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DARK ENERGY

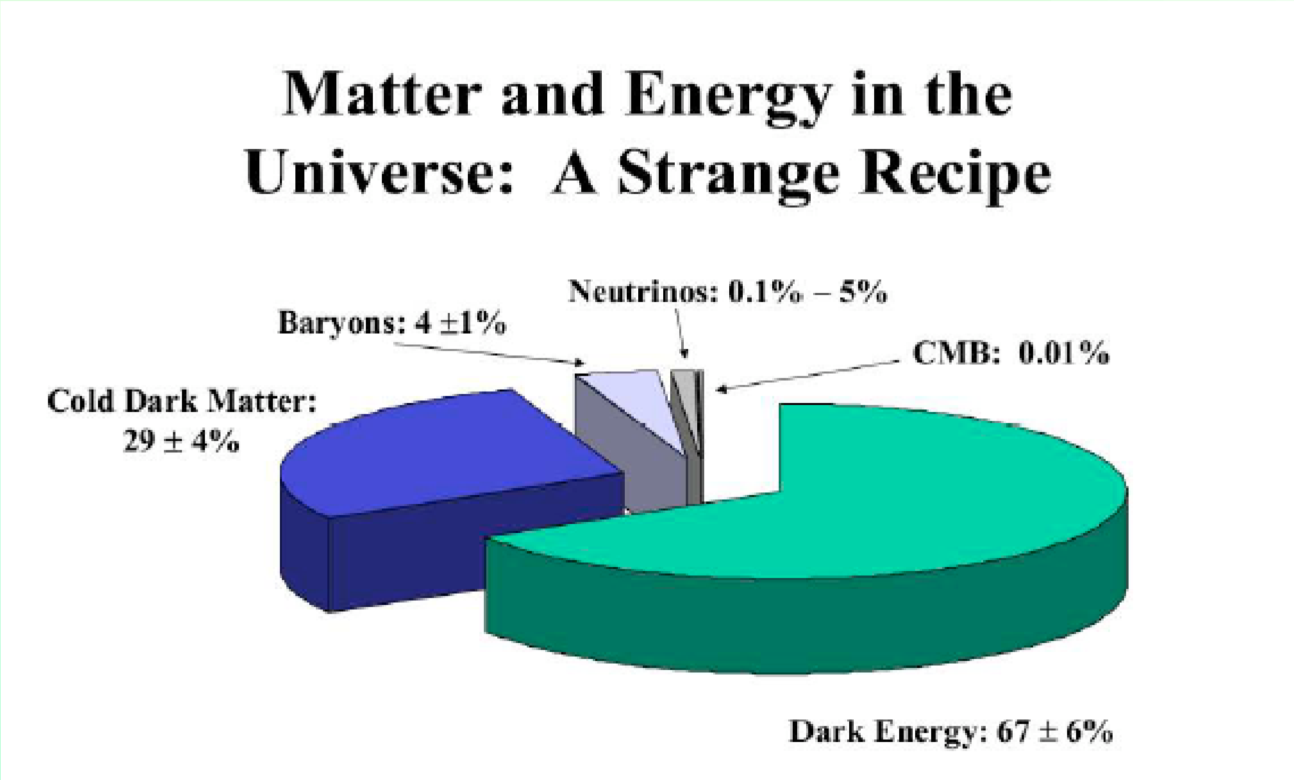
AND

THE DARK MATTER RELIC ABUNDANCE

Porto, July 9th 2004

The cosmological energy budget

W.L. Freedman & M.S. Turner (2003)



COSMOLOGY

Density parameter of component "*i*" $\quad \boxed{\Omega_i = \rho_i/\rho_c}$

Critical energy density $\quad \rho_c \equiv \dfrac{3H^2}{8\pi G}$

FRW metric $\quad ds^2 = dt^2 + a^2(t)\left[\dfrac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]; \quad k = 0, \pm 1$

Cosmological evolution

$$\begin{cases} H^2 \equiv \left(\dot{a}/a\right)^2 = \dfrac{8\pi G}{3}\displaystyle\sum_i \rho_i - \dfrac{k}{a^2} & \text{Hubble parameter} \\ q \equiv -\dfrac{1}{H^2}\dfrac{\ddot{a}}{a} = \dfrac{1}{2}\displaystyle\sum_i \Omega_i\left(1+3w_i\right) & \text{deceleration parameter} \\ \rho_i(a) = \rho_{0i}\,(a/a_0)^{-3(w_i+1)} & \end{cases}$$

Equation of state $\quad \boxed{w_i = p_i/\rho_i} \quad w_m = 0,\ w_r = 1/3,\ w_\Lambda = -1,\ w_\phi = \dfrac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)}$

Scalar field evolution $\quad \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

Scalar field Cosmology

- Runaway potentials $V(\phi) \propto \phi^{-\alpha}$, $e^{-\lambda\phi}$, $\phi^{-\alpha}e^{-\lambda\phi}$

