Reconstructing the dark energy equation of state with varying alpha

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- **In principle**
- **In practice**

NJN, J.E. Lidsey, PRD D69, 123511 (2004)

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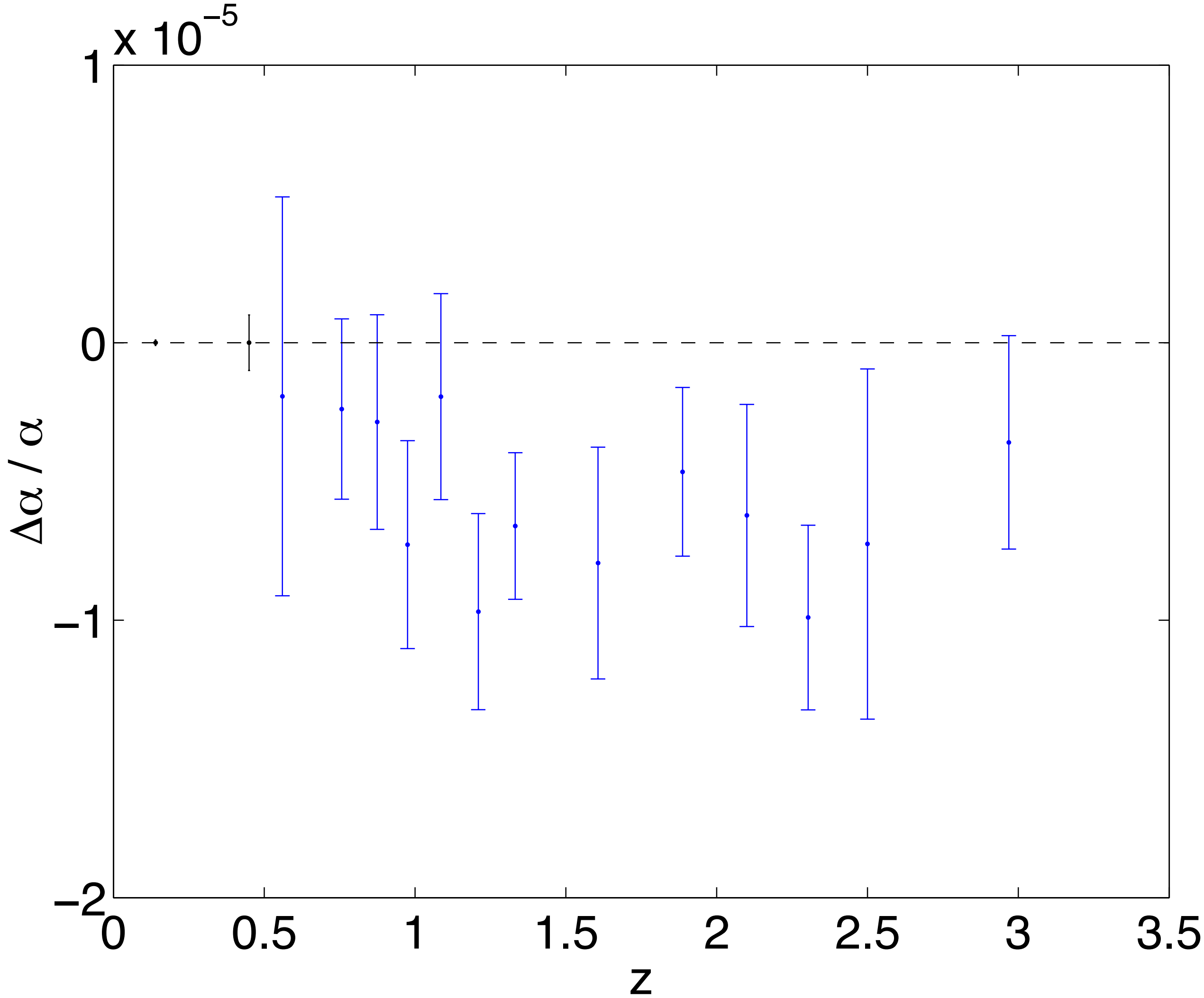
Evolution of α

Data from QSO absorption lines



Murphy et al. (2003)

Evidence for variation in α ?

