Stable gravastars
– an alternative to black holes?

Matt Visser and David L. Wiltshire

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Black holes are ubiquitous

EVIDENCE

- Binary systems, one partner $M \gtrsim 3M_\odot$ – most spectacular examples now from centre of our galaxy
- Energetic objects with accretion disks: proof that Kerr black holes exist – iron emission line redshift from within $r < 6M$, last stable orbit of Schwarzschild black hole [Dabrowski et al, Mon. Not. R. Soc. 288, L11 (1997)]
- Gamma ray bursters
- Active Galactic Nuclei…

But are they really black holes?

- The evidence is from the black hole exterior
- Need to show that an event horizon exists
- Difference between a completely absorbing surface and "something else" difficult to prove beyond all reasonable doubt [Abramowicz, Kluzniak and Lasota, Astron. Astrophys. 396, L31 (2002)]

ALTERNATIVES

- Boson condensate stars
- . . . (many crazy ideas)
- Gravitational vacuum condensate stars (gravastars)

Quantum gravity + black holes $\Rightarrow$ problems

BLACK HOLE INFORMATION PARADOX

- Hawking 1973: in presence of quantum fields black holes radiate with (almost) black body spectrum.
- Heat capacity negative: rate $\propto M^{-1}$, runs away as $M \to 0$
- If it evaporates completely, information is lost

WAYS OUT

- Change quantum mechanics to allow unitarity violation
- Stable remnant black hole remains
- Quantum gravity intervenes near horizon scale; unitarity is preserved

Funny business at the event horizon?

In Schwarzschild geometry

$$\mathrm{d}s^2 = -A(r)\,\mathrm{d}t^2 + A^{-1}(r)\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2),\; A(r) = 1 - \frac{2M}{r}.$$

an infalling observer **locally** measures energies of other infalling things to diverge $P^{\hat{0}} = A(r)^{-1/2} E \to \infty$.
Should we care?

- ’t Hooft: black hole holography, brick wall model …
- Laughlin (et al): quantum gravity phase transitions, “emergent relativity”…
- many less respectable characters…

The Mazur-Mottola gravastar

The gravastar (*GRAvitational VAcuum condensate STAR of Mazur and Mottola*), gr-qc/0109035, is an onion-like construction, with 5 layers:

- An external Schwarzschild vacuum, with energy density, $\rho = 0$, and pressure, $P = 0$.
- A thin shell, with surface density σ_+ and surface tension ϑ_+; with radius $r_+ \gtrsim 2M$.
- A (relatively thin) finite-thickness shell of stiff matter with equation of state $P = \rho$; straddling $r = 2M$ where the horizon would in normal circumstances have formed.
- A second thin shell; with radius $r_- \lesssim 2M$, and with surface density σ_- and surface tension ϑ_-.
- A de Sitter interior, with $P = -\rho$.

The Mazur-Mottola gravastar

- The two thin shells are used to “confine” the stiff matter in a transition layer straddling $r = 2M$, while the energy density in the de Sitter vacuum is chosen to satisfy

$$\frac{4\pi}{3}\rho(2M)^3 = M,$$

- In the approximation where the transition layer is neglected, *all* of the mass of the resulting object can then be traced back to the energy density of the de Sitter vacuum.

The Mazur-Mottola gravastar

- Expect thermodynamic stability
- Solves black hole information paradox

BUT

- In limit $a_+ = 2M(1+\epsilon)$, $\epsilon \to 0$, wouldn't something blow up?
- Are there dynamically stable configurations?