- Attractor solutions exist for $\rho_\phi << \rho_B$ and have constant w_ϕ

$$V(\phi) = M^{4+\alpha}\phi^{-\alpha} \quad \Rightarrow \quad w_\phi = \frac{\alpha w_B - 2}{\alpha + 2} \;; \quad \Omega_\phi \text{ depends on M}$$

$$V(\phi) = M^4 e^{-\lambda\phi} \quad \Rightarrow \quad w_\phi = w_B \;; \quad \Omega_\phi = \frac{3}{\lambda^2}\left(w_B + 1\right) \quad \text{for } \lambda^2 > 3\left(w_B + 1\right)$$

- approximate attractor solutions exist also for more general potentials

  S.C.C. Ng, N. Nunes, F. Rosati (2001)

- the scalar field on the attractor remains subdominant until recent times when it overcomes the matter density

Typical quintessence Cosmology

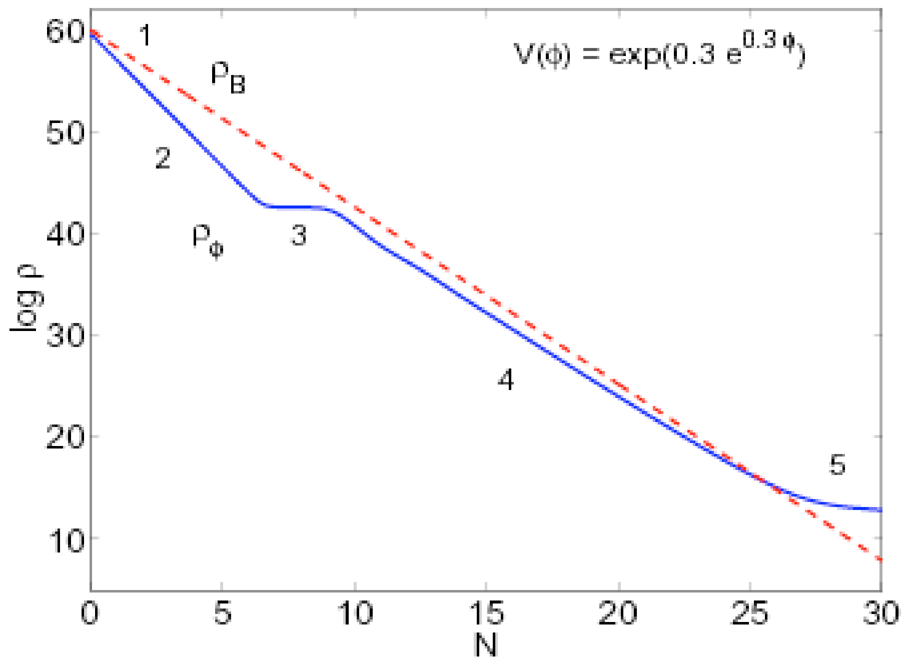


FIG. 1. Evolution of the scalar field energy density ρ_ϕ in a radiation background fluid ρ_B for a double exponential potential. Regions 2 to 5 represent respectively, kination, frozen field, evolution in the attractor and scalar field domination.

2 kination

3 frozen field

4 attractor

5 scalar field domination

S.C.C. Ng, N. Nunes, F. Rosati (2001)

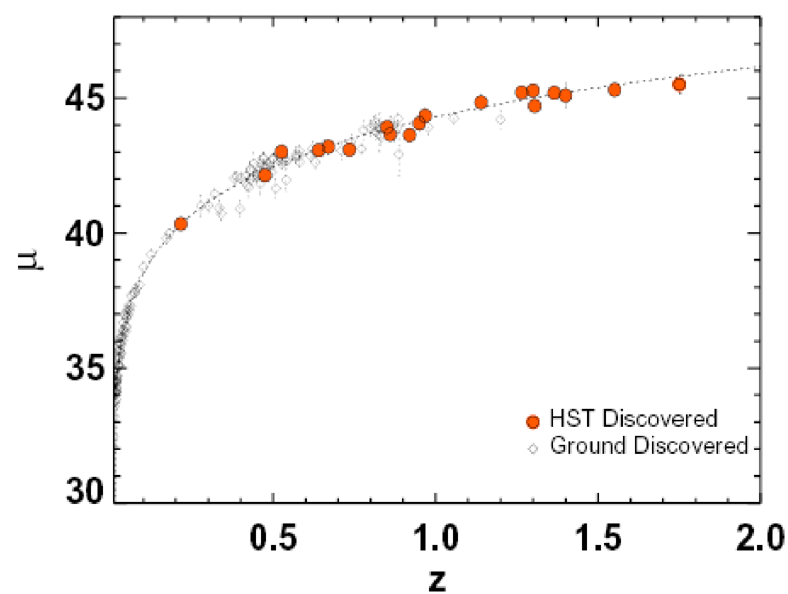
Type Ia Supernova Discoveries at $z > 1$ From the *Hubble Space Telescope:* Evidence for Past Deceleration and Constraints on Dark Energy Evolution[1]