Coupling quintessence to electromagnetism

1. The action

$$S = -\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,R + \int d^4x\sqrt{-g}\,(\mathcal{L}_\phi + \mathcal{L}_M + \mathcal{L}_{\phi F})$$

2. The coupling

$$\mathcal{L}_{\phi F} = -\frac{1}{4}B_F(\phi)F_{\mu\nu}F^{\mu\nu}$$

3. Gauge kinetic function

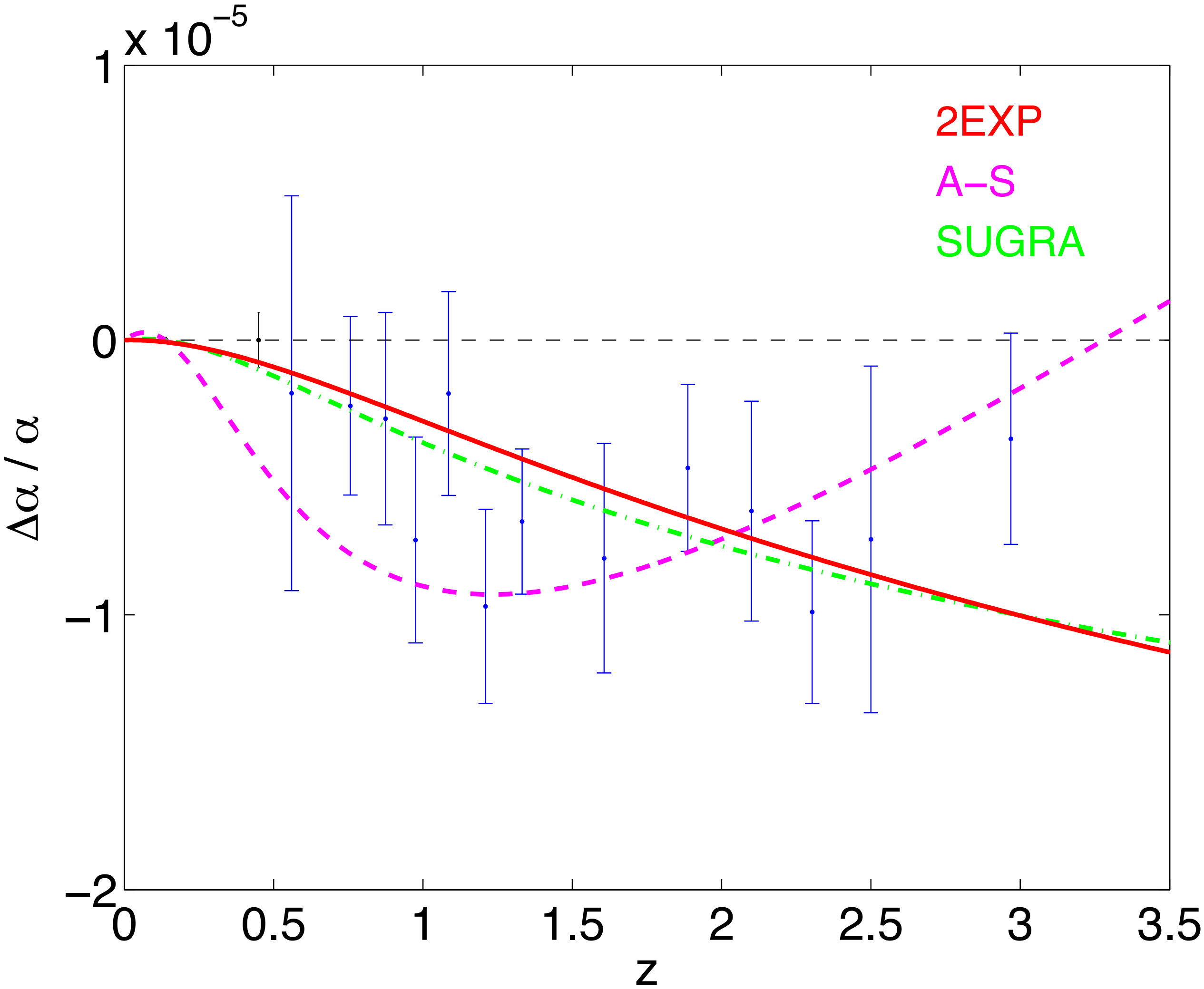
$$B_F(\phi) = 1 - \zeta\kappa(\phi - \phi_0)$$

