Phi in the Sky, Porto

10th July 2004

Inflation and the origin of structure in the Universe

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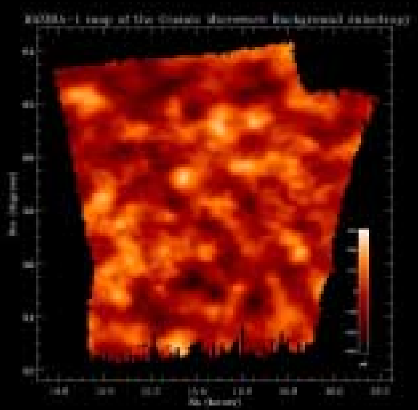
outline

- motivation
- the Primordial Density Perturbation
  - reasons to use zeta
- perturbations from inflation in the early universe
  - inflaton scenario...
  - ...vs curvaton scenario
  - ...or modulated reheating
- relation to observations
- conclusions

Standard model of structure formation

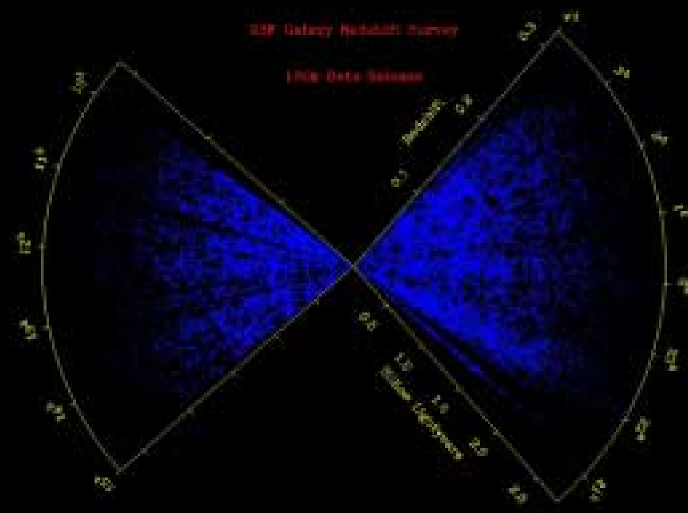
primordial perturbations

in cosmic microwave background



gravitational

instability



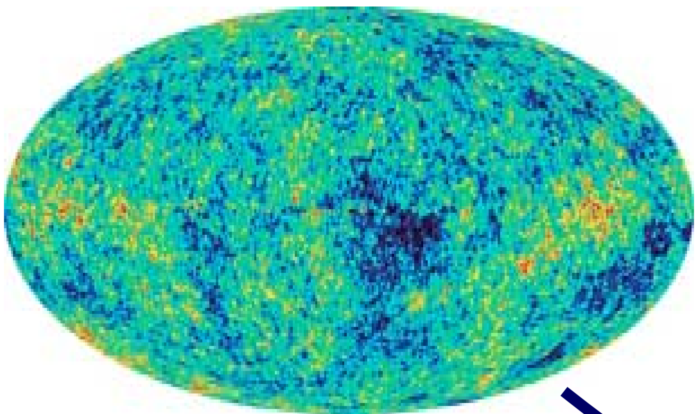
large-scale structure of our Universe

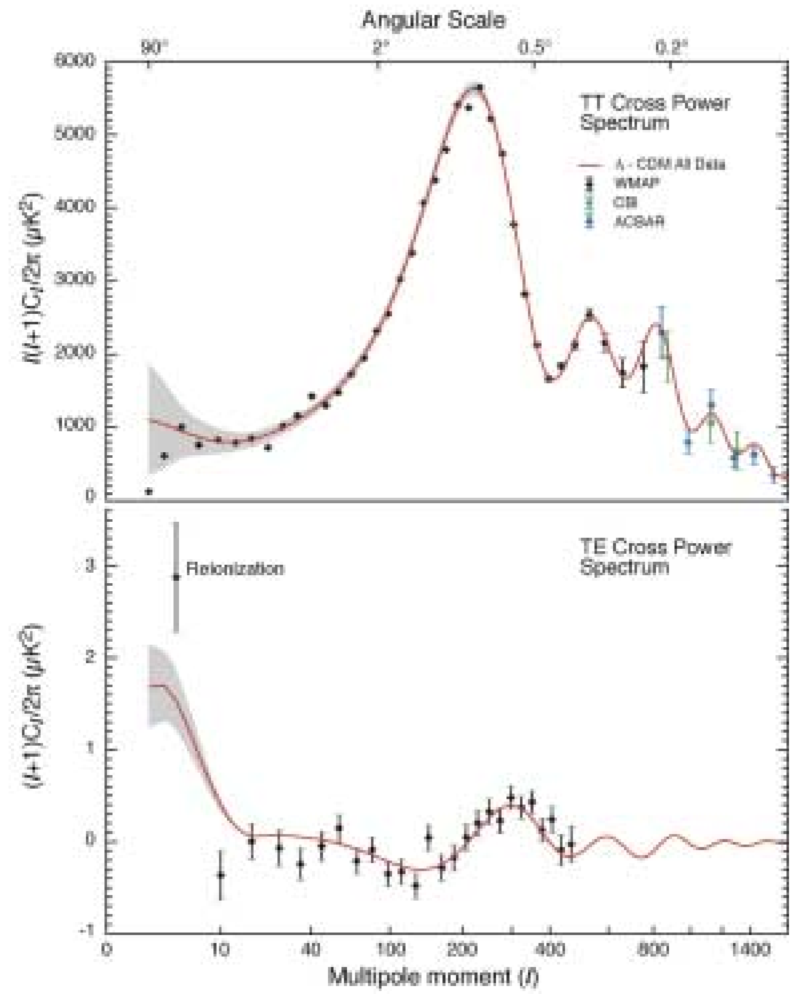
