Quantum Cosmology with a Chaplygin Gas

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- *A* Physical Guide: solutions and quantization

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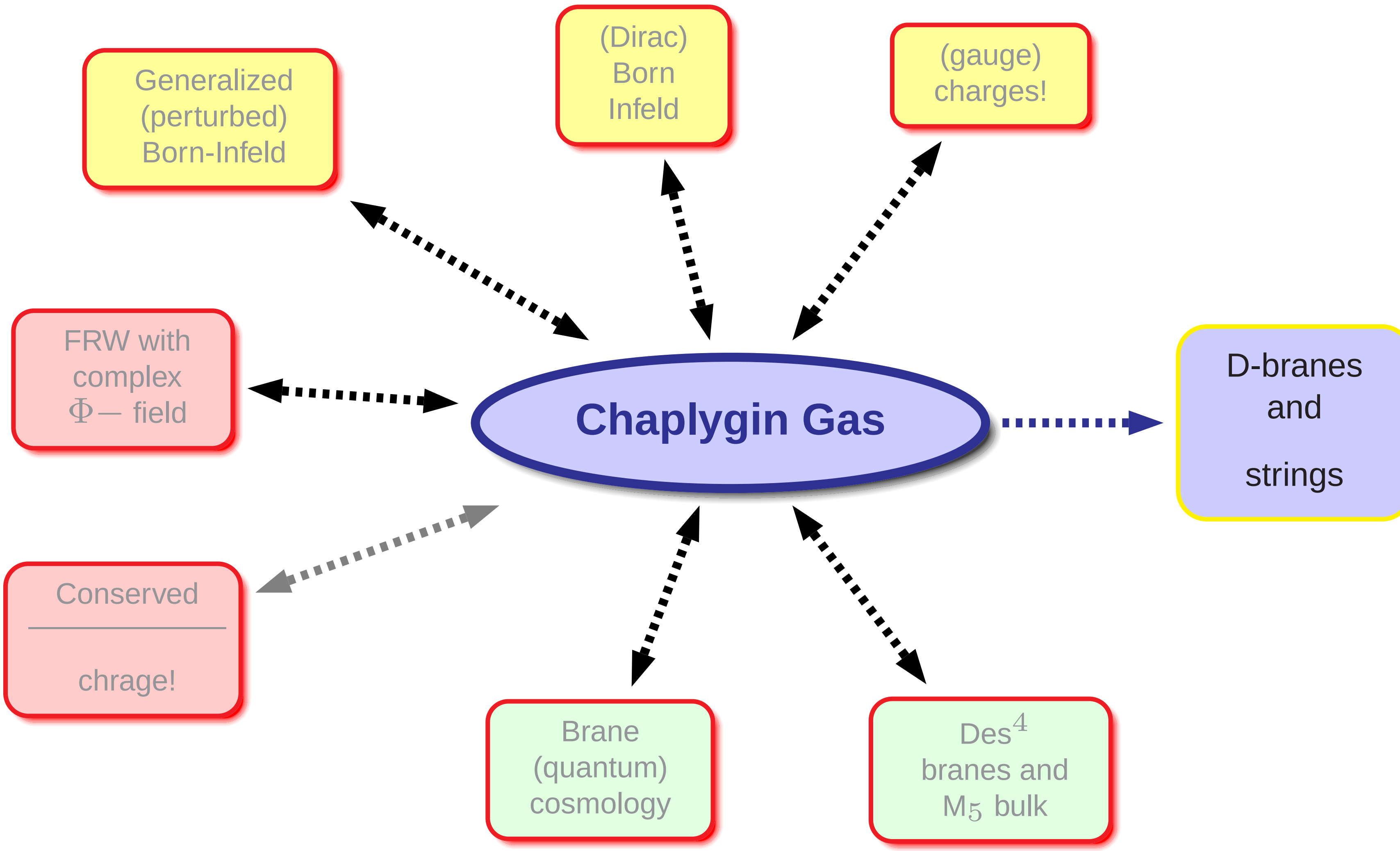
- The theory $\longleftarrow$ from Strings/Branes
- *Mise em Scene*: Chaplygin gas cosmology
- *A* Physical Guide: solutions and quantization
- *Finale* ... well, not yet ...