4. Variation in α

$$\alpha = \frac{\alpha_0}{B_F(\phi)} \qquad \Rightarrow$$

$$\frac{\Delta\alpha}{\alpha} \equiv \frac{\alpha - \alpha_0}{\alpha_0} = \zeta\kappa(\phi - \phi_0)$$

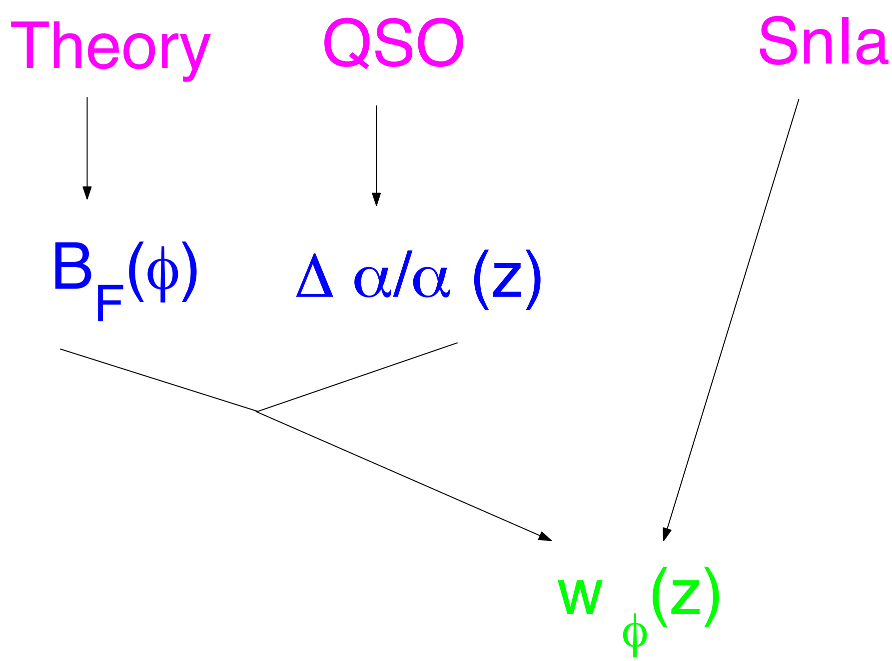
$\Delta\alpha/\alpha$ for some quintessence models