new observational data offers precision tests of

- cosmological parameters
- the nature of the primordial perturbations

Wilkinson Microwave Anisotropy Probe

February 2003





coherent oscillations
in photon-baryon plasma
due to primordial density perturbations
on super-horizon scales

where do the primordial perturbations come from?

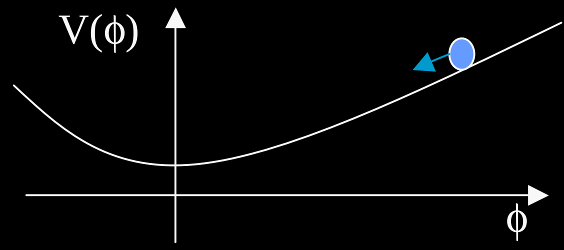
Cosmological inflation:

Starobinsky (1980)

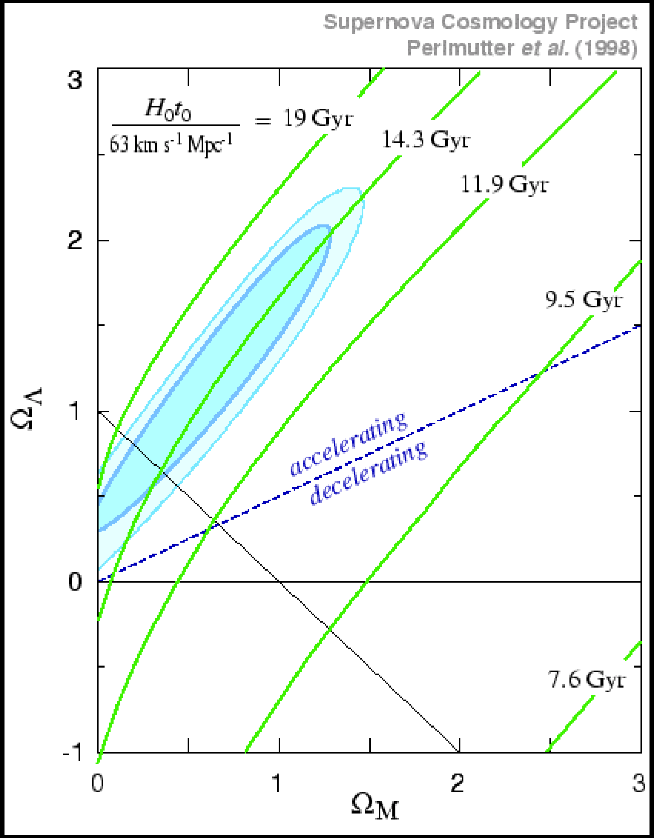
Guth (1981)

- period of accelerated expansion
  in the very early universe
- requires negative pressure

  e.g. self-interacting scalar field

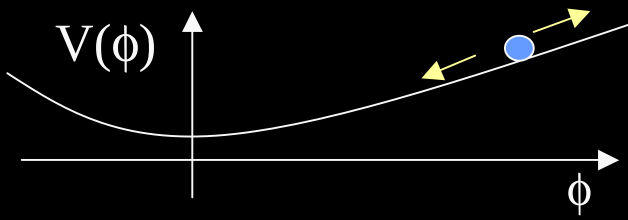


- speculative and uncertain physics
- just the kind of peculiar cosmological
  behaviour we observe today
  dark energy!