To Appear in the Astrophysical Journal, June 2004

Adam G. Riess[2], Louis-Gregory Strolger[2], John Tonry[3], Stefano Casertano[2], Henry C. Ferguson[2], Bahram Mobasher[2], Peter Challis[4], Alexei V. Filippenko[5], Saurabh Jha[5], Weidong Li[5], Ryan Chornock[5], Robert P. Kirshner[4], Bruno Leibundgut[6], Mark Dickinson[2], Mario Livio[2], Mauro Giavalisco[2], Charles C. Steidel[7], Narciso Benitez[8] and Zlatan Tsvetanov[8]

the Hubble diagram luminosity-redshift gives a measure of the expansion rate H and of the acceleration/deceleration parameter q of the Universe

$$d_L \approx \frac{cz}{H_0}\left[1+\frac{1}{2}(1-q_0)z+O(z^2)\right]$$



$$\mu_0 = m - M = 5\log d_L + 25$$

Couplings of the quintessence scalar ⊔ with
other fields in the Universe is strongly suppressed
by experiments on time variation of physical constants
and equivalence principle violations.

BUT EVEN RESPECTING THESE CONSTRAINTS
THE QUINTESSENCE SCALAR CAN PRODUCE
IMPORTANT AND MEASURABLE EFFECTS
ON OTHER COSMOLOGICAL OBSERVABLES
AND IN PARTICULAR
ON THE **DARK MATTER RELIC ABUNDANCE**

KEY IDEA: the presence of a quintessence scalar can have a sizeable effect on those phenomena which have a strong dependence on the Hubble parameter H

RECALL: the earliest constraint we have on H is at the time of BBN

$$\Rightarrow \quad \rho_\phi/\rho_\gamma \lesssim 0.1 \quad \text{at} \quad T \simeq 1\, MeV, z \simeq 10^9$$

TWO WAYS

by which **the mere contribution of the scalar field energy density** to the Hubble parameter can modify the expansion rate in order to produce an enhanced dark matter relic abundance

modify gravity

$$H^2 = \frac{8\pi G}{3}\left[\rho_m + \rho_r + \rho_\phi\right]$$

sizeable scalar contribution (kination)

Salati (2002); Rosati (2002);
Profumo & Ullio (2003);
Catena, Fornengo, Masiero, Petroni, Rosati (2004)

"KINATION" ENHANCEMENT

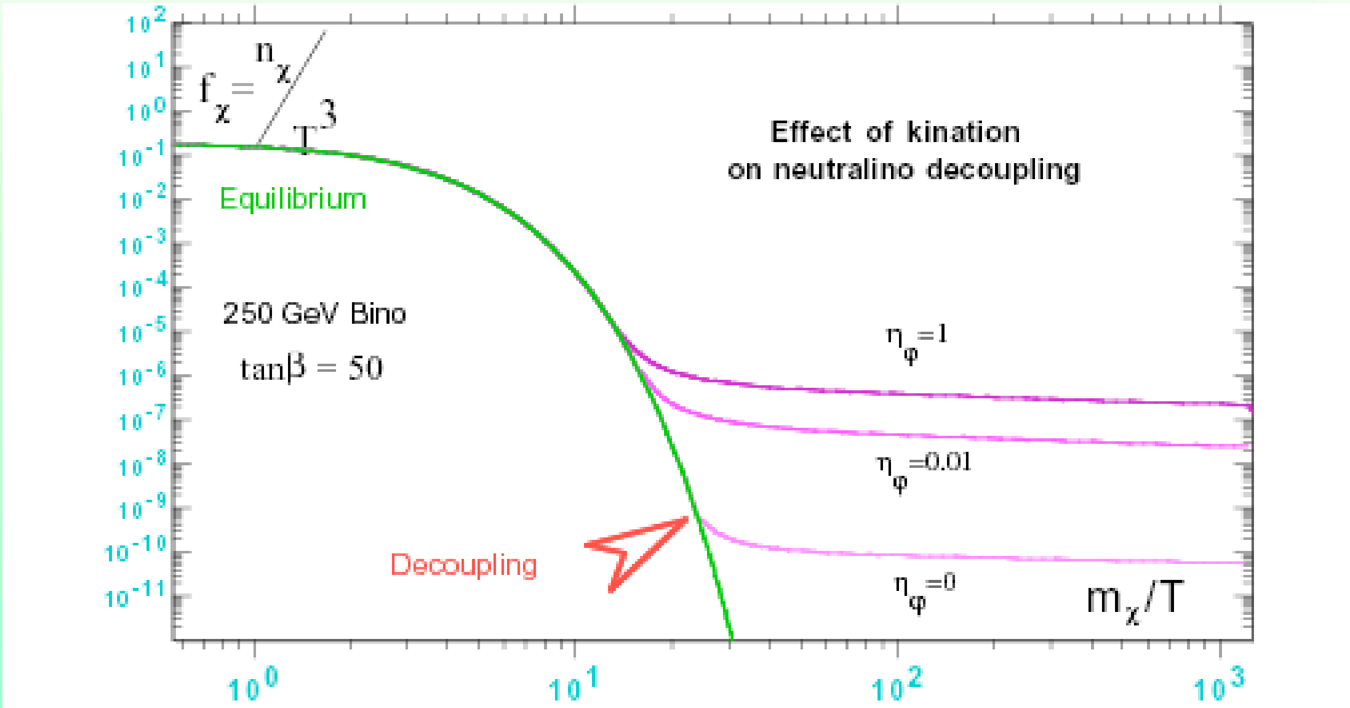
Salati (2002); Rosati (2003); Profumo & Ullio (2003)