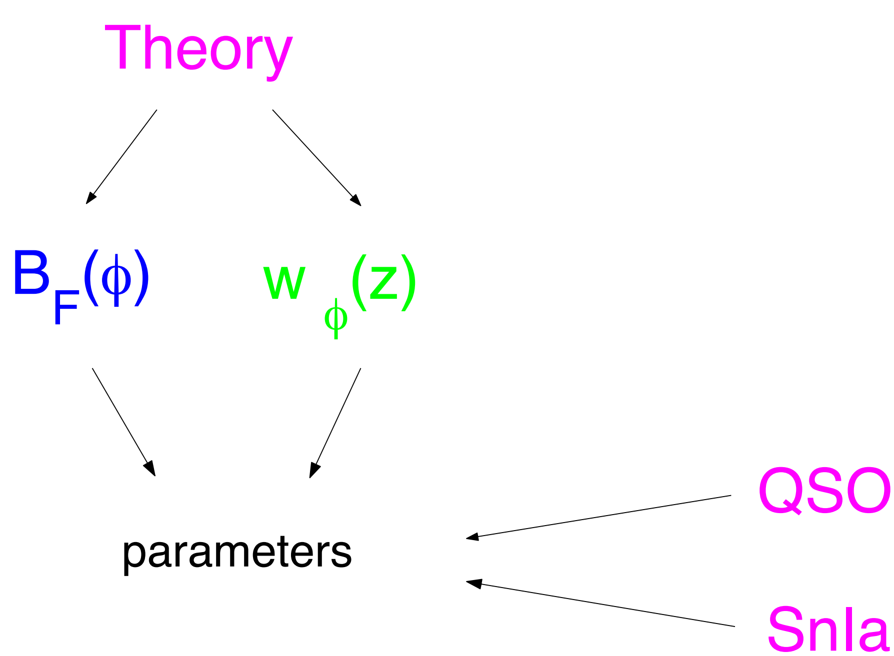
Anchordoqui, Goldberg (2003)
Copeland, NJN, Pospelov (2003)

Reconstructing the equation of state

This talk:



Parkinson, Bassett, Barrow (2003):



Dynamics equations

1. Energy densities

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \qquad \rho_M = \frac{\Omega_{M0}\rho_0}{a^3}$$

2. Equations of motion

$$\dot{H} = -\frac{\kappa^2}{2}(\rho_M + \dot{\phi}^2)\,, \qquad \dot{\rho}_\phi = -3H\dot{\phi}^2$$

3. Equation of state

$$w_\phi = -1 + \frac{\dot{\phi}^2}{\rho_\phi} \;=\; 1 - \frac{2V}{\rho_\phi}$$

4. Rewrite as:

$$\sigma' = -(\kappa\phi')^2(\sigma + a^{-3})$$

where

$$\sigma = \frac{\rho_\phi}{\rho_0\Omega_{M0}}\,, \qquad {}' = \frac{d}{d\ln a}$$

and

$$\begin{aligned}
w &= -1 + \frac{(\kappa\phi')^2}{3}\left(1 + \frac{1}{\sigma a^3}\right) \\
w' &= 2(1+w)\frac{\phi''}{\phi'} + w\left[3(1+w) - (\kappa\phi')^2\right] \\
w'' &= \ldots
\end{aligned}$$

