Potentials and Tracker Fields

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Summary

- Why do we need an attractor (tracker) and which is the precise meaning of this concept?
- Towards $w = -1$
- The asymptotic behaviour
- The right expression for Γ
- What is happening now?
- A general exact solution fits the data
- Conclusions

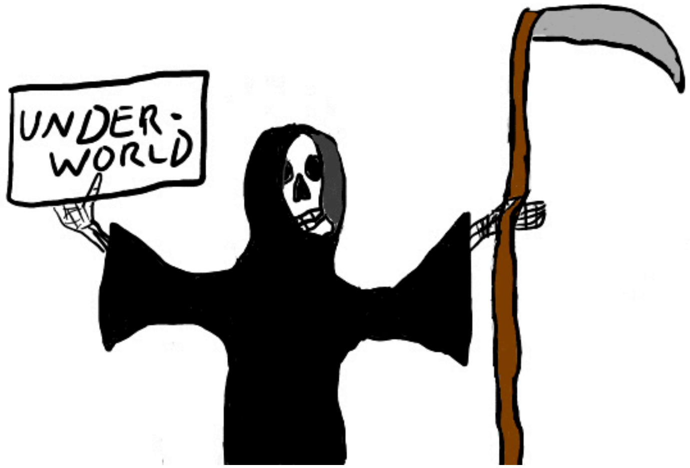
Which is the precise meaning of "tracker"?

In order to solve the problem of fine tuning and coincidence, we need to obtain the present situation as the final result of a very large number of initial conditions

This is achieved usually with the concept of attractor or tracker, but the point is much more subtle than usually understood

The first important question is: assuming the existence of an attractor, has the system reached its asymptotic regime ($t \rightarrow \infty$) ?

Example: there is a well known attractor in everybody's life



Should we conclude that we are probably all dead?

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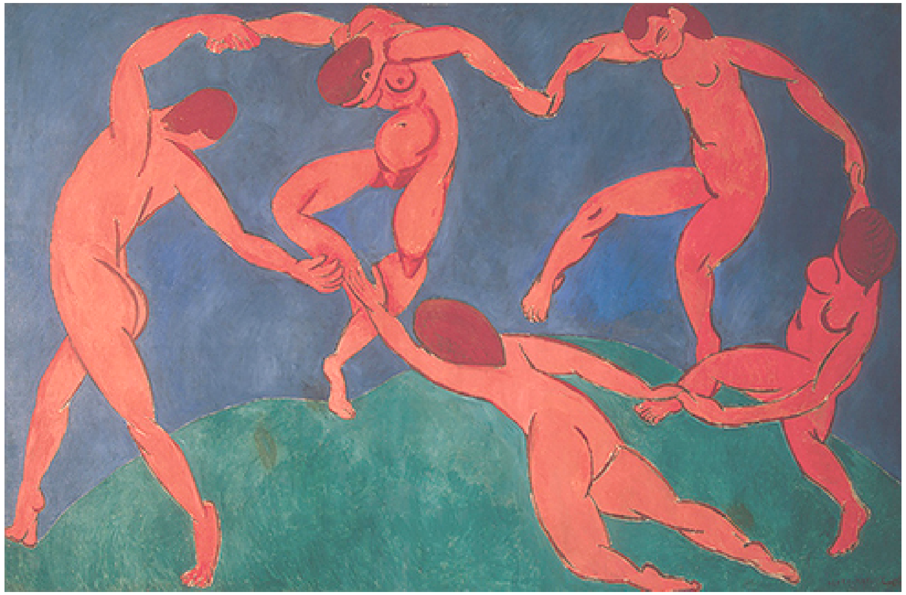


Example: there is a well known attractor in everybody's life

Should we conclude that we are probably all dead?

Considering the attractor misses the most interesting side of the stuff !

The second question is: does an attractor even exist?