New simplified gravastar

- Replace thick stiff matter shell by a thin shell
- Leave equation of state of thin shell free, but look for dynamically stable configurations

Our gravastar is a simple 3-layer model

- An external Schwarzschild vacuum, $\rho = 0 = p$.
- A single thin shell, with surface density σ and surface tension ϑ; with radius $a \gtrsim 2M$.
- A de Sitter interior, $P = -\rho$.
- To avoid forming an event horizon, we shall demand

$$\frac{4\pi}{3}\rho(2M)^3 \lesssim M.$$

The mathematical model

- Consider the class of geometries

$$\mathrm{d}s^2 = -\left[1 - \frac{2m(r)}{r}\right] \mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1 - 2m(r)/r} + r^2(\mathrm{d}\theta^2 + \sin^2\theta \, \mathrm{d}\phi^2).$$

- Less general than the class of all static spherically symmetric geometries but sufficiently general to include both the Schwarzschild and the de Sitter geometries.
- Connect two geometries of this type are connected along a timelike hypersurface at $r = a(t)$, (normal n^a)

$$\mathrm{d}\tau^2 = \left[1 - \frac{2m(a(t))}{a(t)}\right] \mathrm{d}t^2 - \frac{1}{1 - 2m(a(t))/a(t)} \left[\frac{\mathrm{d}a(t)}{\mathrm{d}t}\right]^2 \mathrm{d}t^2,$$

Israel–Lanczos–Sen thin-shell formalism

- Induced metric on the shell, (τ proper time),

$$h_{ab} = g_{ab} - n_a n_b,$$

$$h_{ab}\,\mathrm{d}x^a\,\mathrm{d}x^b = -\mathrm{d}\tau^2 + a(\tau)^2\,(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2).$$

- Extrinsic curvature

$$K_{ab} = h_a{}^c\,h_b{}^d\,\nabla_c n_d$$

- Junction conditions relate discontinuity in extrinsic curvature to the surface stress-energy, S_{ab}:

$$[[K_{ab}]] = -8\pi\left[S_{ab} - \frac{1}{2}S\;h_{ab}\right]; \qquad [[K_{ab} - K\;h_{ab}]] = -8\pi\;S_{ab}.$$

…$[[X]]$ denotes discontinuity in X across the shell.

Dynamical analysis

We find

(1) $$\sigma = -\frac{1}{4\pi a}\left[\left[\sqrt{1-2m(a)/a+\dot{a}^2}\right]\right],$$

(2) $$\vartheta = -\frac{1}{8\pi a}\left[\left[\frac{1-m/a-m'+\dot{a}^2+a\ddot{a}}{\sqrt{1-2m/a+\dot{a}^2}}\right]\right].$$

where $\dot{a} \equiv \frac{da}{d\tau}$ and $m'(a) \equiv \frac{dm}{da}$ etc.

- In fact (2) follows from (1) by energy-momentum conservation
$$\frac{d}{d\tau}(\sigma a^2) = \vartheta \frac{d}{d\tau}(a^2).$$

Master equation

- Dynamic master equation can be rewritten in a form of an "energy equation" for a non-relativistic particle,

$$\frac{1}{2}\dot{a}^2 + V(a) = E,$$

with "potential"

$$V(a) = \frac{1}{2}\left\{1 + \frac{4m_+(a)m_-(a)}{m_s^2(a)} - \left[\frac{m_s(a)}{2a} + \frac{(m_+(a)+m_-(a))}{m_s(a)}\right]^2\right\}$$

$m_-(a) =$ "mass function" for interior geometry;
$m_+(a) =$ "mass function" for exterior geometry;
$m_s = 4\pi\,\sigma\,a^2 =$ mass of thin shell;
and "energy" $E = 0$.

Stability

- $\exists$ strictly stable solution for the shell (against spherically symmetric radial oscillations) iff $\exists$ *some* $m_s(a)$ and *some* a_0 such that we simultaneously have

$$V(a_0) = 0; \qquad V'(a_0) = 0; \qquad V''(a_0) > 0.$$

- Quirk: $E \equiv 0$, the situation where $V(a) \equiv 0$, which in non-relativistic mechanics corresponds to neutral equilibrium, is now converted to a situation of stable equilibrium in this general relativity calculation. (Since now, because one is not free to increase the "energy" E, one has $\dot{a} \equiv 0$.)