If in the very early stages of its evolution, the quintessence scalar undergoes a period of "kination", its energy density would temporarily be of the same order or even larger than the radiation energy density before rapidly becoming subdominant

$$\rho_\phi = \frac{\dot\phi^2}{2} + V(\phi) = \begin{cases} \dot\phi^2/2 \propto a^{-6} & \text{kination} \\ \dfrac{2}{1-w_\phi} V(\phi) & \text{attractor} \\ V(\phi) \simeq const. & \text{freezing} \end{cases} \qquad w_\phi = \frac{\dot\phi^2/2 - V(\phi)}{\dot\phi^2/2 + V(\phi)} = \begin{cases} +1 & \text{kination} \\ \simeq -1/2 & \text{attractor} \\ -1 & \text{freezing} \end{cases}$$

The direct effect of this early kination phase is to enhance the expansion rate H at the time of neutralino decoupling, causing an earlier freeze-out and hence a larger relic abundance

Salati (2002)



$$\eta_\Phi = \frac{\rho_\Phi^0}{\rho_\gamma^0}.$$

the BBN bound requires $\eta_\Phi \equiv \frac{\rho_\Phi}{\rho_{rad}} \lesssim 0.1$

Rosati (2003)

BUT, a scalar field which has dominant initial conditions, does not provide a good quintessence candidate, since it would not be able to reach the attractor before the present epoch

REMEDY:

we can modify the standard quintessence scenario introducing more scalars or an interaction term in the scalar potential

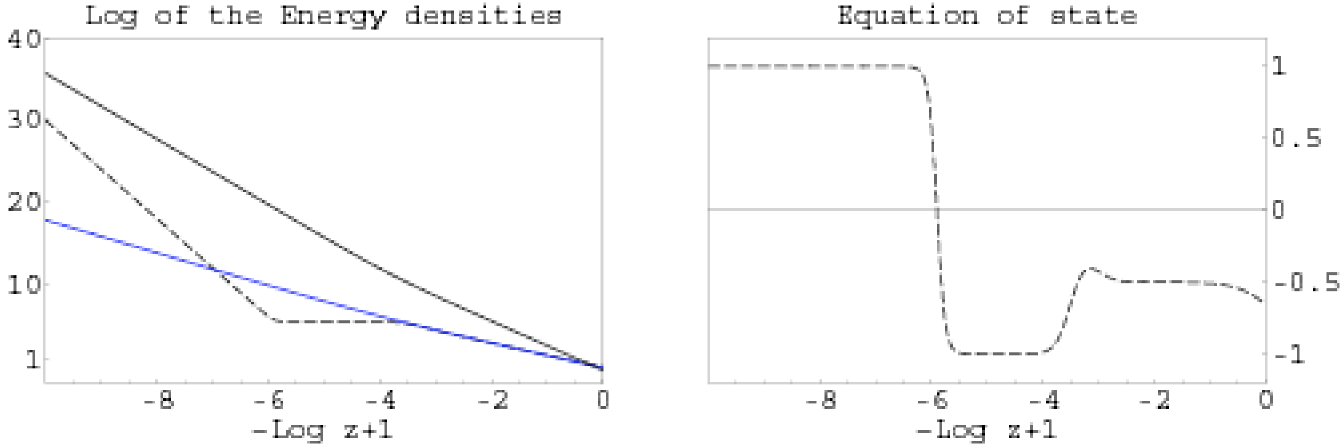


Figure 1: The figures show the typical evolution of the relevant energy densities (w.r.t. the present critical energy density $\rho_c^o \simeq 10^{-47} GeV^4$) and of the scalar equation of state, for a cosmological scalar field with potential $V \sim \phi^{-2}$. The black curve corresponds to the background (radiation plus matter) and the blue curve to the attractor. The dashed lines show the scalar energy density and equation of state: it can be easily seen that after an initial stage of 'kination' ($w_\phi = 1$), the field is 'freezing' ($w_\phi = -1$) and subsequently joins the attractor until it overtakes the background energy density. On the attractor the scalar equation of state is $w_\phi = -1/2$.