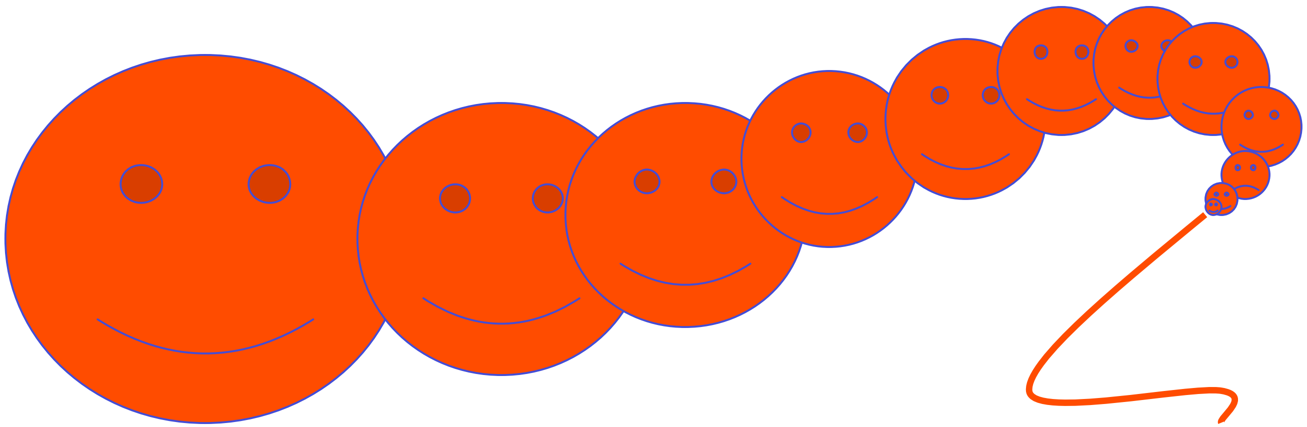
The Friedmann and Klein-Gordon equations can be derived from a Lagrangian (PRD **69**, 103510)

$$L = 3a\dot{a}^2 - \frac{1}{2}a^3\dot{\varphi}^2 + a^3V(\varphi)$$

As a consequence, Liouville's theorem holds and there is no attractor at all!

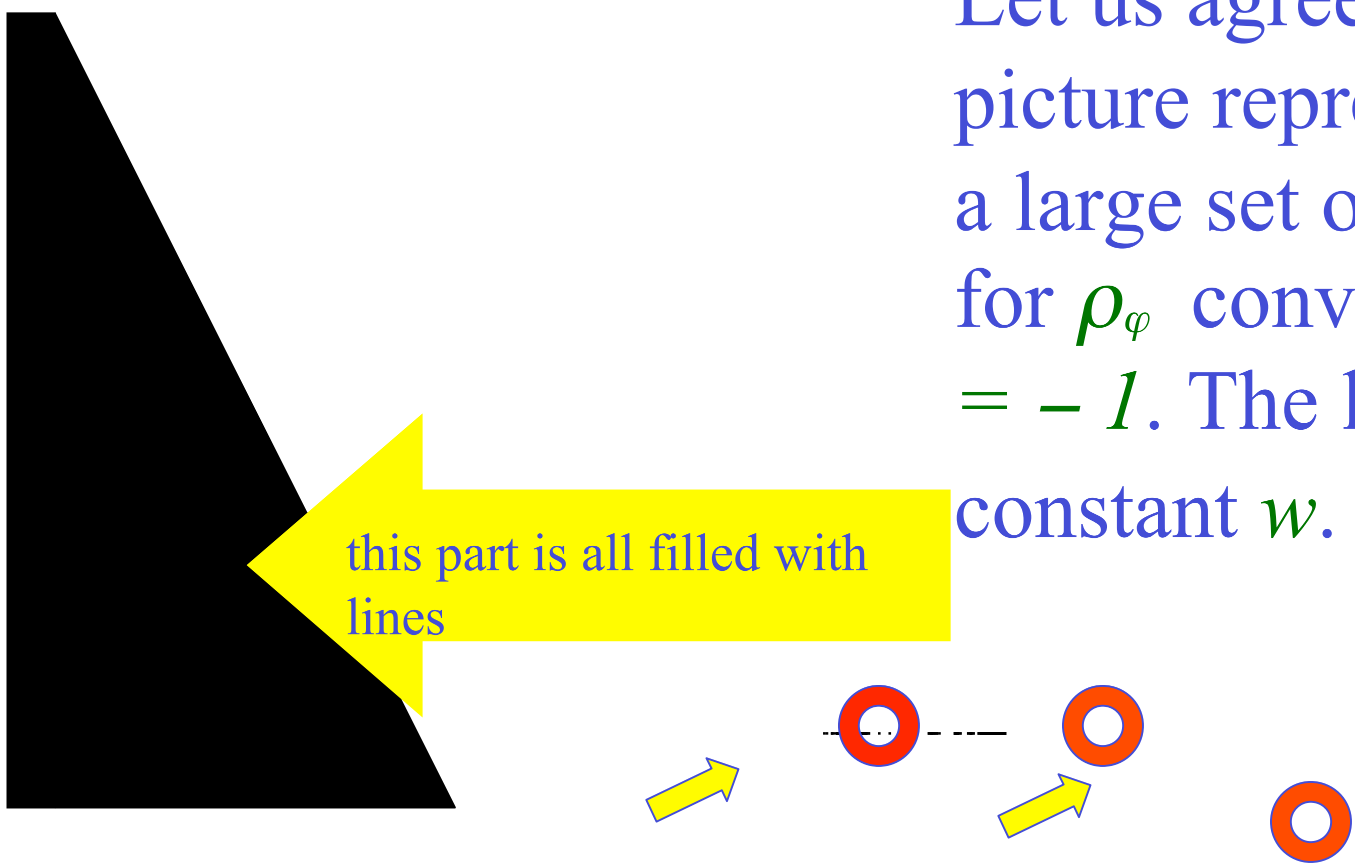
How can we get out of this *cul de sac* ?