... and framework:



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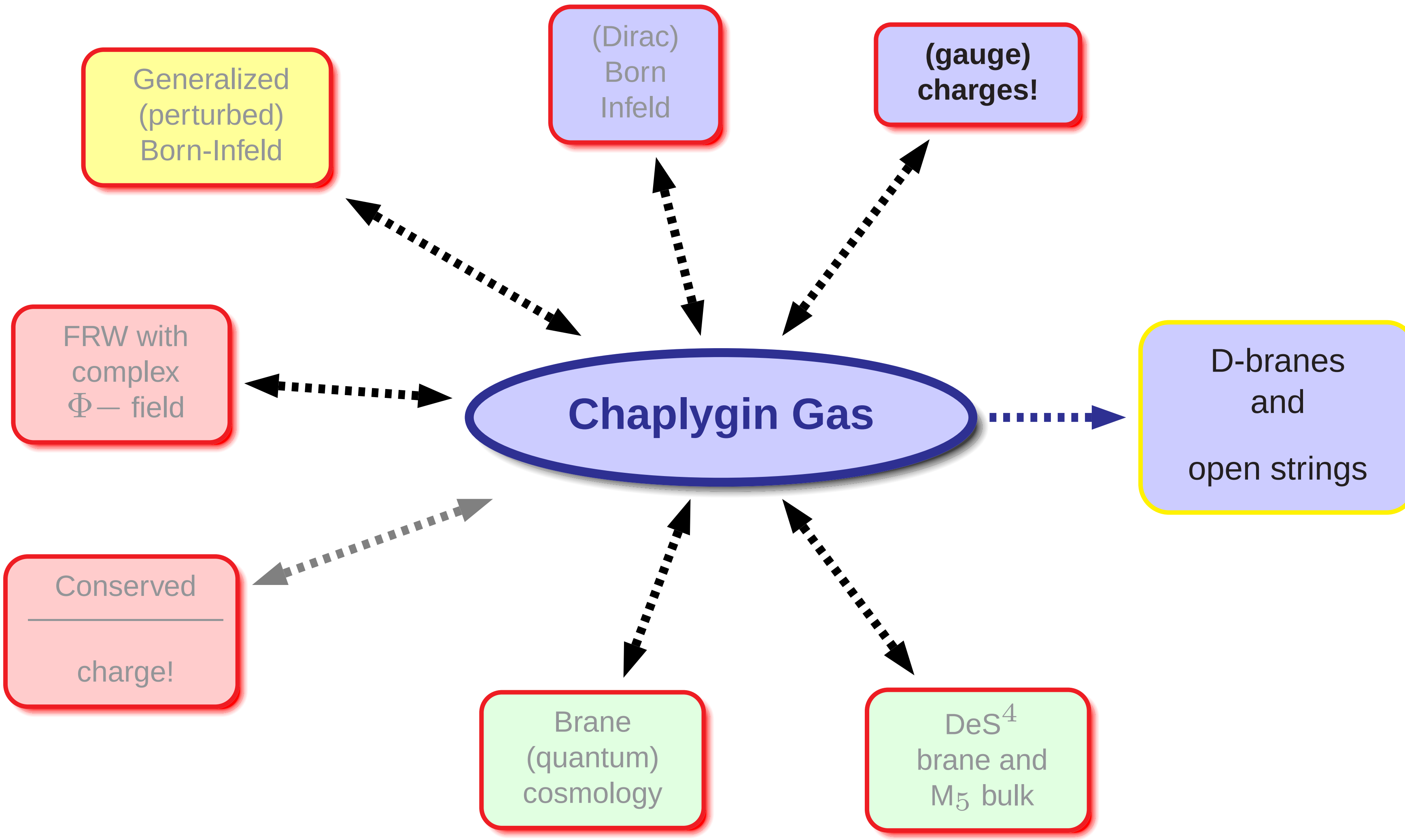

