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Cosmological Perturbations Through a Simple Bounce

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Motivation

- ✭ String and M-theory have motivated consideration of models of the universe in which there was a contracting phase before the big bang. However, there is much controversy over how perturbations evolve through such a transition point.
- ✭ Many of these models (such as the ekpyrotic or pre-big bang scenarios) have a discontinuous change from contraction to expansion - arguments over matching conditions.
- ✭ Try models in which there is a smooth transition from contraction to expansion, a bounce, where this problem is avoided. (Cartier, 2004. Gasperini, Giovannini, Veneziano, 2003.)
- ✭ Claims that only one of the well known gauge-invariant perturbations $\mathcal{R}$ and Φ can evolve smoothly through the bounce. (Cartier, Durrer and Copeland, 2003.)
- ✭ We would like to shed light on this problem by considering the evolution of perturbations within a simple model which bounces.

The Model

Two scalar fields:

(i) φ has positive kinetic energy + positive exponential potential, $V = V_0 \exp(-\lambda\kappa\varphi)$.

(ii) χ is massless and has negative kinetic energy.

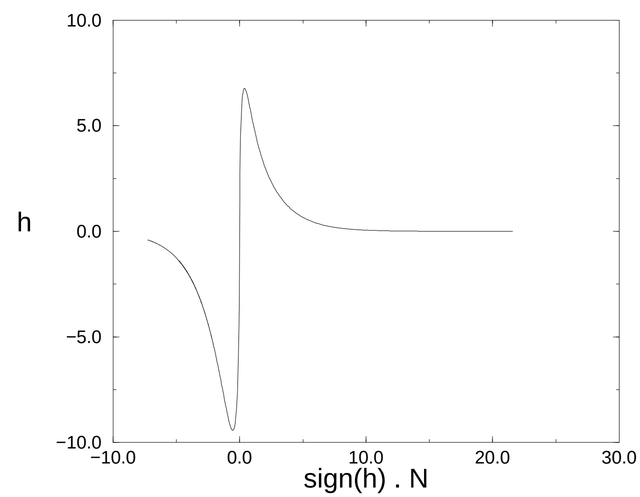
Usual Klein-Gordon and Friedmann equations

$$\varphi'' + 2h\varphi' + a^2 V_{,\varphi} = 0$$

$$\chi'' + 2h\chi' = 0$$

$$h^2 = \frac{\kappa^2}{3}\left(\frac{1}{2}\varphi'^2 - \frac{1}{2}\chi'^2 + a^2 V\right)$$