The parameter space of the system is four dimensional. If we think of a large "ball" of possible initial conditions, we may imagine that this "ball", during the evolution is deformed to a "pencil", so that its projection on a suitable 2-dimensional subset of the parameter space gives approximately a point, fixed or moving along a stable trajectory.



Remarks

- The choice of the set of parameters and projection is crucial
- The gain in information in one couple of parameters entails a loss in the other couple
- Once the projection is reduced to a point, the tracker condition is reached, but the shape of the trajectory is still undetermined

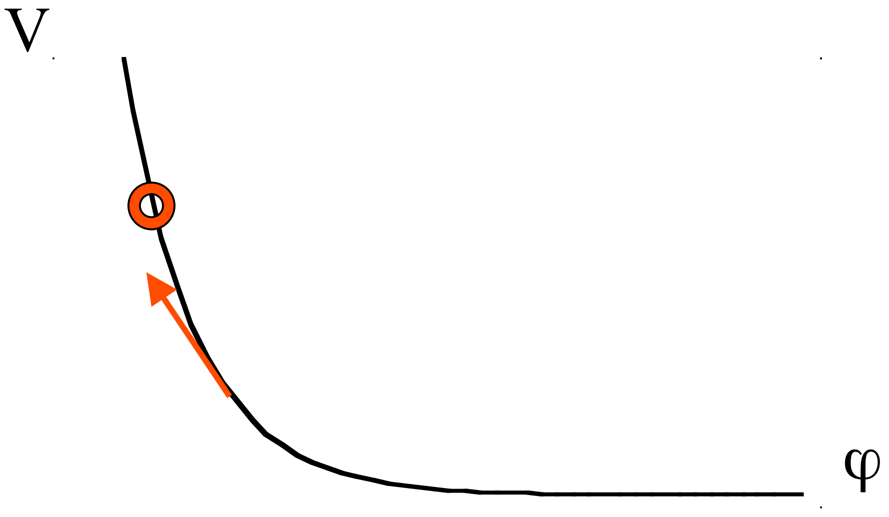


Let us agree that this picture represents our goal: a large set of initial values for ρ_φ converges towards $w = -1$. The last period has constant w.

The scales are intentionally blank. Which is the time of the two transitions?

and where is "now"?

Towards *w* = – *1*



The only assumptions we need are $dV/d\varphi < 0$

and $\dot{\varphi}(t_{in}) < 0$

Rolling up over the potential, φ reaches a maximum point, where $w = -1$, and comes back. We can thus safely start from this point, provided it is reached sufficiently early. Another condition can be the arbitrary normalization of the scale factor. We are thus left with two free parameters: Ω_{m0} and H_0

The asymptotic behaviour

Under which condition the final part of the graph is a straight line? It is easy to show that, if $w_\infty = const$ and the scalar field dominates, then necessarily

$$w_\infty = \frac{\left(\frac{V_0'}{V}\right)^2_\infty - 3}{3}$$

In other words, only a potential which behaves like an exponential towards ∞ can give a constant w. This seems to go against a widely accepted result

The right expression for *Γ*

It is generally reported that, for the exponential potential, the asymptotic value of *w* is the same as that of the background (zero in our case).