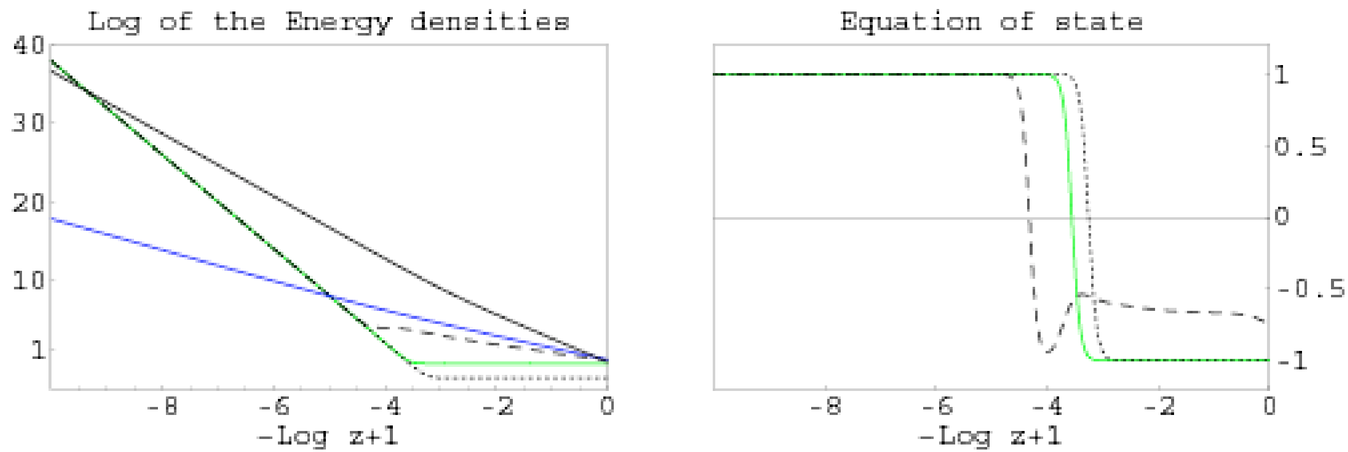


Figure 2: The figures show the evolution of the relevant energy densities (w.r.t. the present critical energy density $\rho_c^o \simeq 10^{-47} GeV^4$) and of the scalar equation of state, depending on the initial conditions, in the case of two scalar fields ϕ_1 and ϕ_2 with potential $V \sim (\phi_1\phi_2)^{-1}$. The black curve corresponds to the background (radiation plus matter) and the blue curve to the attractor. The green line is the case in which the two fields start with the same initial conditions with a total energy density corresponding to the overshooting case. If we vary the fields' values at the beginning (keeping the total energy density fixed), we can obtain two situations: if both the fields are still $\ll 1$ at the beginning (dashed line) then the attractor is reached in advance w.r.t the equal fields' case; if instead one of the fields is > 1 (dotted line), then the attractor is reached later. In the examples shown, at $z = 10^{10}$ we have $\rho_\phi = 10^{38}$ and for the fields: $\phi_1 = \phi_2 = 10^{-19}$ (green); $\phi_1 = 10^{-16}$, $\phi_2 = 10^{-22}$ (dashed) and $\phi_1 = 100$, $\phi_2 = 10^{-40}$ (dotted).