Stability

- Less stringent notion of stability, "bounded excursion", also useful. Suppose we have $a_2 > a_1$ such that

$$V(a_1) = 0; \quad V'(a_1) \le 0; \quad V(a_2) = 0; \quad V'(a_2) \ge 0;$$

with $V(a) < 0$ for $a \in (a_1, a_2)$.

- In this situation the motion of the shell remains bounded by the interval (a_1, a_2). Although not strictly stable, since the shell does in fact move, this notion of "bounded excursion" more accurately reflects some of the aspects of stability naturally arising in non-relativistic mechanics.
- Adding a small negative offset to a strictly stable potential converts it to one exhibiting "bounded excursion"

$$V(a) \to V(a) - \epsilon^2$$

Inverting the potential

- Assume $V(a)$, $m_-(a)$ and $m_+(a)$ given, and invert to find $m_s(a)$ or $\sigma(a)$:

$$\sigma(a) = -\frac{1}{4\pi a}\left\{\sqrt{1 - 2V(a) - \frac{2m_+(a)}{a}} - \sqrt{1 - 2V(a) - \frac{2m_-(a)}{a}}\right\}$$

- For our case $m_+(a) = M/a$ (Schwarzschild), and $m_-(a) = (4\pi/3)\rho a^3 \equiv ka^3$ (de Sitter), so that

$$\sigma(a) \equiv \frac{1}{4\pi a}\left\{\sqrt{1 - 2V(a) - 2ka^2} - \sqrt{1 - 2V(a) - \frac{2M}{a}}\right\},$$

Inverting the potential

- The surface tension $\vartheta(a)$ is found as a result

$$\vartheta(a) \equiv \frac{1}{8\pi a}\left\{\frac{1-2V(a)-a\,V'(a)-4ka^2}{\sqrt{1-2V(a)-2ka^2}} - \frac{1-2V(a)-a\,V'(a)-M/a}{\sqrt{1-2V(a)-2M/a}}\right\}.$$

Cases of interest

- $V(a) \equiv 0$, a degenerate, but physically important case of static shell $\dot{a} \equiv 0$.
- $V(a) = \frac{1}{2}(a-a_0)^2\, f(a)$, where $f(a)$ is an arbitrary positive function which is regular at a_0. Trivial: master equation has unique solution at $a = a_0$ and $\dot{a} = 0$, and all possibility of motion is excluded by fiat.