This originates from a wrong evaluation of the function

$$\Gamma \stackrel{\mathrm{def}}{=} \frac{V'' V}{(V')^2}$$

which, in the case of $w \cong const$, reads

$$\Gamma = 1 + \frac{w_B - w_\varphi}{2(1+w_\varphi)}$$

The correct expression is instead (PRD **69**, 103510)

$$\Gamma = 1 + \frac{w_B - w_\varphi}{2(1+w_\varphi)}(1 - \Omega_\varphi)$$

The asymptotic behaviour

From the above expressions we can conclude, for the most popular potentials

$$V = V_0 e^{-\lambda\varphi} \rightarrow w_\infty = \frac{\lambda^2 - 3}{3}$$

$$V = \frac{\Lambda^{4+\alpha}}{\varphi^\alpha} \rightarrow w_\infty = -1$$

$$V = \frac{V_0 e^{-\lambda\varphi}}{\varphi^\alpha} \rightarrow w_\infty = \frac{\lambda^2 - 3}{3}$$

$$V = \frac{V_0 e^{\varphi^2/2}}{\varphi^\alpha} \rightarrow w = -1$$

How long is $w \cong -1$?

There is not a definite answer to this question. It depends on the shape of the potential and the free parameters left. In the literature, there is not so much attention paid tho this problem.

The general opinion seems to be that the second transition is situated far in the past

We shall see that the situation can be very different!

What is happening now?

In other words, we are still left with the most important question: Where is "now"?

The following example illustrates a non trivial possibility:

$$V = B^{2} e^{à\, û '}$$

A general exact solution

Let us consider the potential

$$V = B^2 e^{-\sigma\varphi}$$

with $\sigma^2 = 3/2$. It is then possible to obtain a general exact solution (GRG **34**, 307)

$$a = \sqrt[3]{uv} \qquad \varphi = -\frac{1}{\sigma}\log\frac{u}{v}$$

where

$$u = u_1 t + u_2 \qquad \omega = \sigma^2 B^2$$

$$v = \left(\frac{1}{6}u_1 \omega\, t^3 + \frac{1}{2}u_2 \omega\, t^2 + v_1 t + v_2\right)$$

A complete control of the situation

The existence of such a solution gives us the opportunity to have the complete control of the parameters.

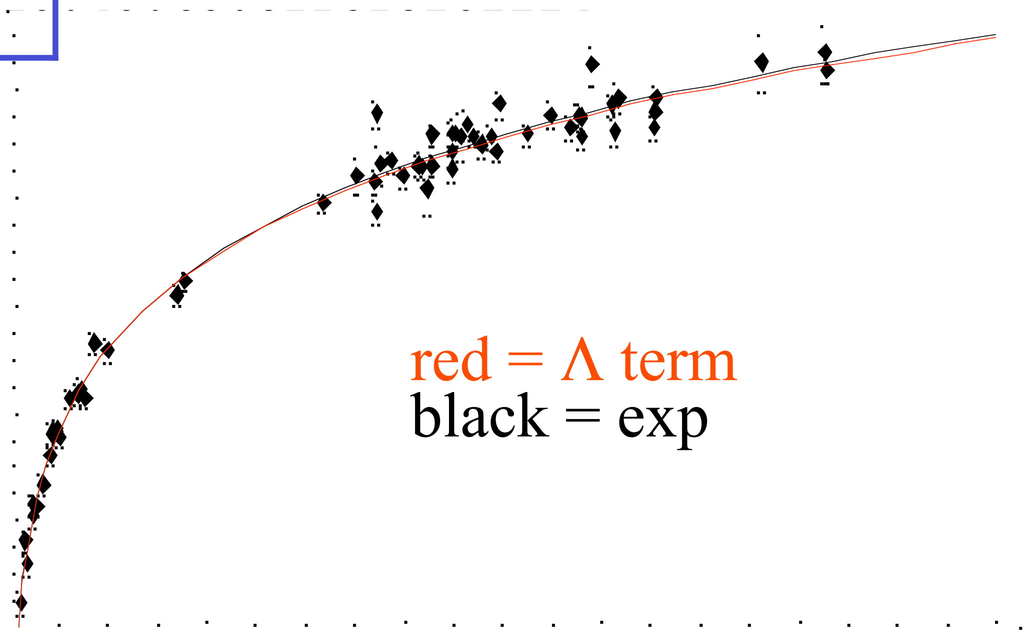
After fixing some, according to the above requirements, we performed a best fit procedure with WMAP and PERLMUTTER data, using CAMB and COSMOMC programs, with the help of one of the authors (thanks to A. Lewis - Toronto)