Vacuum fluctuations

Hawking '82, Starobinsky '82, Guth & Pi '82



- *small-scale/underdamped zero-point fluctuations* $\delta\phi_k \approx \frac{e^{-ik\eta}}{a\sqrt{2k}}$
- *large-scale/overdamped perturbations in growing mode*

  linear evolution $\Rightarrow$ *Gaussian random field*

$$\left\langle \delta\phi^2 \right\rangle_{k=aH} \approx \frac{4\pi k^3 \left|\delta\phi_k\right|^2}{(2\pi)^3} = \left(\frac{H}{2\pi}\right)^2$$

fluctuations of any scalar light fields (m<3H/2) \`frozen-in' on large scales

how do we relate scalar field fluctuations during inflation to primordial density perturbations of photons, baryons, etc...?

describing the primordial density perturbation

use zeta!

$$\zeta_i = -\frac{H}{\dot{\rho}_i}\delta\rho_i - \psi$$

- **gauge-invariant** combination of the gauge-dependent density perturbation, $\delta\rho_i$, and spatial curvature, ψ
- dimensionless density perturbation on spatially flat hypersurfaces
- equal to the perturbed expansion $\delta N = -H\delta t$
- first introduced for total density perturbation by Bardeen et al (1983)
- constant on large scales for perfect fluid with equation of state, $P_i(\rho_i)$

Wands, Malik, Lyth & Liddle (2000)

Reasons to use ζ:

1. constant on large scales for adiabatic perturbations
2. physically-defined quantity
   - hence gauge-invariant
   - naturally extended to non-linear perturbations

gauge-invariance at first- and second-order:

Malik & Wands, Class Quantum Grav (2004)

$$\zeta = \zeta_1 + \frac{1}{2}\zeta_2 + \ldots$$

$$\zeta_1 = -\frac{H}{\rho'}\delta\rho_1 - \psi_1$$

$$\zeta_2 = -\frac{H}{\rho'}\delta\rho_2 - \psi_2 + 2\frac{H}{\rho'^2}\delta\rho_1'\delta\rho_1 + 2\frac{\delta\rho_1}{\rho'}\left(\psi_1' + 2H\psi_1\right)$$
$$+\frac{\delta\rho_1{}^2}{\rho_1'^2}\left(2H^2 + H' - H\frac{\rho''}{\rho'}\right) - 2\beta^i\partial_i\left(\psi_1 + H\frac{\delta\rho_1}{\rho'}\right)$$

Reasons to use ζ:

1. constant on large scales for adiabatic perturbations
2. physically-defined quantity
   - hence gauge-invariant
   - naturally extended to non-linear perturbations
3. constancy on large scales follows directly from conservation equation
   - valid even in alternative gravity theories
   - e.g., brane inflation

conserved cosmological perturbations

Lyth & Wands, Phys Rev D (2003)

For every quantity, y, that obeys a **local conservation equation**

$$\frac{dy}{dt} = H \; f(y)$$

where H is the locally-defined Hubble rate along comoving worldlines

there is a **conserved perturbation**
(local integration constant)

$$\zeta_x = H \, \delta t = \frac{\delta y}{f(y)}$$

where δy is a evaluated on hypersurfaces separated by uniform expansion $\Delta N = \int H \, dt$ (such as $\psi=0$ hypersurfaces on large scales)

Primordial Density Perturbation

perturbed cosmic fluid consists of

- photons, ζ_γ, neutrinos, ζ_ν, baryons, ζ_B, cold dark matter, ζ_{CDM}, (and dark energy, ζ_X)

total density perturbation, or **curvature** perturbation

$$\zeta = \sum_i \left(\frac{\dot{\rho}_i}{\dot{\rho}} \right) \zeta_i$$

relative density perturbations, or **isocurvature** perturbtns

$$S_i = 3\left(\zeta_i - \zeta_\gamma\right)$$

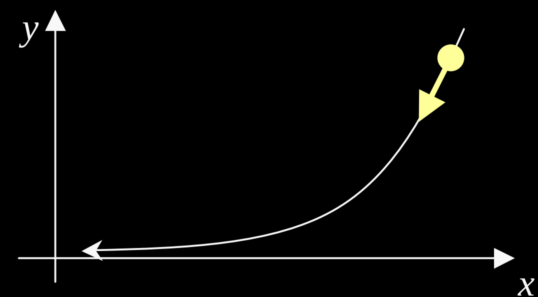
special case of **adiabatic** perturbations:

$$S_i = 0 \; \forall \, i \quad \Rightarrow \quad \zeta = \sum_i \left(\frac{\dot{\rho}_i}{\dot{\rho}} \right) \zeta_\gamma = \zeta_\gamma = \text{constant}$$