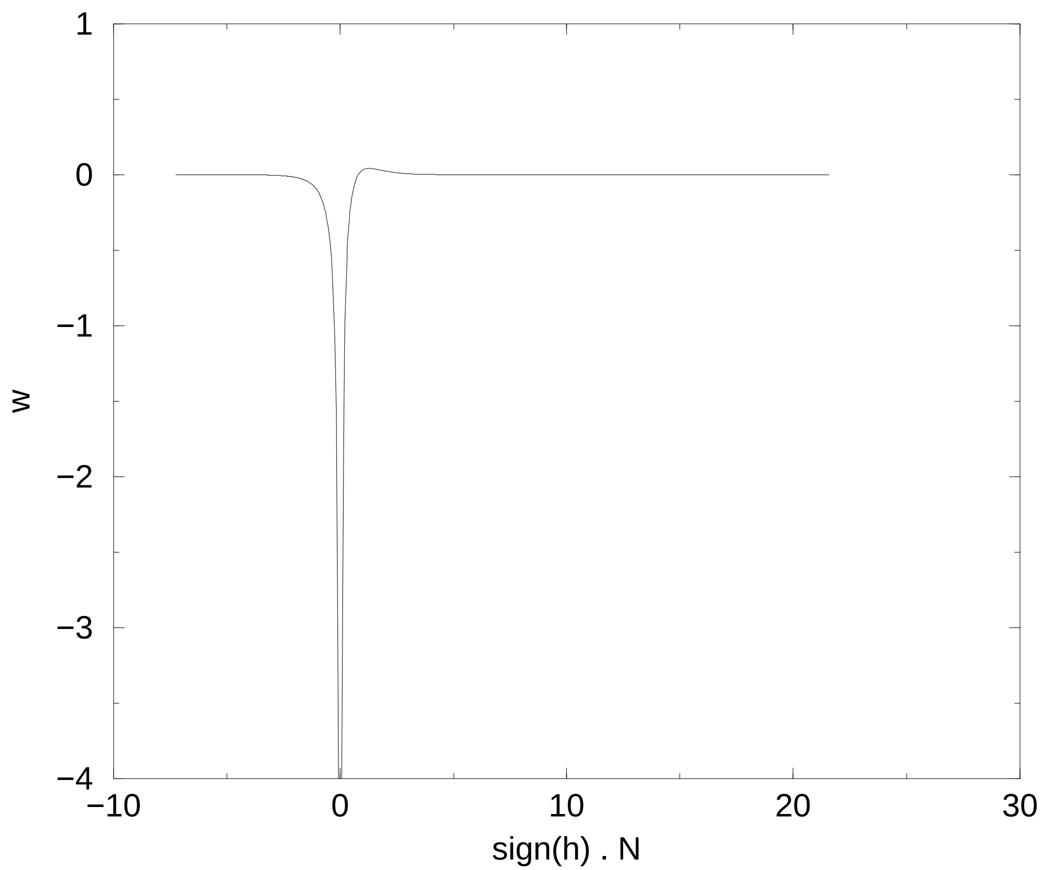
The negative K.E of the phantom field allows h to be zero so a bounce is possible.



The equation of state of the system is

$$w = -1 + \frac{\varphi'^2 - \chi'^2}{\rho}$$

The Universe will become phantom $(w < -1)$ when $|\chi'| > |\varphi'|$.



The phase plane

Dimensionless Variables:

$$\alpha = \frac{\varphi'}{\chi'}, \quad \beta = \frac{a\sqrt{2V}}{\chi'}, \quad \gamma = \frac{\sqrt{6}h}{\kappa\chi'}$$

The Friedmann equation gives the constraint

$$\alpha^2 + \beta^2 - \gamma^2 = 1$$

- solutions lie on a hyperboloid in the phase space.

In the (α, γ) plane solutions evolve along straight lines

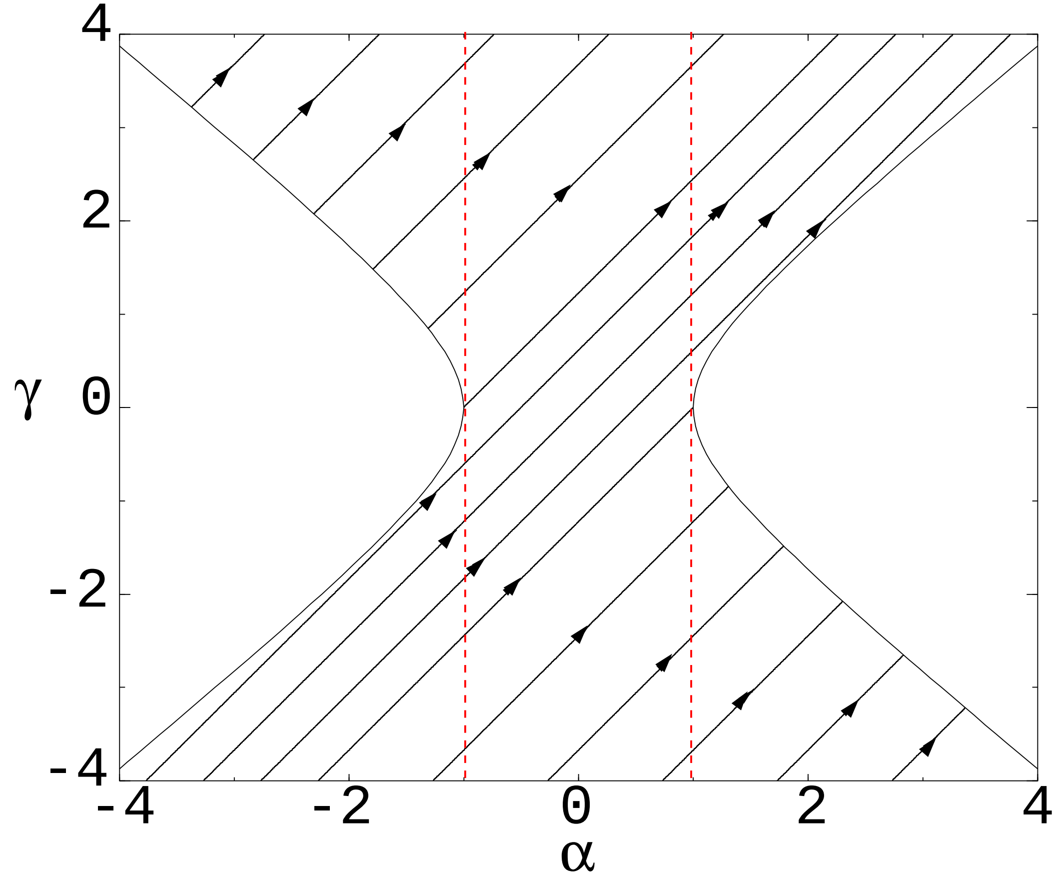
$$\alpha - \frac{\lambda}{\sqrt{6}}\gamma = C$$