Comparison with data

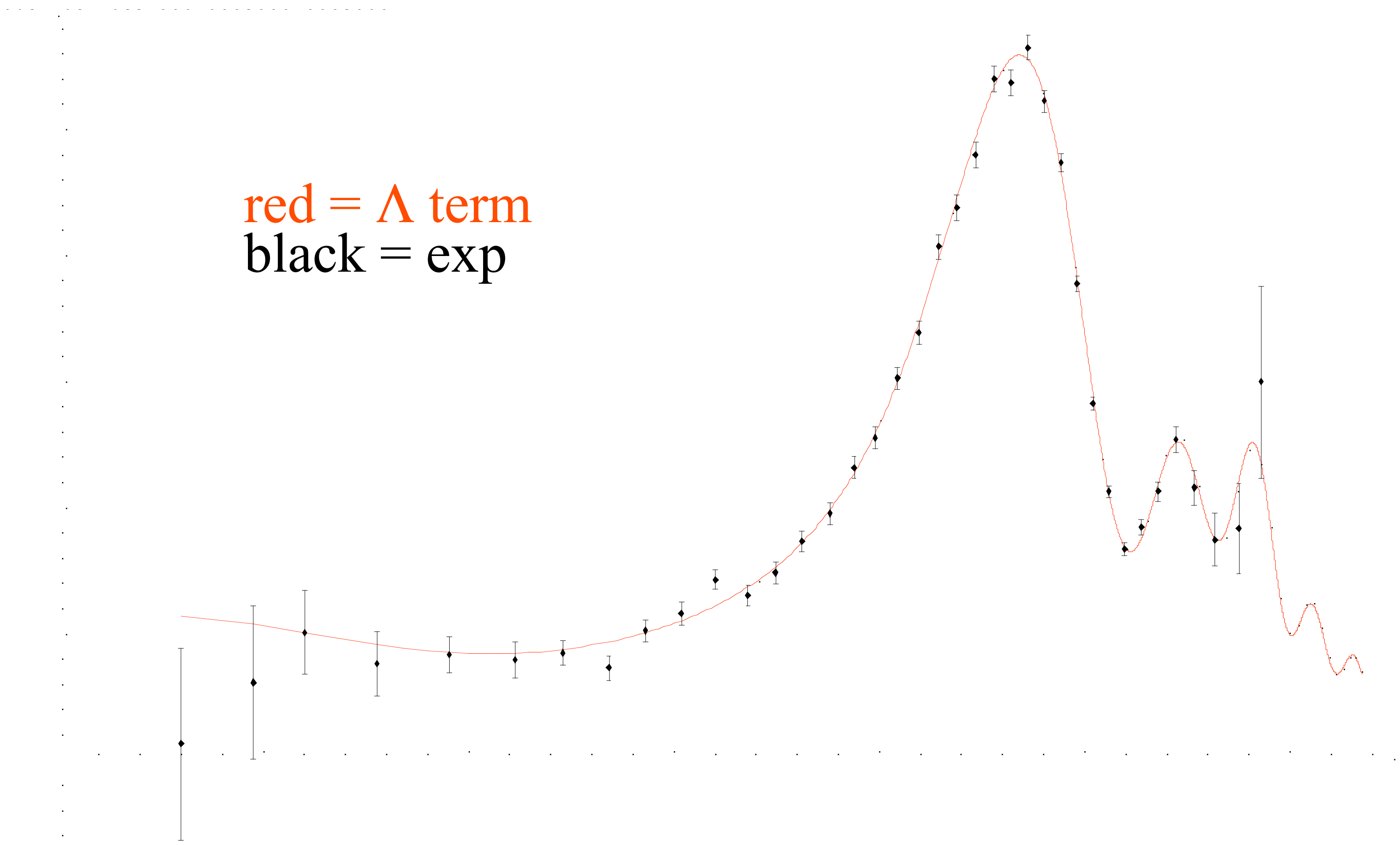
param.	Λ term	Exp.
Ω_m	0.30	0.30
n	0.96	0.95
h	0.69	0.67
LH	765	767