Reconstruction example

Assume $B_F(\phi)$ & $\Delta\alpha/\alpha(z) \rightarrow$

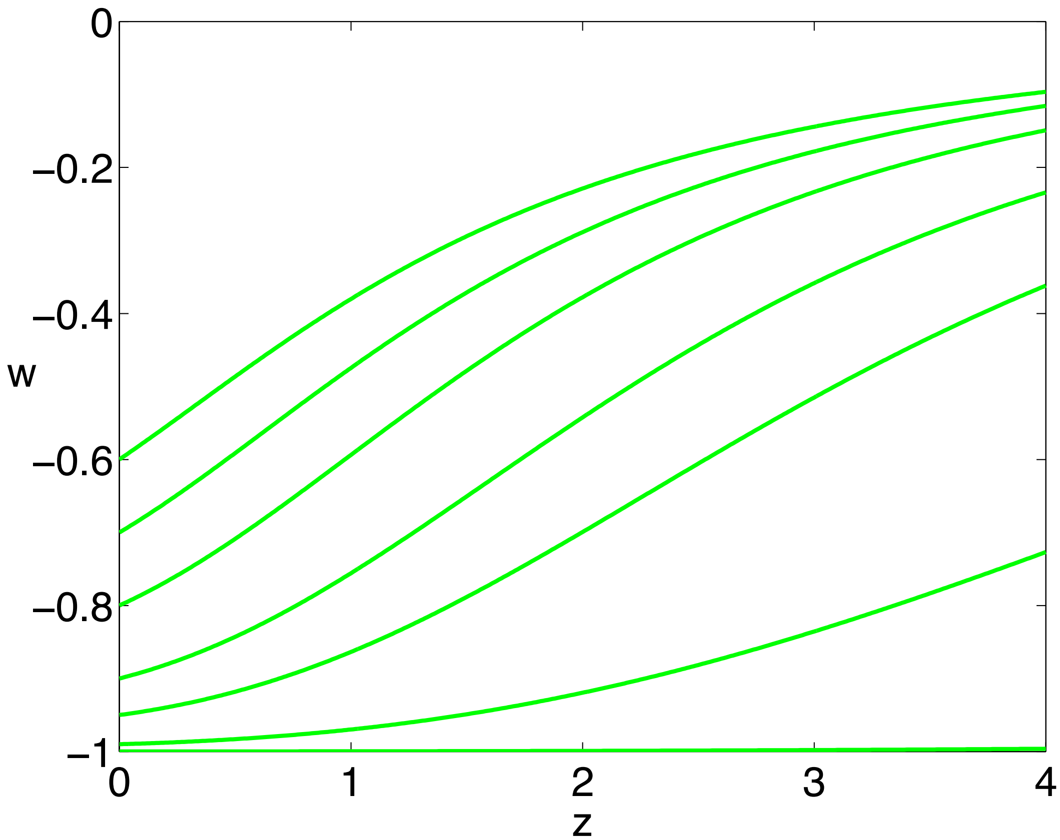
$$(\kappa\phi')^2 = \lambda^2 = 3\Omega_{\phi 0}(1 + w_0)$$

Integration of equation of motion σ' gives

$$\sigma = \left(\frac{\Omega_{\phi 0}}{\Omega_{M0}} + \frac{\lambda^2}{\lambda^2 - 3}\right) e^{-\lambda^2 N} - \left(\frac{\lambda^2}{\lambda^2 - 3}\right) e^{-3N}$$

$$w = (\lambda^2 - 3)\left[3 - \frac{\lambda^2}{w_0}\frac{\Omega_{M0}}{\Omega_{\phi 0}} \exp\left((\lambda^2 - 3)N\right)\right]^{-1}$$

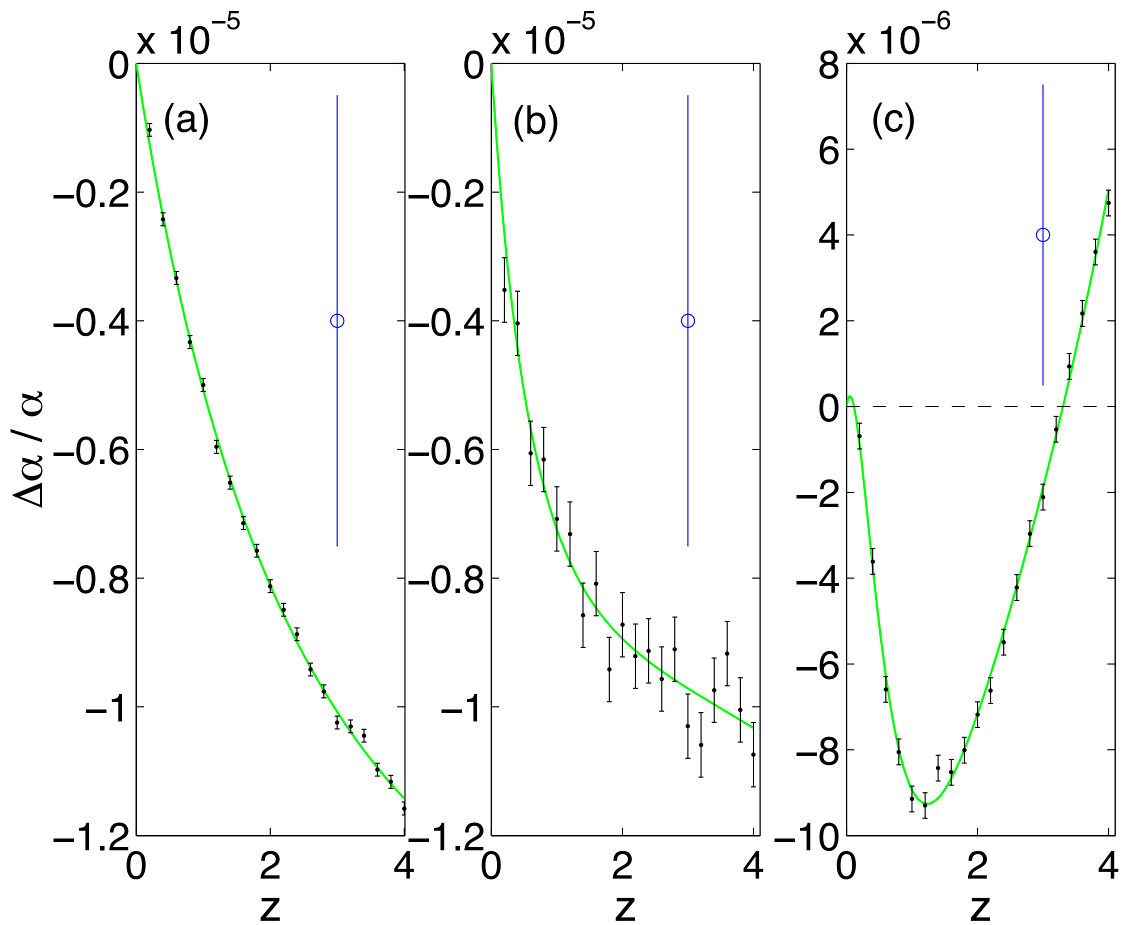
$$V = Ae^{-\frac{3}{\lambda}\kappa\phi} - Be^{-\lambda\kappa\phi}$$