$$V(\phi_1,\phi_2) = M^{n+4}\,(\phi_1 \cdot \phi_2)^{-n/2}$$

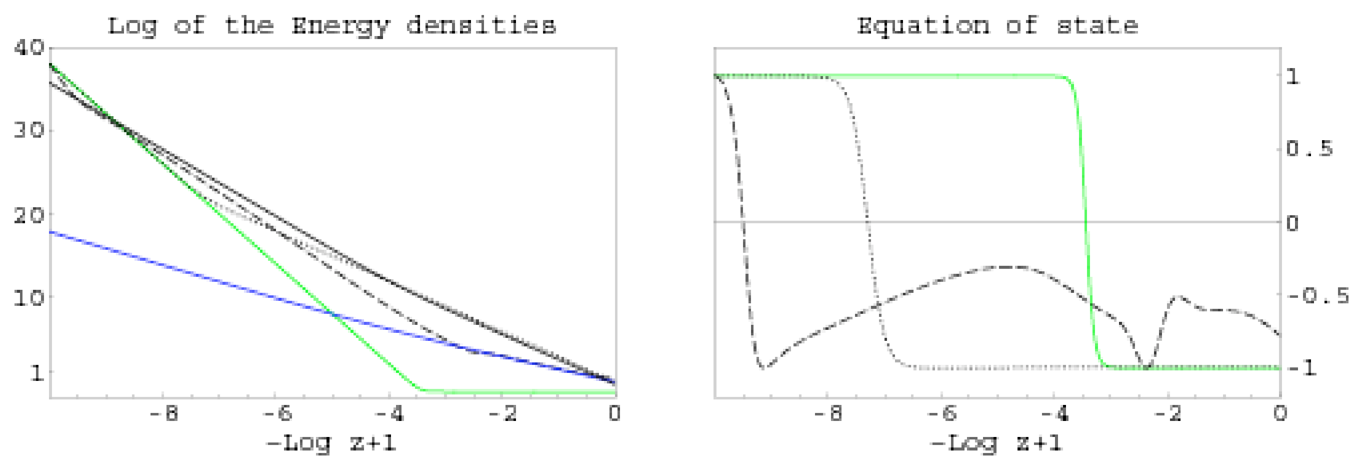


Figure 3: The figures show the evolution of the relevant energy densities (w.r.t. the present critical energy density $\rho_c^o \simeq 10^{-47} GeV^4$) and of the scalar equation of state, depending on the interaction of the Quintessence field with the dark matter fields. The black curve corresponds to the background (radiation plus matter) and the blue curve to the attractor. The green line is the Quintessence field evolution in the overshooting case ($\rho_\phi = 10^{38}$ at $z = 10^{10}$), with potential $V \sim \phi^{-2}$ and no interaction. If we switch on an additional term in the potential, the cosmological evolution will change correspondingly. The dashed line shows the case of a coupling $V_b = \frac{1}{2} b H^2 \phi^2$ with $b = 0.25$; the dotted line shows the case of a coupling $V_c = c\rho_m\phi$ with $c = 0.5$. Please note that the values chosen for b and c in the figure are purely illustrative. Coupling constants two orders of magnitude smaller than the ones considered here are sufficient to ensure the desired effect.

$$V_b = \frac{b}{2} H^2 \phi^2 \quad , \quad V_c = c\rho_m\phi$$

“SCALAR-TENSOR” ENHANCEMENT

Catena, Fornengo, Masiero, Pietroni, Rosati (2004)

Scalar-tensor theories of gravity

- provide a natural scalar candidate for the quintessence field
- respect the equivalence principle by construction
- can benefit from a double attractor mechanism that drives the scalar $\sqcup$ to a tracking regime and gravity towards general relativity

Basic equations

In the Einstein frame:

$$S_g = M^2 \int d^4x \sqrt{-g} \left[-R + g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{2}{M^2} V(\phi) \right], \quad S_m = S_m\left[\psi_m, A^2(\phi) g_{\mu\nu} \right]$$

and we define: $\alpha(\phi) = \dfrac{d \log A(\phi)}{d\phi}$

The scalar evolution equation gets a source term :

$$\ddot{\phi} + 3H\dot{\phi} + \frac{1}{M^2} V'(\phi) = -\frac{\alpha(\phi)}{M^2 \sqrt{2}} (\rho - 3p)$$

The effect of the early presence of a scalar field comes through the Jordan-frame Hubble parameter $\tilde{H} = H \dfrac{1 + \alpha(\phi)\phi'}{A(\phi)}$

Will choose:

$$A(\varphi) = 1 + B\, e^{-\beta\varphi} \quad \Rightarrow \quad \alpha(\varphi) = -\frac{\beta B\, e^{-\beta\varphi}}{1 + B\, e^{-\beta\varphi}}$$

and for the scalar potential $V(\varphi) = \Lambda^4\, \varphi^{-\alpha}$

With the constraint:

BBN $\frac{A(\varphi_{BBN})}{A(\varphi_0)} < 1.08$

also considered GR tests (Cassini) and CMB

We have considered the maximal allowed value of $\tilde{H}$ in the early Universe, given the avaliable more recent constraints

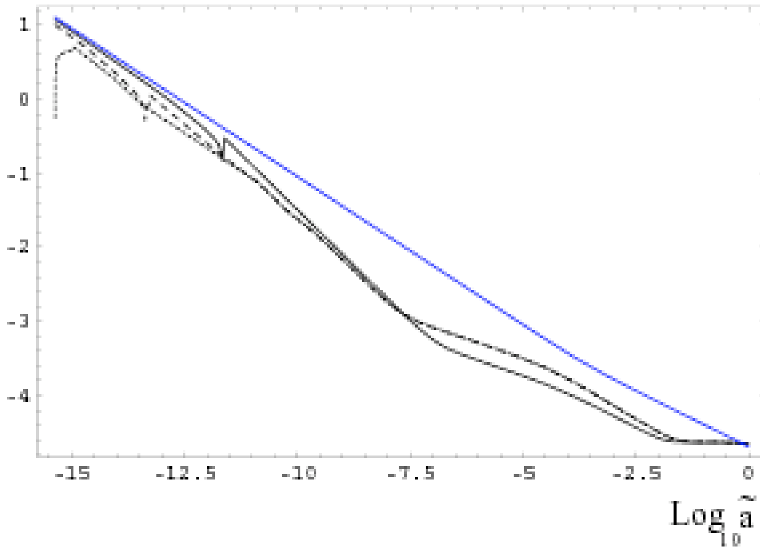


FIG. 2. Evolution of the energy density of the background (upper solid line) and of three typical solutions for the scalar field. We see that different initial conditions converge to the same solution.