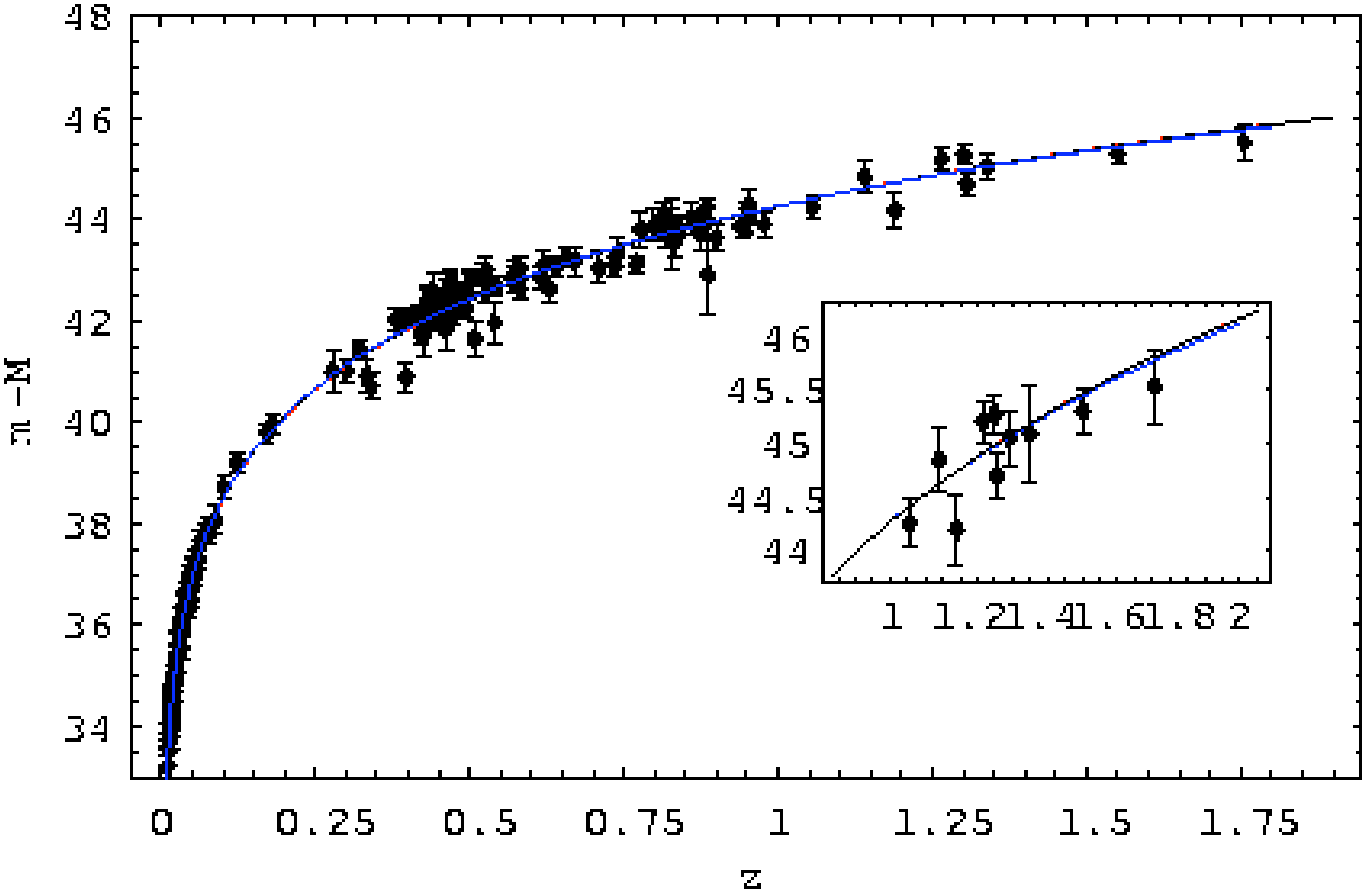
Comparison shows that our potential can perfectly emulate a Λ term