Require $\lambda < \sqrt{6}$ for non-singular bouncing solutions to exist:

power-law contraction $(a \propto |t|^{2/\lambda^2}) \longrightarrow$ bounce $\longrightarrow$ power-law expansion $(a \propto |t|^{2/\lambda^2})$

$\lambda = \sqrt{3} \quad \Rightarrow \quad a \propto |t|^{2/3}$ during power-law evolution

- mimics matter dominated evolution away from the bounce.


4
2
0
-2
-4
γ
-4
-2
0
2
4
α

Perturbations

The scalar part of the perturbed metric

$$g_{\mu\nu} = a^2(\eta) \begin{pmatrix} -(1+2\phi) & B_{|i} \\ B_{|j} & (1-2\psi)\gamma_{ij} + 2E_{|ij} \end{pmatrix}$$

- ϕ : perturbation to the lapse function.
- ψ : curvature perturbation.
- σ : shear, $\sigma = -B + E'$.

Field perturbations:

- ✯ $\varphi = \varphi^{(0)} + \delta\varphi$
- ✯ $\chi = \chi^{(0)} + \delta\chi$

Use the perturbed Einstein and Klein-Gordon equations to get the perturbation equations.

It is easiest to work in 2 regimes by making 2 different gauge choices

- ✯ one away from the bounce - the uniform curvature gauge.
- ✯ one near and through the bounce - the uniform χ-field gauge.

Perturbations away from the bounce

Away from the bounce we work in the uniform curvature gauge ($\tilde{\psi} = 0$).

This corresponds to a temporal gauge transformation from an arbitrary gauge

$$\tilde{\eta} = \eta - \frac{\psi}{h}$$

2 coupled second order equations for the field perturbations

$$\begin{aligned}
\delta\varphi'' - \nabla^2\delta\varphi + 2h\delta\varphi' + \left[a^2 V_{,\varphi\varphi} + \kappa^2\varphi'\left(\varphi'(\frac{h'}{h^2} - 2) - 2\frac{\varphi''}{h}\right)\right]\delta\varphi &= \frac{\kappa^2\chi'}{h}(\varphi'\frac{h'}{h} - \varphi'')\delta\chi\,, \\
\delta\chi'' - \nabla^2\delta\chi + 2h\delta\chi' - \kappa^2{\chi'}^2(2 + \frac{h'}{h^2})\delta\chi &= \frac{\kappa^2\chi'}{h}(\varphi'' - \frac{h'}{h}\varphi')\delta\varphi\,.
\end{aligned}$$

Away from the bounce $\varphi' \propto h$ and the right hand sides of these equations disappear.