$\bar{\square} = \square^4 / \square_m^0$

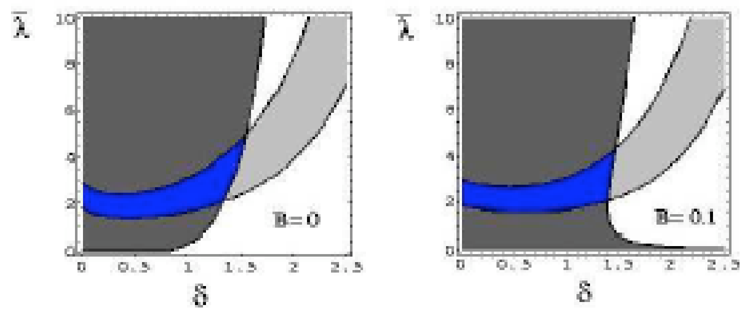


FIG. 3. The regions in the $\bar{\lambda}$-δ parameter plane giving $w_\varphi < -0.7$ (dark grey) and $0.65 < \Omega_\varphi < 0.75$ (light grey). The left plot is the pure GR case ($B = 0$) while the right one is for ST with $B = 0.1$, $\beta = 8$.

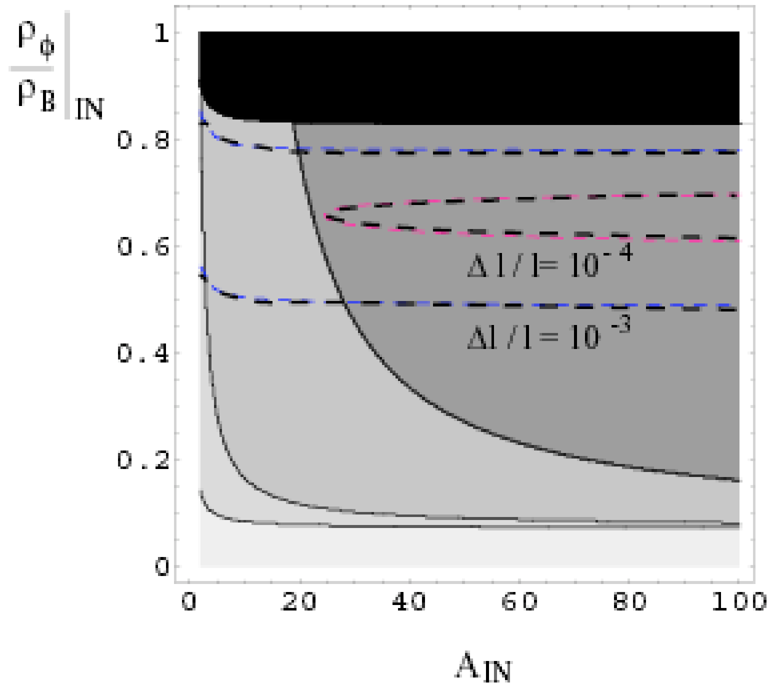


FIG. 5. The contours show the expansion rate enhancement $\tilde{H}/\tilde{H}_{GR}$ at $T = 10$ GeV obtained in the ST model, as a function of the initial values of the factor $A(\varphi)$ and of the ratio of the scalar to background energy density ρ_φ/ρ_B. We considered for the initial conditions a temperature of $T = 500$ GeV. The black area represents initial conditions which are excluded by nuclesynthesis. The grey contours represent enhancements of $1 \div 10^2$, $10^2 \div 10^3$, $10^3 \div 10^4$, $10^4 \div 10^5$ from the lightest to the darkest. The dashed lines show the shifts of the CMB doppler peaks obtained in the ST model.