This is illustrated both by the distance modulus of SNIa

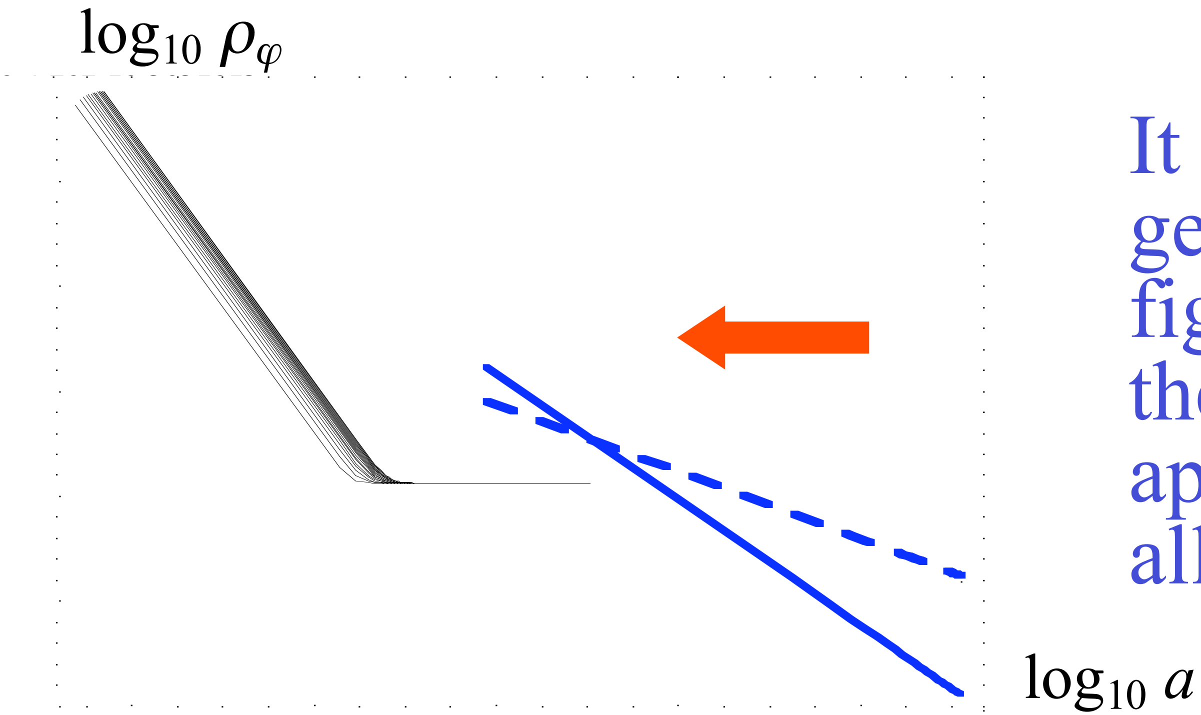


and by the CMBR spectrum



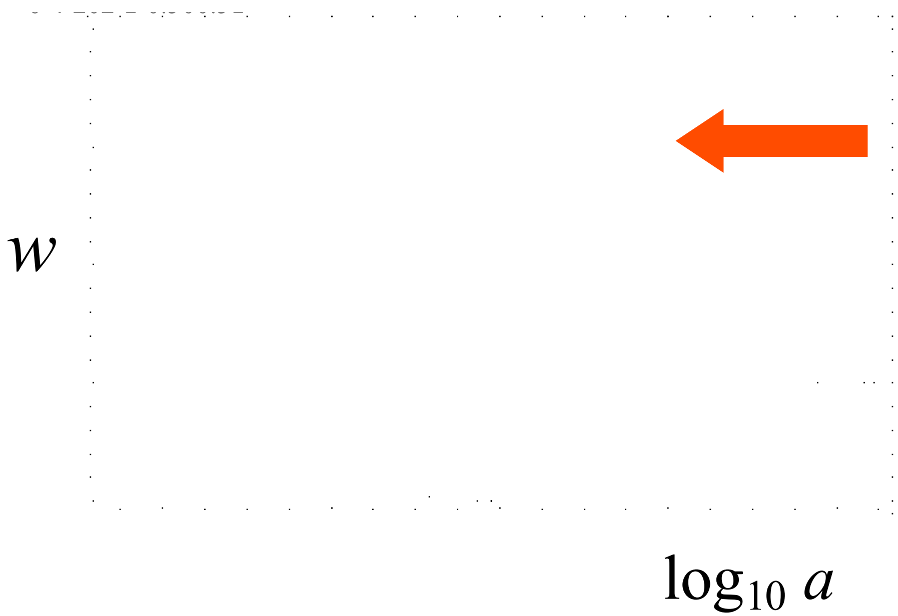


The agreement is excellent also with the new set of data (Riess et al. ApJ **607**, 665)



It is possible to generate this figure. I stress that there are no approximations at all.