Reconstruction in practice I

STEP 1:

- Observational data
- Generate data based on numerical solution of eqs. of motion for ϕ
  - Data points equally spaced in redshift intervals $\Delta z = 0.2$
  - Normal distribution with mean $\Delta\alpha/\alpha = \zeta\kappa(\phi - \phi_0)$.



Reconstruction in practice II

STEP 2: Fitting data

$$g(N) = \frac{\Delta\alpha}{\alpha} = g_1 N + g_2 N^2 + \ldots$$

$$\kappa\phi' = g'/\zeta$$

STEP 3: Estimating ζ

$$\zeta^2 = \frac{1}{3}\frac{g'^2}{\Omega_\phi(1+w)}$$

Typical values:

$$\begin{aligned} &\Omega_{\phi 0} \sim 0.7, \\ &w_0 \sim [-0.99, -0.6], \qquad \rightarrow \qquad \zeta \sim 10^{-7} - 10^{-4} \\ &g'_0 \sim 10^{-7} - 10^{-5} \end{aligned}$$

Equivalence Principle tests $\quad \rightarrow \quad |\zeta| < 10^{-3}$

Olive, Pospelov (2002)

Reconstruction in practice III

$$\zeta^2 = \frac{1}{3}\,\frac{g'^2}{\Omega_\phi(1+w)}$$

$w \sim -1 \rightarrow$ small uncertainty in w leads to large uncertainty in ζ

(i) Use 2nd order consistency equation

$$\zeta^2 = \frac{g'^2}{w'}\,\frac{\Omega_M}{\Omega_\phi}\,\left(w + \frac{2}{3}\frac{1}{\Omega_M}\frac{g''}{g'}\right)$$