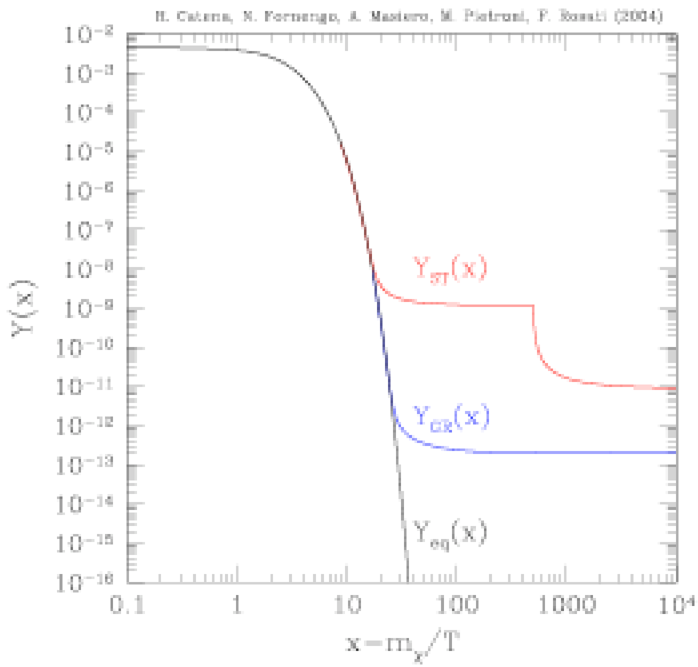


FIG. 7. Numerical solution of the Boltzmann equation Eq. (31) in a ST cosmology for a toy–model of a DM WIMP of mass $m = 50$ GeV and constant annihilation cross-section $\langle \sigma_{\rm ann} v \rangle = 1 \times 10^{-7}$ GeV^{-2}. The temperature evolution of the WIMP abundance $Y(x)$ clearly shows that freeze–out is anticipated, since the expansion rate of the Universe is largely enchaced by the presence of the scalar field φ. At a value $x = m/T_\varphi$ a re–annihilation phase occurs and $Y(x)$ drops to the present day value.

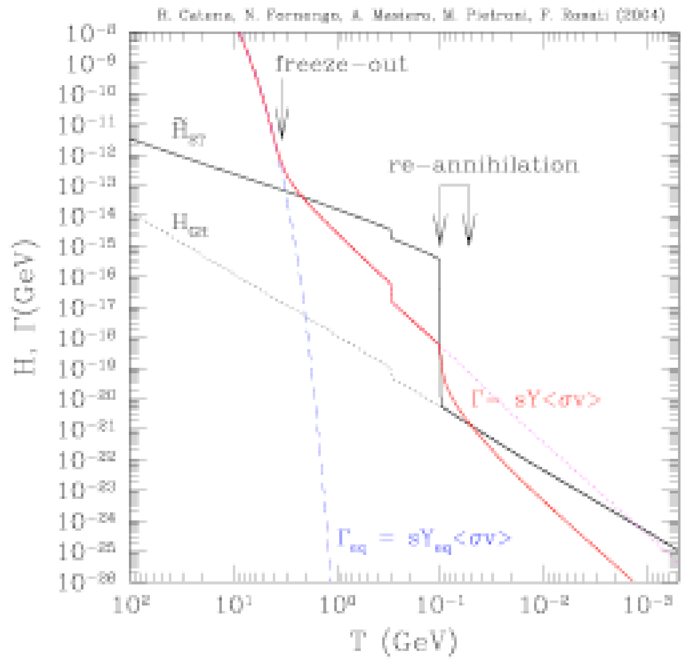


FIG. 8. The Expansion rate of the Universe $\tilde{H}$ and the WIMP interaction rate $\Gamma = Y\, s\, \langle\sigma_{\rm ann} v\rangle$ are plotted as a function of the temperature. The re-annihilation effect discussed in the text is outlined. The small drop in the rates at $T = 300$ MeV is due to the quark–hadron phase transition.

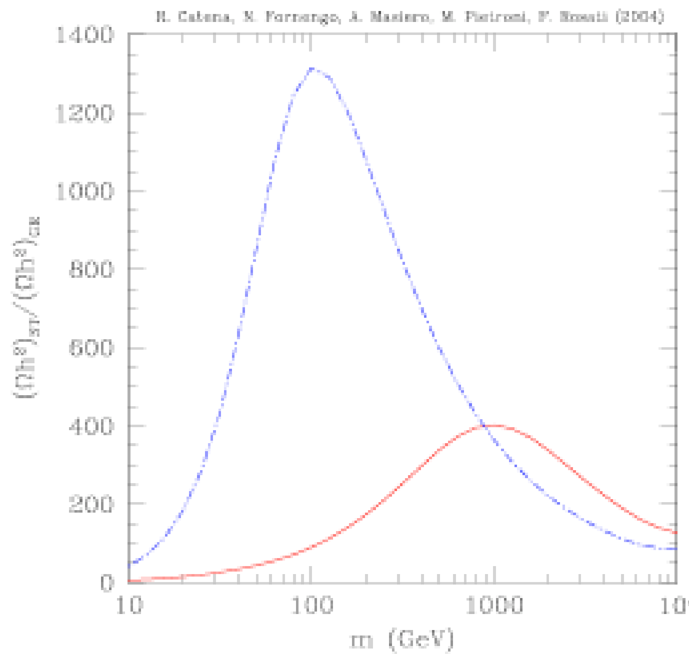


FIG. 10. Increase in the WIMP relic abundance in ST cosmology with respect to the GR case. The solid curve refers to an annihilation cross section constant in temperature, i.e. $\langle\sigma_{\rm ann}v\rangle = a = 10^{-7}$ GeV^{-2}, while the dashed line stands for an annihilation cross section which evolves with temperature as $\langle\sigma_{\rm ann}v\rangle = b/x = 10^{-7}$ GeV$^{-2}/x$.

CONCLUSIONS

Dark Energy is there

and

if in the form of a cosmological scalar

could have a measurable impact

on the DM relic abundance