The main point is that the present time is situated just in the middle of the transition.
This is better seen in the plot of w



The last figure seems to show a striking coincidence, but it is possible to show that, for any reasonable value of Ω_{0m} (0,01 - 0.99), the situation is almost unchanged

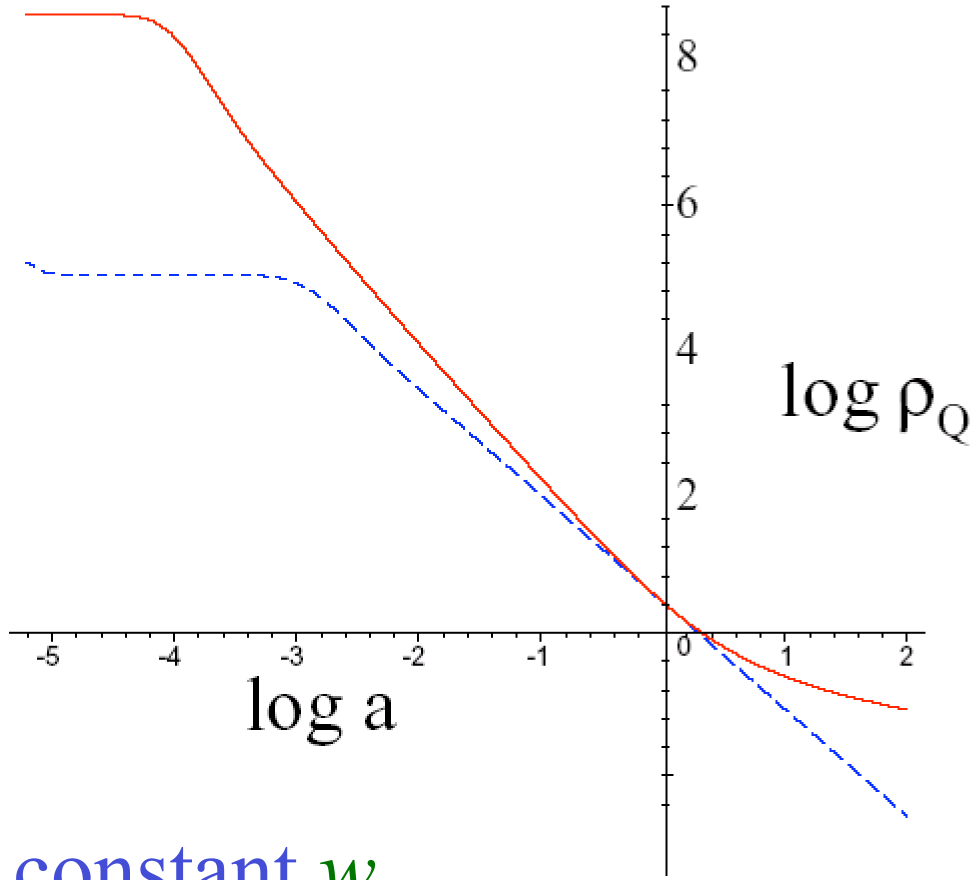
In any case, other potentials do not give better results

Let us take as an example the SUSY potential

$$V = \frac{\Lambda^{4+\alpha}}{\varphi^{\alpha}} \rightarrow w_{\infty} = -1$$

Clearly, we cannot situate the present time when $w = w_{\infty}$.

(Figure from: Bludman & Roos - PRD 65, 043503)



The red line is SUSY potential. The blue dotted is

$$V\ /\ (\mathrm{sinh}'\ )^{\text{àì}}$$

which gives an exactly constant *w*.

We see that, for SUSY, *w* stays constant for a while, around $z = 0$, and then goes to $w = -1$.

The coincidence is not avoided!

Conclusions

- The conditions for tracking are rather loose
- This helps for the fine tuning problem, but seems not useful for the cosmic coincidence
- The analysis of a general exact solutions shows that, at $z = 0$, w may be mostly variable
- An exponential potential can perfectly emulate a cosmological constant and fit all the present data
- Together with other considerations, all this suggests that there will be little hope to discriminate among the models even when much more precise data will be available