Cosmological perturbations on large scales

- **adiabatic perturbations** *e.g.,* $$\delta\left(\frac{n_\gamma}{n_B}\right) \propto \frac{\delta n_\gamma}{n_\gamma} - \frac{\delta n_B}{n_B} = 0$$
  - *perturb along the background trajectory*

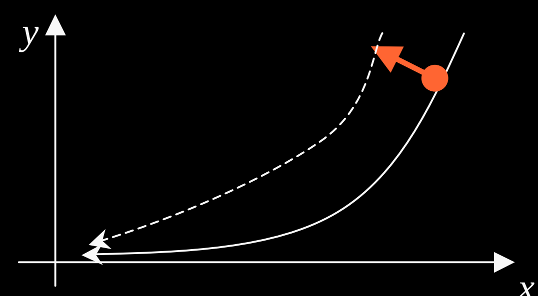
$$H\frac{\delta x}{\dot{x}} = H\frac{\delta y}{\dot{y}} = H\delta T$$



  - ***adiabatic perturbations stay adiabatic***

- **entropy perturbations**
  - *perturb off the background trajectory*

$$H\frac{\delta x}{\dot{x}} \neq H\frac{\delta y}{\dot{y}}$$



  - *e.g., baryon-photon* ***isocurvature*** *perturbation:*

inflaton scenario:

necessarily adiabatic primordial perturbations

$$\left(\zeta_\gamma = \zeta_B = \zeta_\nu = \zeta_{cdm}\right) = \zeta_\sigma$$

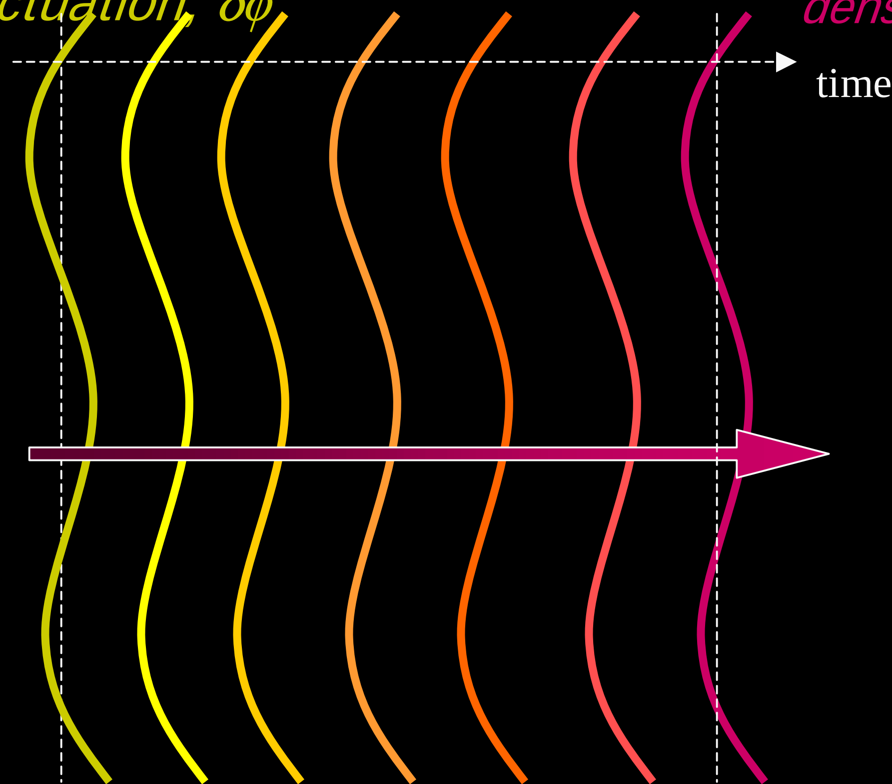
during inflation

scalar field fluctuation, $\delta\phi$

during matter+radiation era

density perturbation, $\delta\rho$

$$\zeta = \frac{H\delta\sigma}{\dot{\sigma}}$$



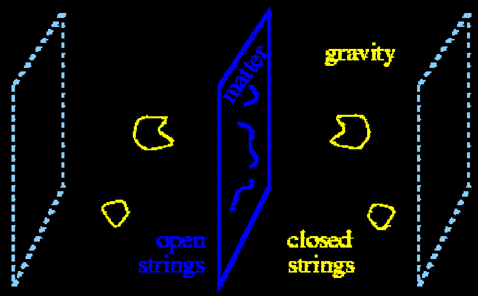
$$\zeta = \frac{H\delta\rho}{\dot{\rho}}$$

for adiabatic perturbations on super-horizon scales $\dot{\zeta} = 0$

brane-world inflaton scenario:

Maartens, Wands, Bassett & Heard (2000)

- 4D inflaton on our brane-world



- 5D metric perturbations too complicated to solve at high energies
- Calculate zeta during slow-roll inflation on the brane (negligible metric back-reaction for V=constant)
- zeta conserved on large scales until low energies where we can use GR
- same basic calculation works in different brane-world gravity models – relies only on energy conservation on the brane

new ideas in inflation:

- why should the field that drives inflation also be responsible for the primordial density perturbations?

the division of labour

"THE GREATEST improvement in the productive powers of labour, and the greater part of the skill, dexterity, and judgment with which it is any where directed, or applied, seem to have been the effects of the **division of labour**."

Adam Smith, The Wealth of Nations (1776)

new scenarios for origin of structure:

- ***the curvaton scenario***

  weakly-coupled late-decaying scalar field

  Enqvist & Sloth; Lyth & Wands; Moroi & Takahashi (2001+)

- ***inhomogeneous / modulated reheating***

  inflaton decay-rate modulated by another light field

  Dvali, Gruzinov & Zaldariaga; Kofman (2003+)

curvaton scenario:

(i) during inflation

- curvaton χ light during inflation (m<<H)

- spectrum of isocurvature perturbations from quantum fluctuations

$$\zeta_\chi = \frac{1}{3}\left\langle \frac{\delta\rho_\chi}{\rho_\chi} \right\rangle \approx \frac{2}{3}\left\langle \frac{\delta\chi}{\chi} \right\rangle \approx \left(\frac{H}{3\pi\langle\chi\rangle} \right)_*$$

- spectral tilt:

$$\Delta n_\chi = \frac{d \ln\langle\delta\chi^2\rangle}{d \ln k} \approx -2\varepsilon + 2\eta \quad \text{where } \varepsilon \equiv -\frac{\dot{H}}{H^2}, \eta \equiv \frac{m_\chi^{\ 2}}{3H^2}$$

- assume negligible inflaton density perturbation

$$\zeta_* \approx \frac{H^2}{\varepsilon M_{Pl}^{\ 2}} << 10^{-5}$$

$\Rightarrow$ negligible gravitational waves $H << 10^{-5} M_{Pl}^{\ 2}$ for $\varepsilon < 1$

curvaton scenario:

(ii) after inflation

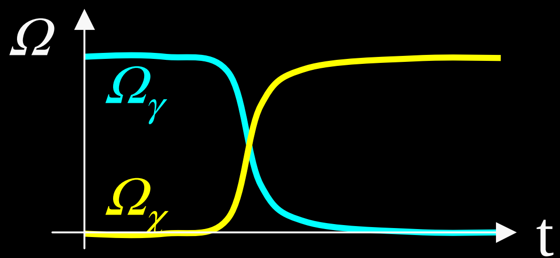
- radiation $\Omega_\gamma \approx 1$ plus curvaton $\Omega_\chi << 1$
- total density perturbation initially negligible

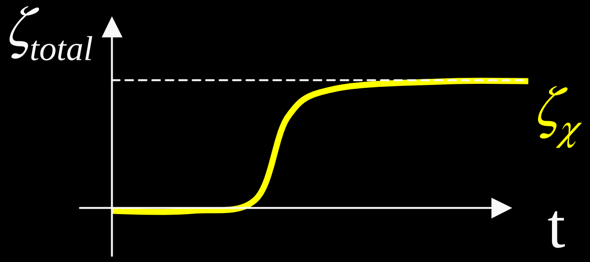
$$\zeta_{total} = \sum \frac{\dot{\rho}_i}{\dot{\rho}_{total}} \zeta_i$$

$$\approx \frac{3\Omega_\chi}{4\Omega_\gamma + 3\Omega_\chi} \zeta_\chi$$

but (neglecting curvaton decay)