Leaves the well known single field solutions for each field.

$$\delta\varphi,\ \delta\chi \ = \frac{\sqrt{\pi}}{2}\frac{e^{i(\nu+\frac{1}{2})\frac{\pi}{2}}}{a}|\eta|^{\frac{1}{2}}(J^{(1)}_{|\nu|}(|k\eta|) + iY^{(1)}_{|\nu|}(|k\eta|))$$

where $\nu \equiv \frac{3}{2} + \frac{1}{p-1}$.

Perturbations near and through the bounce

Need a gauge which doesn't break down at the bounce and in which perturbations remain small.

The uniform χ-field gauge.

$$\tilde{\eta} = \eta + \frac{\delta\chi}{\chi'}$$

(No good away from the bounce as $\chi' \to 0$)

Four coupled evolution equations for the perturbations

$$\begin{aligned}
\delta\varphi'' + 2h\delta\varphi' - \nabla^2\delta\varphi + a^2 V_{\varphi\varphi}\delta\varphi + 2a^2 V_\varphi \phi &= 0\,, \\
\psi'' + 2h\psi' + (2h' + h^2 + \frac{\kappa^2}{2}({\phi_0'}^2 - {\chi_0'}^2))\phi + h\phi' - \frac{\kappa^2}{2}(\varphi_0'\delta\varphi' - a^2 V_\varphi \delta\varphi) &= 0\,, \\
\phi' + 3\psi' - \nabla^2\sigma &= 0\,, \\
\sigma' + 2h\sigma - \phi + \psi &= 0\,,
\end{aligned}$$

and 2 constraints

$$\begin{aligned}
h\phi + \psi' &= \frac{\kappa^2}{2}\varphi_0'\delta\varphi\,, \\
\nabla^2[\psi + h\sigma] &= -\frac{\kappa^2}{2}(({\varphi_0'}^2 - {\chi_0'}^2)\phi - \varphi_0'\delta\varphi' + (3h\varphi_0' - a^2 V_\varphi)\delta\varphi)\,.
\end{aligned}$$

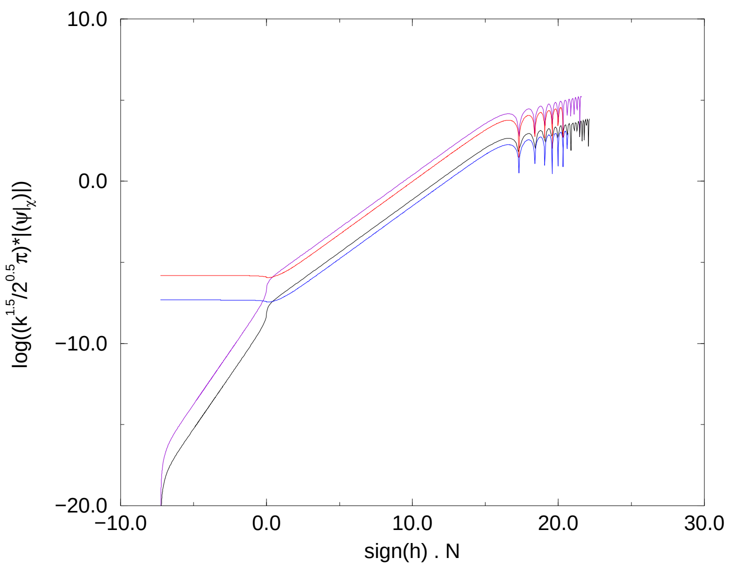
Numerical evolution

Use the analytical solutions found for the uniform curvature perturbations to set the initial conditions for the numerical evolution of the uniform field perturbations.