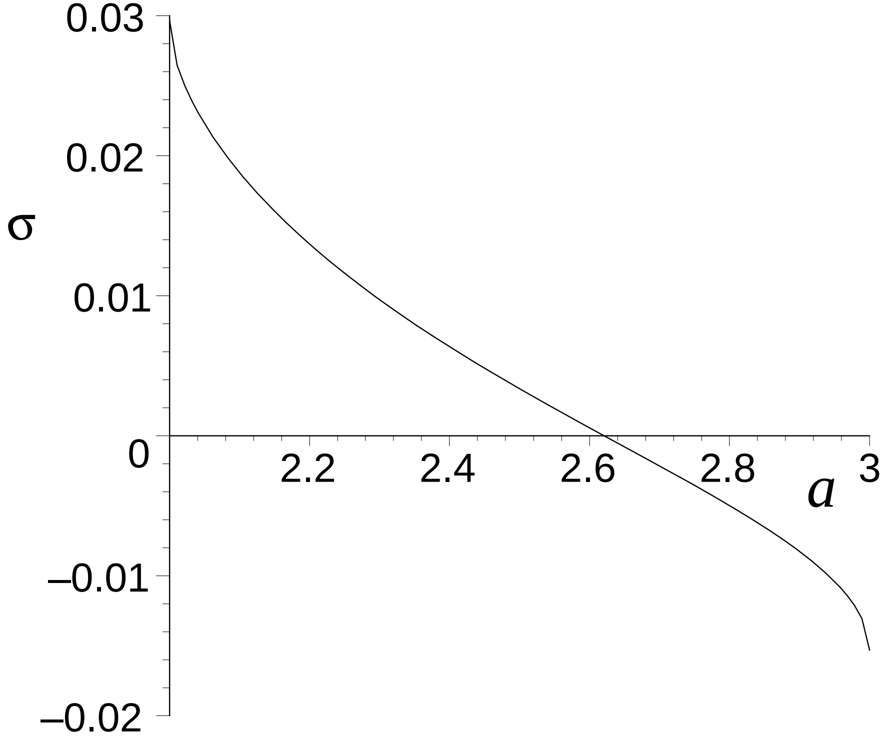
Inverting the potential

- $V(a) = \frac{1}{2}(a - a_0)^2\, f(a) - \epsilon^2$ gives models stable under "bounded excursion".

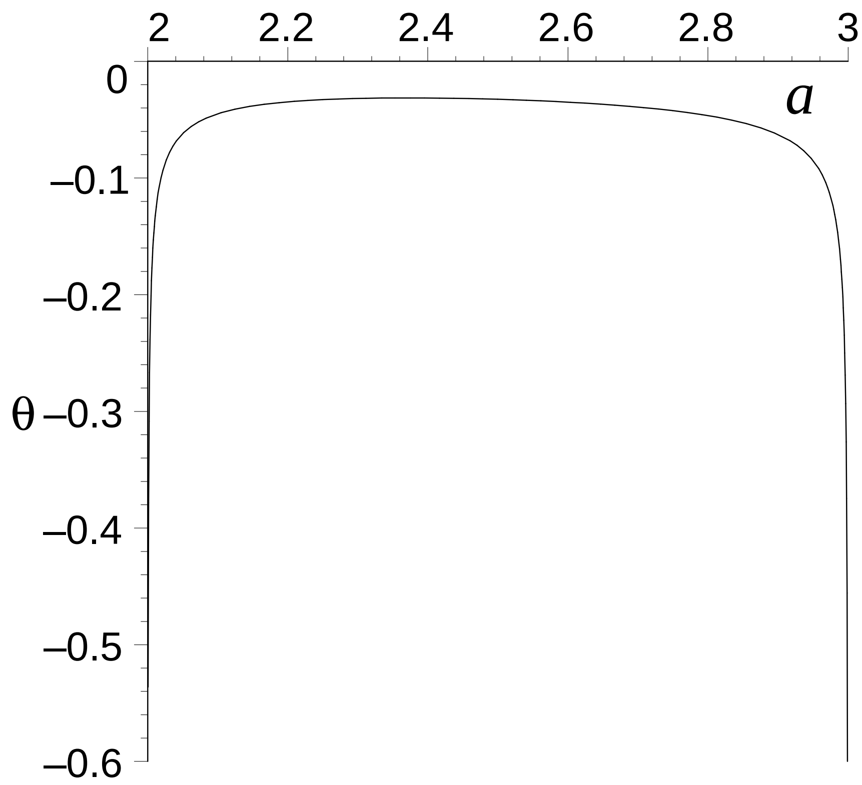
We consider just the $V(a) \equiv 0$ (purely static shell) in what follows.

- Stable solutions with a shell satisfying the Dominant Energy Condition exist if $0 < kM^2 < \lambda_{\rm cr}$, where
$$(400000000\,\lambda_{\rm cr}^4 - 1054320000\,\lambda_{\rm cr}^3 \\ +257041039\lambda_{\rm cr}^2 - 19516500\,\lambda_{\rm cr} + 337500) = 0$$
i.e., $\lambda_{\rm cr} = 0.0243045493773\dots$
- For parameter values $0 < kM^2 < \lambda_{\rm cr}$, there will be a range of values $a_1 < a < a_2$ over which the dominant energy condition is satisfied.