$\rho_\gamma \propto a^{-4}$ and $\rho_\chi \propto a^{-3}$, hence $\Omega_\chi / \Omega_\gamma \propto a$





(iii) curvaton decay:

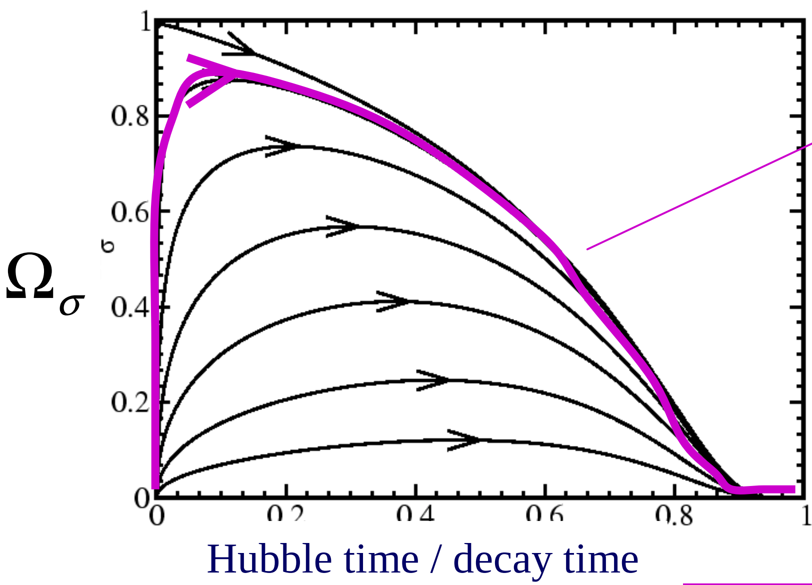
Malik, Wands & Ungarelli (2003)

Gupta, Malik & Wands (2004)

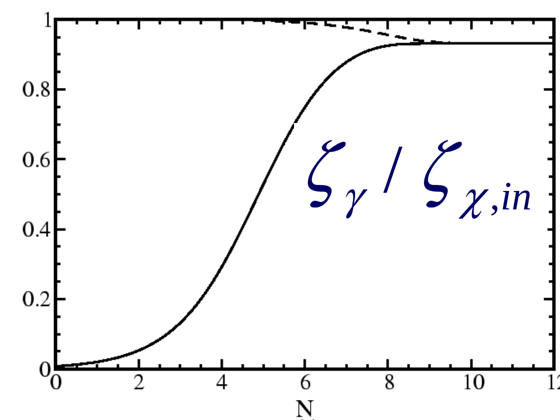
decay to radiation (before primordial nucleosynthesis):

$$\dot{\rho}_\gamma = -4H\rho_\gamma + \Gamma\rho_\chi$$
$$\dot{\rho}_\chi = -3H\rho_\chi - \Gamma\rho_\chi$$

$$\dot{\zeta}_\gamma = \frac{\Gamma}{2}\frac{\dot{\rho}_\chi}{\dot{\rho}_\gamma}\left(2 - \frac{\rho_\chi}{\rho}\right)\left(\zeta_\chi - \zeta_\gamma\right)$$
$$\dot{\zeta}_\chi = \frac{\Gamma}{2}\frac{\dot{\rho}_\gamma}{\dot{\rho}_\chi}\frac{\rho_\chi}{\rho}\left(\zeta_\chi - \zeta_\gamma\right)$$



$$\boxed{p = \left(\Omega_\chi / (\Gamma / H)^{1/2}\right)_{in}}$$



$$\boxed{\zeta_{\gamma,\,final} = r(p)\,\zeta_{\chi,in} \quad \text{and} \quad r(p) \approx \Omega_{\chi,decay}}$$

why might theorists want a curvaton?

- many candidate curvatons already exist (in the literature)
  - pseudo-Nambu Goldstone bosons, the Affleck-Dine field (baryogenesis), sneutrino (leptogenesis)
- MSSM flat directions
  - Enqvist et al
- inflaton may be decoupled from ordinary matter
  - no need for conventional reheating after inflation

why should observers care?