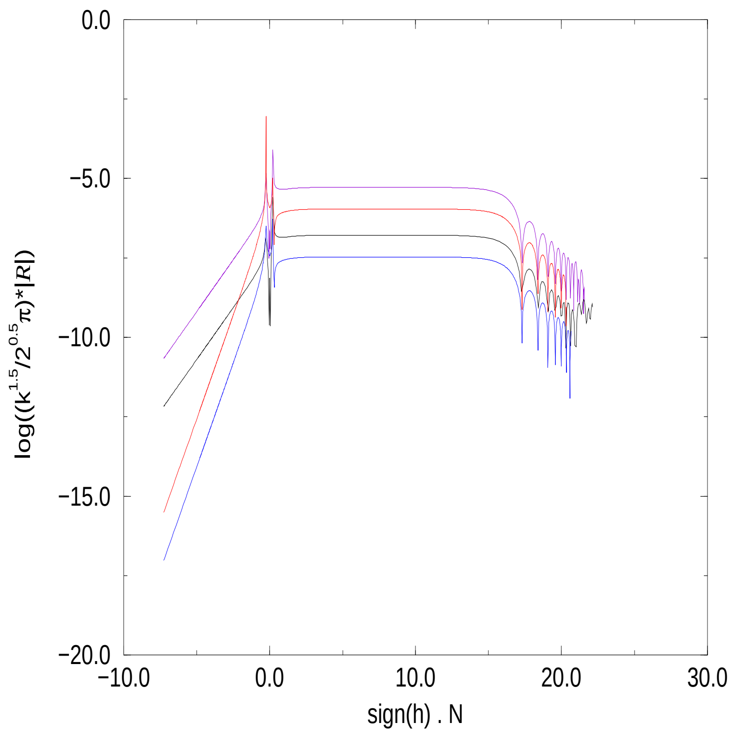
Four different modes of initial conditions in the uniform curvature gauge

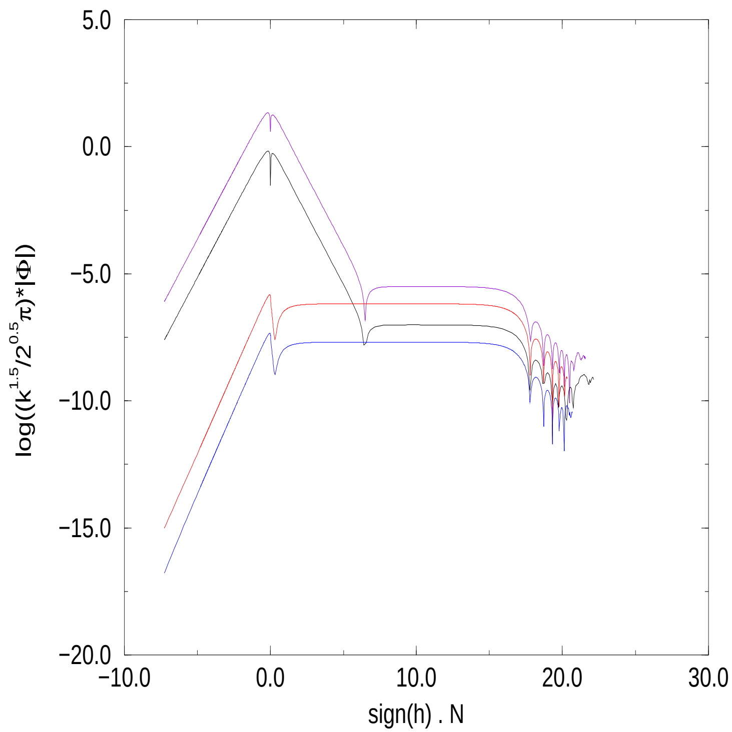
- ✭ $[\varphi_Y$ and $\varphi_J]$: $\delta\chi|_\psi = \delta\chi'|_\psi = 0, \quad \delta\varphi|_\psi = Y$-soln. or J-soln.
- ✭ $[\chi_Y$ and $\chi_J]$: $\delta\varphi|_\psi = \delta\varphi'|_\psi = 0, \quad \delta\chi|_\psi = Y$-soln. or J-soln.

Transform these initial conditions to the $\delta\chi = 0$ gauge and evaluate at initial value of N to set numerical initial conditions.