(ii) ζ from fundamental particle physics

Reconstruction examples

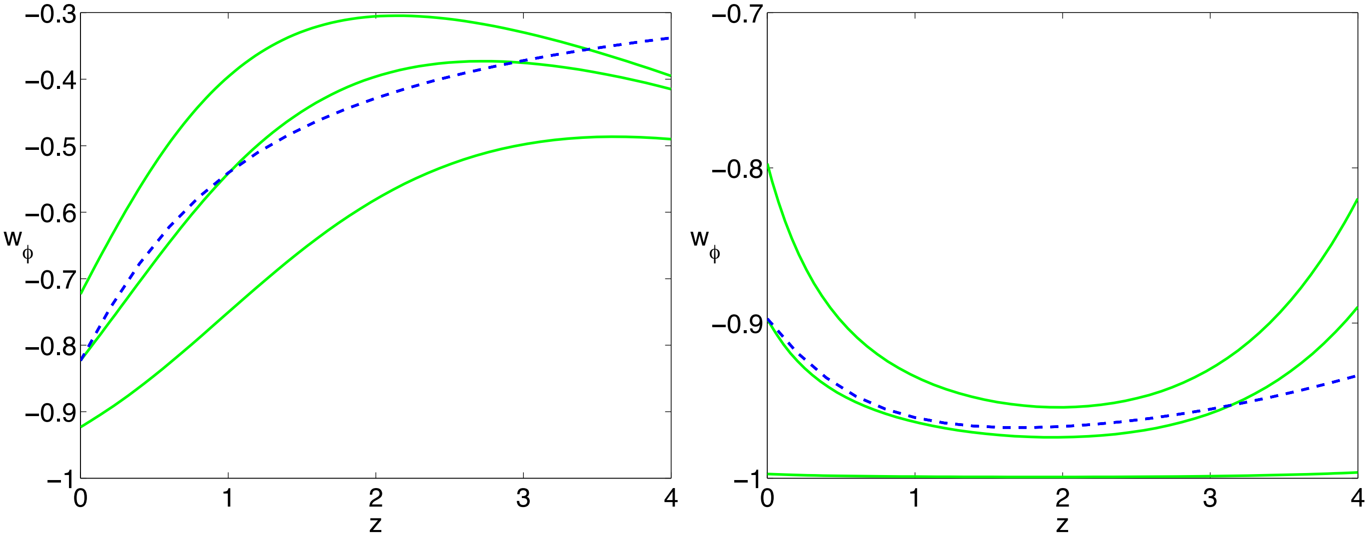


Figure 1: $V(\phi) = V_0 \exp((\kappa\phi)^2/2)/\phi^{11}$, $V(\phi) = V_0(\exp(50\kappa\phi) + \exp(0.8\kappa\phi))$