Surface energy density and tension

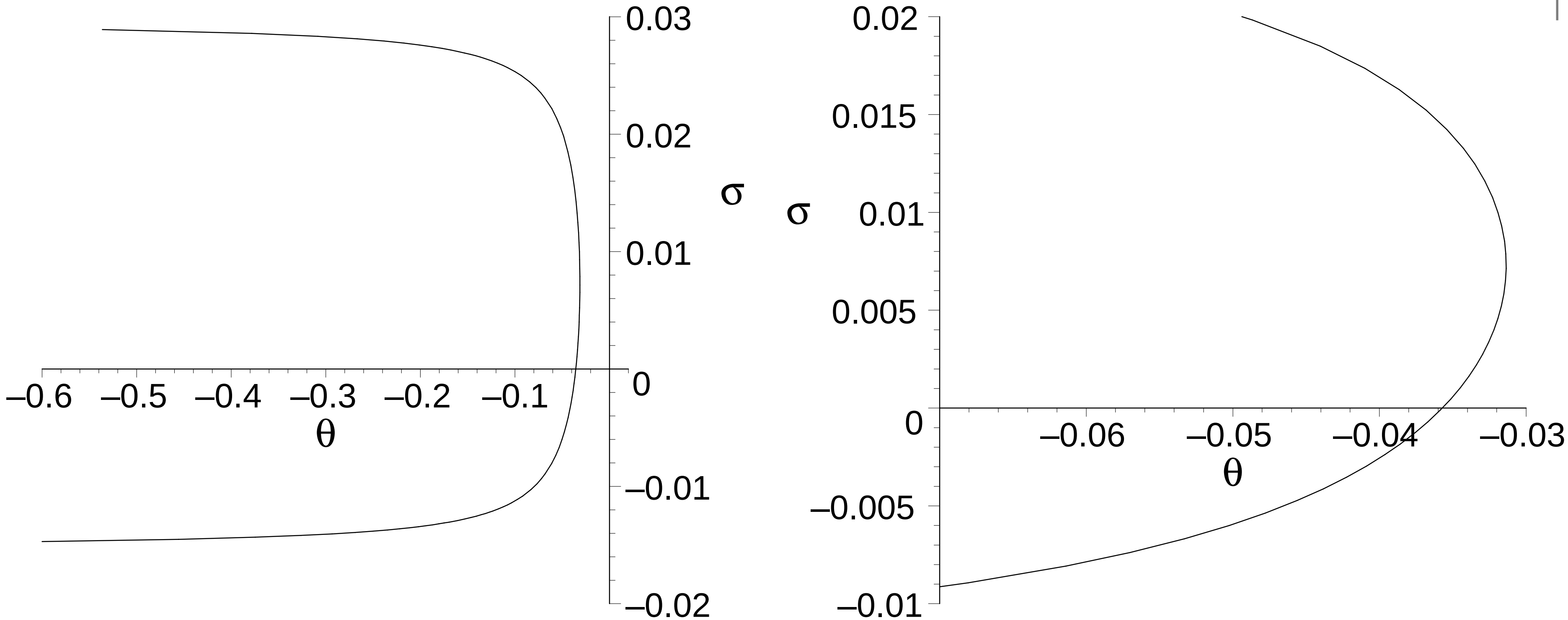


Surface energy density σ (in units M^{-1}), as a function of radius, a (in units M). ($kM^2 = 1/18$; $V(a) \equiv 0$.)



Surface tension, ϑ (in units M^{-1}), as a function of radius, a (in units M). ($kM^2 = 1/18$; $V(a) \equiv 0$.)