The evolution of $\mathcal{R}$ and Φ





$$\mathcal{R} = \psi + h\frac{\varphi'\delta\varphi - \chi'\delta\chi}{\varphi'^2 - \chi'^2}$$

$$\Phi = \psi + h\sigma$$

Spectral tilts

Power spectrum:

$$\mathcal{P}_x = \frac{k^3|x|^2}{2\pi^2} = \sum_{i=1}^{4} \frac{k^3|x_i|^2}{2\pi^2}$$

Spectral tilt:

$$\Delta n_x \equiv \frac{d\ln(\mathcal{P}_x)}{d\ln(k)}$$

$(\Delta n_x \equiv n - 1)$

- mode φ_Y:
  Initial $\Delta n_{\mathcal{R}} = 0, \quad \Delta n_\Phi = -4, \quad \Longrightarrow$ Final $\Delta n_{\mathcal{R}} = \Delta n_\Phi = 0.$
- mode φ_J:
  Initial $\Delta n_{\mathcal{R}} = 2, \quad \Delta n_\Phi = -2, \quad \Longrightarrow$ Final $\Delta n_{\mathcal{R}} = \Delta n_\Phi = 2.$
- mode χ_Y:
  Initial $\Delta n_{\mathcal{R}} = 0, \quad \Delta n_\Phi = -4, \quad \Longrightarrow$ Final $\Delta n_{\mathcal{R}} = \Delta n_\Phi = 0.$
- mode χ_J:
  Initial $\Delta n_{\mathcal{R}} = 2, \quad \Delta n_\Phi = -2, \quad \Longrightarrow$ Final $\Delta n_{\mathcal{R}} = \Delta n_\Phi = 2.$

Tensor Perturbations

h_{ij} - tensor part of the perturbed metric (transverse, traceless and invariant under gauge-transformations).

Expand in plane-waves

$$h_{ij}(\mathbf{x},\eta) = (2\pi)^{-3}\int d^3\mathbf{k}\sum_{p=\times,+}\delta g^p_{\mathbf{k}}(\eta)\,\epsilon^p_{ij}(\mathbf{k},\mathbf{x})$$

ϵ^p_{ij} - polarisation tensor,

$p=\times,\ +$ - the two independent polarisation states,

$\delta g^{\times,+}_{\mathbf{k}}$ - scalar amplitude of each state for wave-mode $\mathbf{k}$.