- distinctive observational predictions
  - no gravitational waves
  - possibly detectable non-Gaussianity
  - possibly correlated isocurvature perturbations

non-Gaussianity

simplest kind of non-Gaussianity:

$$\zeta \approx \zeta_1 + f_{NL}\zeta_1^{\;2}$$

Komatsu & Spergel (2001)

Wang & Kamiokowski (2000)

recall that for curvaton

$$\zeta \approx \Omega_{\chi,\mathrm{decay}}\,\zeta_\chi \approx \Omega_{\chi,\mathrm{decay}}\left(\frac{\delta\chi}{\chi} + \left(\frac{\delta\chi}{\chi}\right)^2\right)$$

corresponds to

$$\zeta_1 \approx \Omega_{\chi,\mathrm{decay}}\left(\frac{\delta\chi}{\chi}\right) \quad , \quad f_{NL} \approx \frac{1}{\Omega_{\chi,\mathrm{decay}}}$$

Lyth, Ungarelli & Wands '02

constraints on f_{NL} from WMAP $f_{NL} < 134$

hence $\Omega_{\chi,\mathrm{decay}} > 0.01$ and $10^{-5} < \delta\chi/\chi < 10^{-3}$

c.f. non-Gaussianity from inflaton scenario

single-field consistency relation:

$$f_{NL} = \frac{n_s - 1}{4} \approx \frac{-3\varepsilon + \eta}{2} << 1$$

Maldacena (2002)

Gruzinov; Creminelli & Zaldarriaga (2004)

"easy" to disprove the (simplest) inflaton scenario

inhomogeneous/modulated reheating:

Dvali, Gruzinov & Zaldariaga; Kofman (2003)

- light field during inflation, χ,
  modulates inflaton decay rate, $\Gamma(\chi)$
- non-adiabatic perturbations (inhomogeneous equation of state) change zeta on large scales
- after reheating

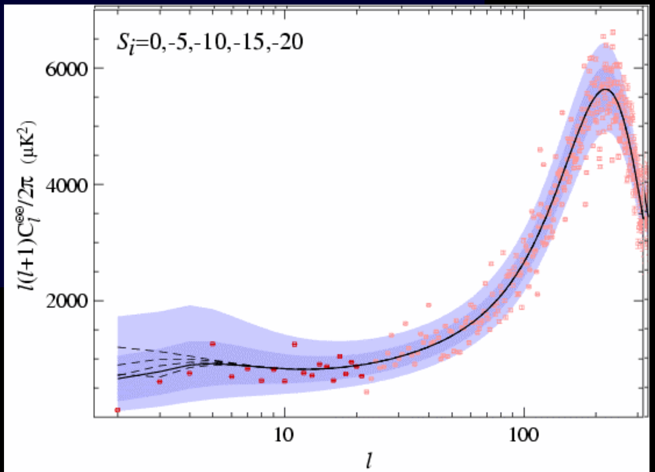
$$\zeta_\gamma \approx \frac{1}{6}\frac{\delta\Gamma}{\Gamma} \approx \frac{1}{12}\zeta_\chi$$

- many similarities with curvaton
  - isocurvature fluctuations during inflation, hence same spectral tilt
  - negligible gravitational waves (at least for slow-roll inflation)
  - possible non-Gaussianity
  - possible isocurvature perturbations?

correlated dark energy and the CMB quadrupole?

Moroi & Takahashi (2003); Gordon & Hu (July 2004)

- large dark energy fluctuations affect only lowest multipoles of the CMB (if $c_x^2=1$)
- correlated isocurvature perturbations in the dark energy can reduce quadrupole $\Delta T/T$ (but leave CMB polarisation largely unaffected)
- Gordon & Hu claim 2-sigma evidence for correlated dark energy perturbations
- best-fit value? $$\zeta_X = 12\,\zeta_\gamma$$



Conclusions:

1. ***Precision cosmology*** (especially cosmic microwave background data) offer detailed measurements of ***primordial density perturbations***
2. ***Inflaton*** or ***curvaton/modulated reheating*** fields during inflation could produce ***adiabatic density perturbations***
3. ***Primordial isocurvature*** perturbations and/or ***non-Gaussianity*** may provide valuable info about origin of perturbations
4. ***Late-time curvature perturbation*** on large scales may depends upon dark energy isocurvature perturbation
5. ***More precise data*** allows/requires us to study more detailed models of the inflation and cosmological perturbations