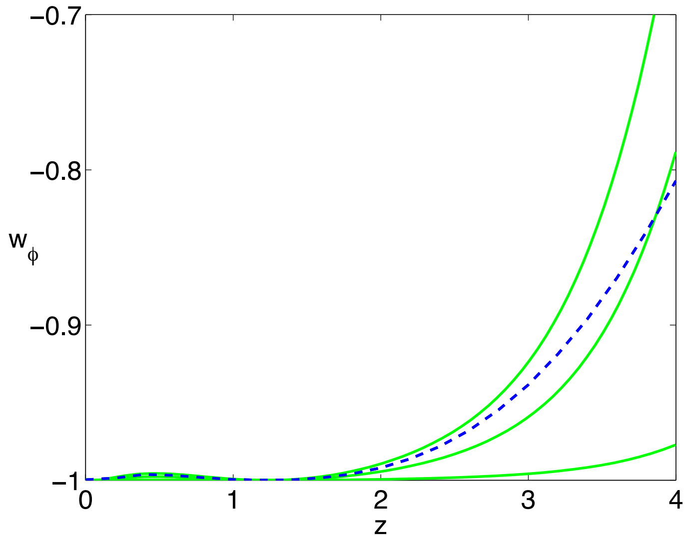
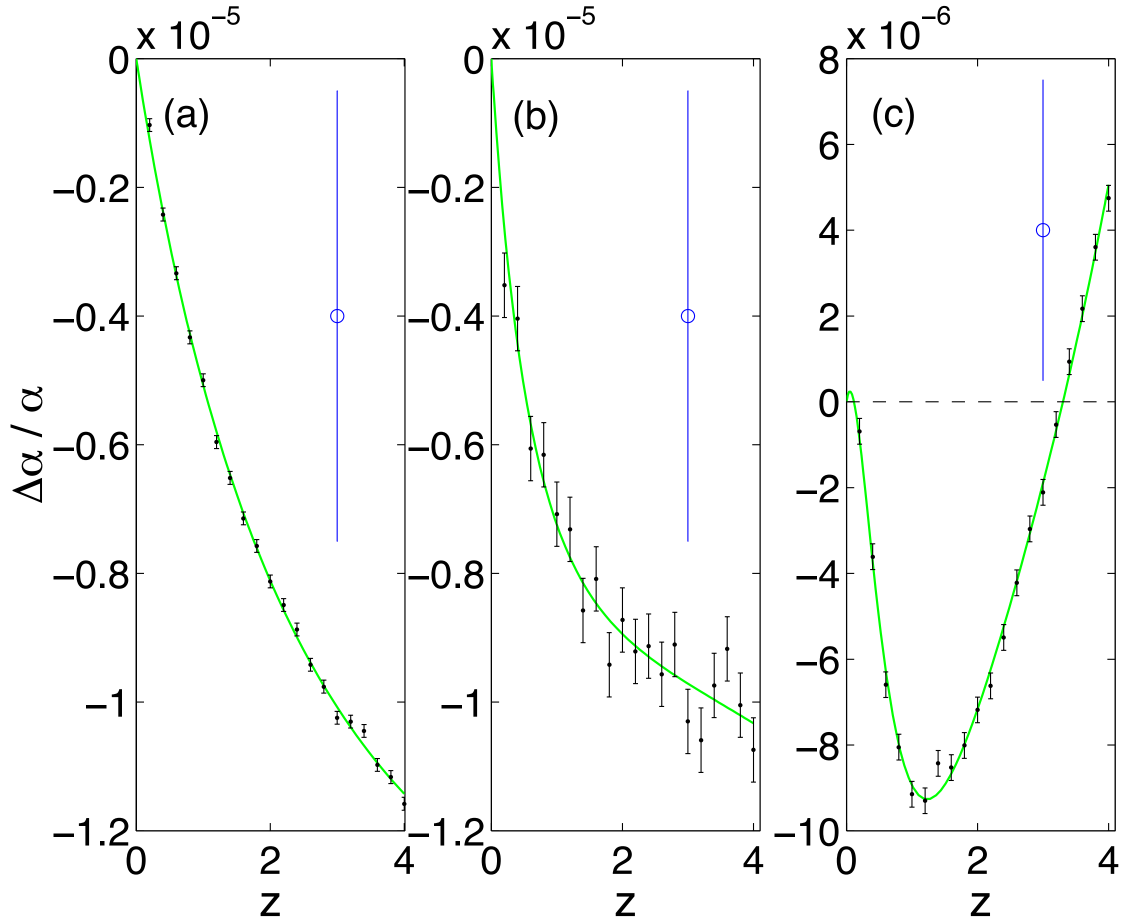


Figure 2: $V(\phi) = \kappa^{-4} e^{-A\kappa\phi} \left[(\kappa\phi - C)^2 + B)\right]$

Error bars



(a) $\delta\left(\frac{\Delta\alpha}{\alpha}\right) < 10^{-7}$

(b) $\delta\left(\frac{\Delta\alpha}{\alpha}\right) < 5 \times 10^{-7}$

(c) $\delta\left(\frac{\Delta\alpha}{\alpha}\right) < 5 \times 10^{-7}$

Conclusions

- Qualitative shape of $w(z)$ can be deduced;
- In particular sign of the running w'. Tracking quintessence? Creeping quintessence? K–essence?
- Information on $w(z)$ up to very high redshifts;
- When $w \approx -1$ information on w'_0 is helpful;
- Need in general $\delta\left(\frac{\Delta\alpha}{\alpha}\right) < 5 \times 10^{-7}$.