For each polarisation state

$$\delta g'' + 2h\delta g' + k^2\delta g = 0$$

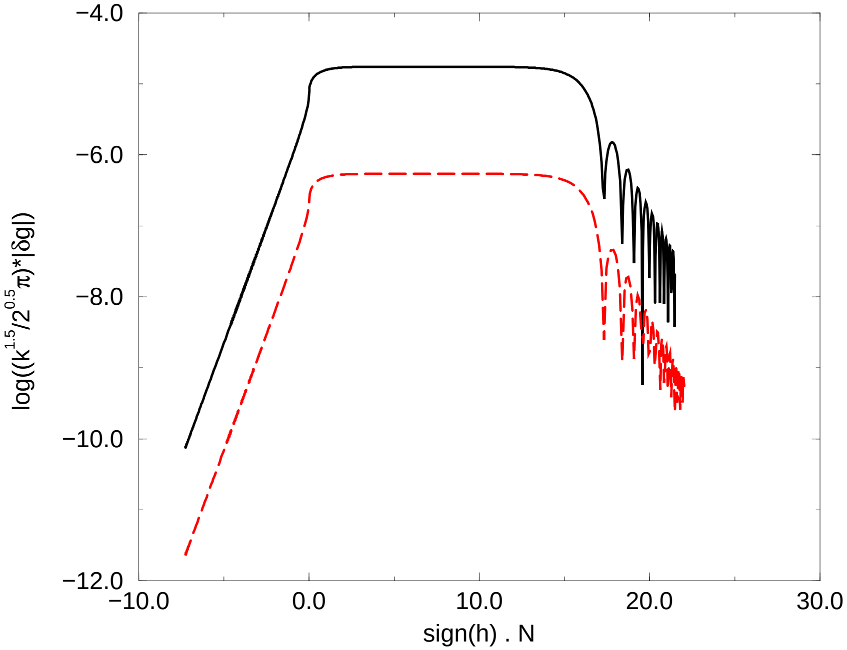
Solution

$$\delta g = \frac{\sqrt{\pi}e^{i(\nu+\frac{1}{2})\frac{\pi}{2}}}{a}|\eta|^{\frac{1}{2}}(J^{(1)}_{|\nu|}(|k\eta|) + iY^{(1)}_{|\nu|}(|k\eta|))$$

where $\nu \equiv \frac{3}{2} + \frac{1}{p-1}$.

Two modes: g_Y and g_J.

Tensor evolution



$$\mathcal{P}_g = 2\frac{k^3|\delta g|^2}{2\pi^2}$$

For each mode

$$\Delta n_g \equiv \Delta n_{\mathcal{R}}$$

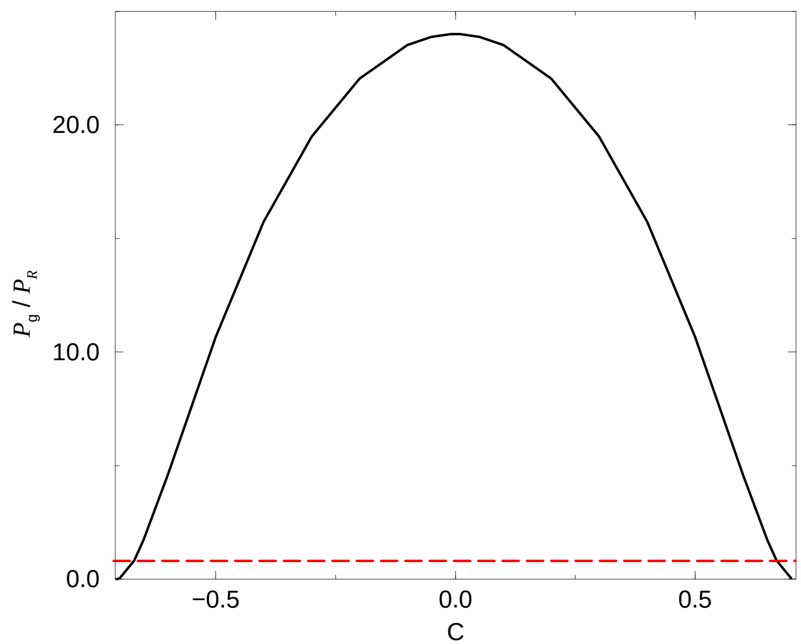
i.e. tensor power spectrum retains its tilt throughout the evolution and the dominant Y-mode is scale invariant.

Tensor-scalar ratio

During power-law collapse

$$\frac{\mathcal{P}_g}{\mathcal{P}_\mathcal{R}} = \frac{16}{p} = 24$$

After the bounce tensor-scalar ratio dependent on value of C, which sets the phase-space trajectory



To fit observational limits and background constraint require $0.67 < |C| < 0.71$!

Summary and Conclusions

- ✯ It is possible to have smooth perturbative evolution through a bounce.
- ✯ $\mathcal{R}$ is not constant on super-horizon scales but all the long wavelength modes share the same time evolution.
- ✯ All the perturbation modes consistently retain the same spectral tilt for $\mathcal{R}$ through the bounce

$$\Delta n_{\mathcal{R}out} = \Delta n_{\mathcal{R}in}$$

but the spectral tilt of Φ is changed

$$\Delta n_{\Phi out} = \Delta n_{\Phi in} + 4 = \Delta n_{\mathcal{R}}$$

- ✯ Picking the right value of the potential slope, $\lambda = \sqrt{3}$, does result in a scale-invariant spectrum of primordial curvature perturbations after the bounce.
- ✯ Fine tuning- to keep the tensor-scalar ratio within acceptable limits we require an asymmetric bounce.