Equation of state: Case 1



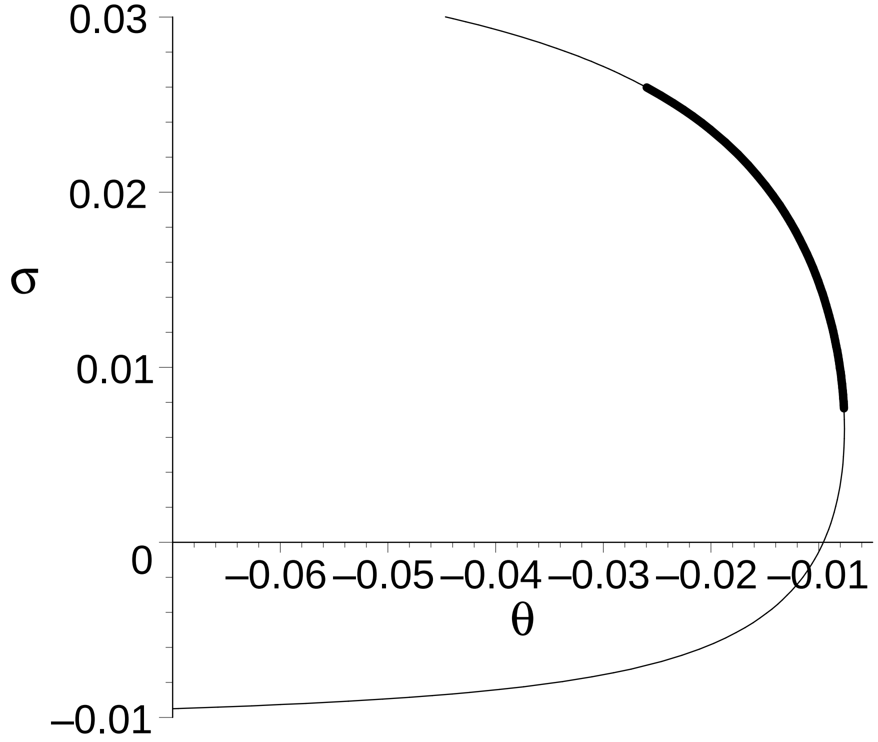
Example: $kM^2 = 1/18$; $V(a) \equiv 0$.

Surface energy density as a function of surface tension.

Right hand panel shows an enlargement of central region.

The dominant energy condition is violated always.

Equation of state: Case 2



Example: $kM^2 = 1/72$; $V(a) \equiv 0$. Parameter values for which the dominant energy condition is violated are shown by a thin line, and parameter values for which the dominant energy condition is satisfied, viz., $2.124319\,M < a < 3\,M$, are shown by a thick line.

Special geometries: Mazur-Mottola 1

- $k \to 1/(8M^2)$ gives the "Mazur–Mottola limit" $k(2M)^3 = M$.
- To understand the nature of this limit it is convenient to write

$$k = \frac{1}{8M^2(1+\epsilon)^2}, \qquad \epsilon \gtrsim 0.$$

- Energy density and surface tension are both real for

$$a \in (2M, 2M[1+\epsilon]).$$

- On kinematic grounds, we have a severely restricted range of possible motions for the shell.
- In limit $\epsilon \to 0$ $\sigma \to 0$ and $\vartheta \to -\infty$. OUCH!

Stiff shell gravastar

We propose a new stable 3-layer limit of Mazur-Mottola model

- Forget about equating exterior mass with that of de Sitter vacuum
- Take a thin shell at $a > 2M$ with stiff equation of state $P = \rho$ or $\vartheta = -\sigma$.
- For $kM^2 < 0.0243045493773\ldots$ there are two values of a at which we can place a stiff shell in stable gravastars, the lower value, a_1, being in the range $2M < a_1 < 2.3005600972496\, M$.
- The inner stiff shell case is certainly so close to the putative horizon that any test to distinguish such an object from a true Schwarzschild black hole would be extremely difficult in astrophysical contexts.

Do I buy it?

PERSONAL PREJUDICES

- Something funny ought to happen at the horizon, but we should preserve the “holey” properties of black holes
- Hawking evaporation is a process which has more to do with quantum field theory than quantum gravity per se
- The fundamental quantum dynamics which explains black hole entropy remain to be found (despite much work)
- As a quantum positivist I advocate the view that classical space should not exist inside a black hole: we want a sum over all possible interior geometries consistent with the surface boundary data
- BH holographic principle would be consistent with such a view; but gravastars not ostensibly so.

Conclusion

Gravastars are

- interesting
- better than we expected at the outset
- maybe good enough to convince a number of people
- would have to be firmly placed in a quantum gravity context (why is a de Sitter fluid natural?) to convince me