this kind of problem, which does not exist! Depending on the particular problem, the frictional force can point in the direction of motion, or opposite to it, or it can even happen that there is no frictional force at all!

Besides, the normal force is not always applied to the body at the same point. This is a direct consequence of the torque of each force acting on the body and therefore it is absolutely necessary to think simultaneously in terms of translation and rotation, for a more precise description in solving rigid body problems.

Taking this into account, we suggest that **before** basic concepts such as *angular speed* and *acceleration*, *moment of inertia* and *torque* are introduced in the classroom, teachers should **avoid**:

- using definitions for the direction of the frictional force that apply only to specific situations such as sliding (e.g. 'the frictional force is resistant to motion' or 'the frictional force points in the opposite direction to the motion' (Caldas *et al* 2001, Sousa and Pina 1999));
- giving exercises or practical work involving wheels, spheres or cylinders, particularly when students must apply equations or draw diagrams to describe the motion of bodies that will be treated as material particles.

On the other hand, whenever bodies are treated as material particles in order to simplify the problem, this should be clearly explained and discussed with students to avoid any misconceptions.

Later, when the rigid body is studied in secondary school, teachers should encourage their students to draw free body diagrams, including not only the rigid body itself but also the surface (Puri 1996, Arons 1997, Viennot 2003), so that they can discuss both the forces acting on the surface and on the body as well as the corresponding pairs of forces (Newton's third law force pairs).

Finally, editors should also be encouraged to have their textbooks on physics reviewed by experts other than the authors, in order to guarantee scientifically independent work that will help to eliminate potential situations leading to misconceptions—this does not seem to be a general rule. If this were done, textbooks would no longer be, as in many cases, an obstacle to learning, but a powerful instrument for teaching.

Received 1 October 2004, in final form 5 January 2005
doi:10.1088/0031-9120/40/3/001

References

Alonso M and Finn A J 1973 *Fundamental University Physics, Volume I—Mechanics* 4th edn (New York: Addison-Wesley) p 163 (Portuguese version 1999 (Brazil: Addison-Wesley) problem 7.2, p 104)

Arons A B 1997 *Teaching Introductory Physics* (New York: John Wiley)

Bartoš J and Musilová J 2004 Small surprises in 'rolling-physics' experiments *Eur. J. Phys.* **25** 675–87

Bloomfield L A 1997 *How Things Work: the physics of everyday life* (New York: John Wiley) pp 224–34

Caldas H and Saltiel E 1999a Direction frictional forces and motion sense—I *Revista Brasileira de Ensino de Física* **21** 359–65

Caldas H and Saltiel E 1999b Frictional forces sense and motion—II: an analysis of the books used in Brazilian higher educations *Revista Brasileira de Ensino de Física* **21** 542–9

Caldas H, Saltiel E and Cunha A L 2001 Atrito entre sólidos—discussão de algumas dificuldades dos alunos *Gazeta de Física* **24** 12–19

Carvalho P S and Sousa A S S 2002 A critical view about applied forces and contact forces (workshop) *12th Iberia Meeting for Physics Teaching, Évora (Portugal)*

Doménech A, Doménech T and Cebrián J 1987 Introduction to the study of rolling friction *Am. J. Phys.* **55** 231–5

Driver R *et al* (ed) 1985 *Children's Ideas in Science* (Milton Keynes: Open University Press)

Hewitt P G 2002 *Conceptual Physics, Ninth Edition—Practicing Physics* (San Francisco: Addison Wesley)

Mungan C E 2001 Acceleration of a pulled spool *Phys. Teacher* **39** 481–5

Pinto A and Fiolhais M 2001 Rolling cylinder on a horizontal plane *Phys. Educ.* **36** 250–4

Puri A 1996 The art of free-body diagrams *Phys. Educ.* **31** 155–7

Robinson M C 1979 Undergraduate laboratories in physics: Two philosophies *Am. J. Phys.* **47** 859–62

Salazar A, Sánchez-Lavega A and Arriandiaga M A 1990 Is the frictional force always opposed to the motion? *Phys. Educ.* **25** 82–5

Shaw D E 1979 Frictional force on rolling objects *Am. J. Phys.* **47** 887–8

Sousa C A and Pina E P 1999 Aspects of mechanics and thermodynamics in introductory physics: an illustration in the context of friction and rolling *Eur. J. Phys.* **18** 334–7

Sousa C A and Pina E P 1999 A lei de conservação da energia: aplicação ao rolamento com e sem deslizamento *Gazeta de Física* **22** 10–15

Viennot L 2003 *Teaching Physics* 2nd edn (Dordrecht: Kluwer Academic) pp 41–65

Wehrbein W M 2004 Wheelies and headers, or how to keep both bicycle wheels on the ground *Phys. Teacher* **42** 27–30

Zacharia Z 2003 Beliefs, attitudes, and intentions of science teachers regarding the educational use of computer simulations and inquiry-based experiments in physics *J. Res. Sci. Educ.* **40** 792–823

Paulo Simeão Carvalho is an assistant professor at Porto University Physics Department. His research interests are highly polar liquid crystals, liquid crystal–polymer composites and physics education.

Adriano Sampaio e Sousa is a teaching assistant at Porto University Physics Department. His research interests are mainly in teaching strategies and conceptual learning. He has worked in teacher training (initial and continuous) for the last 25 years.