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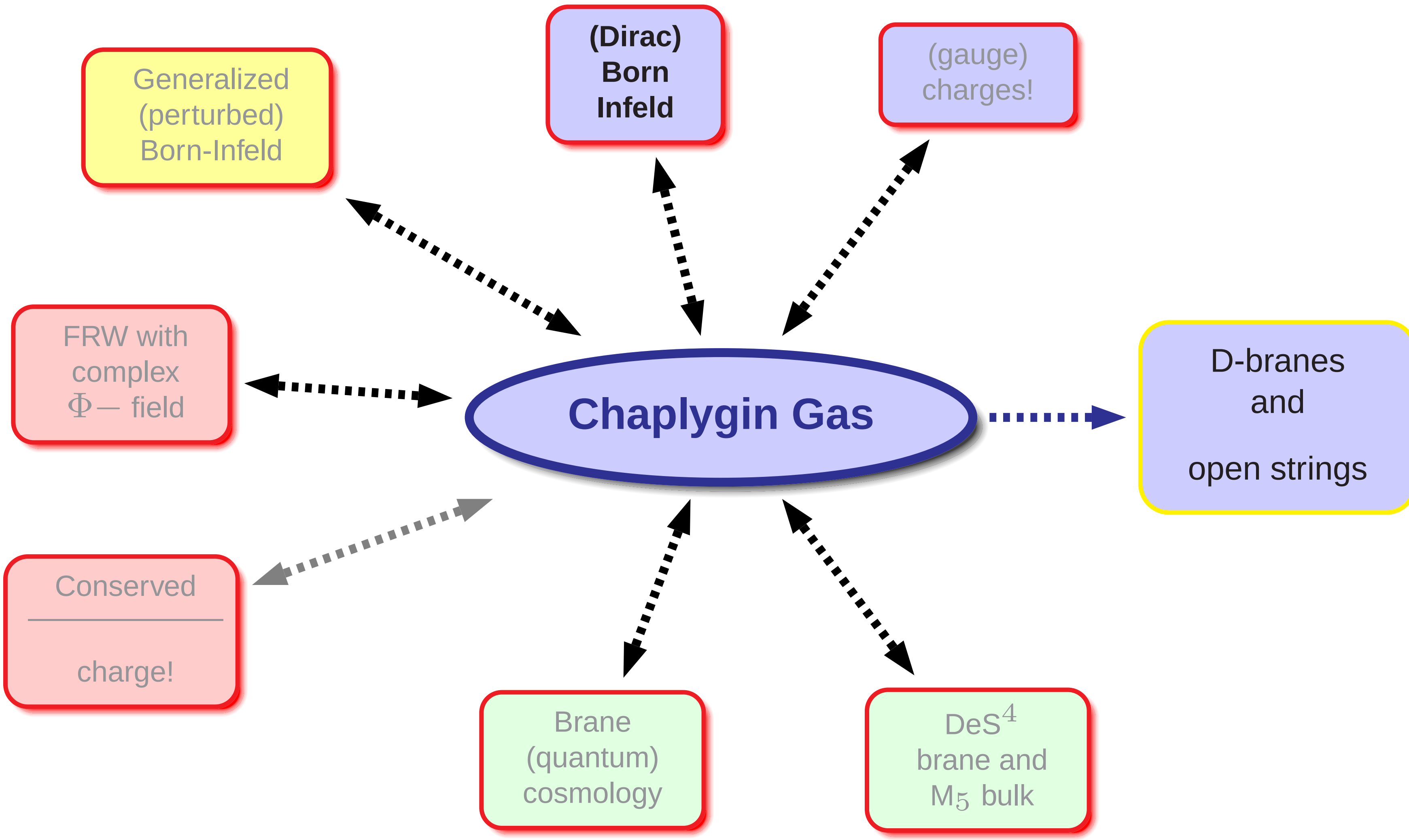

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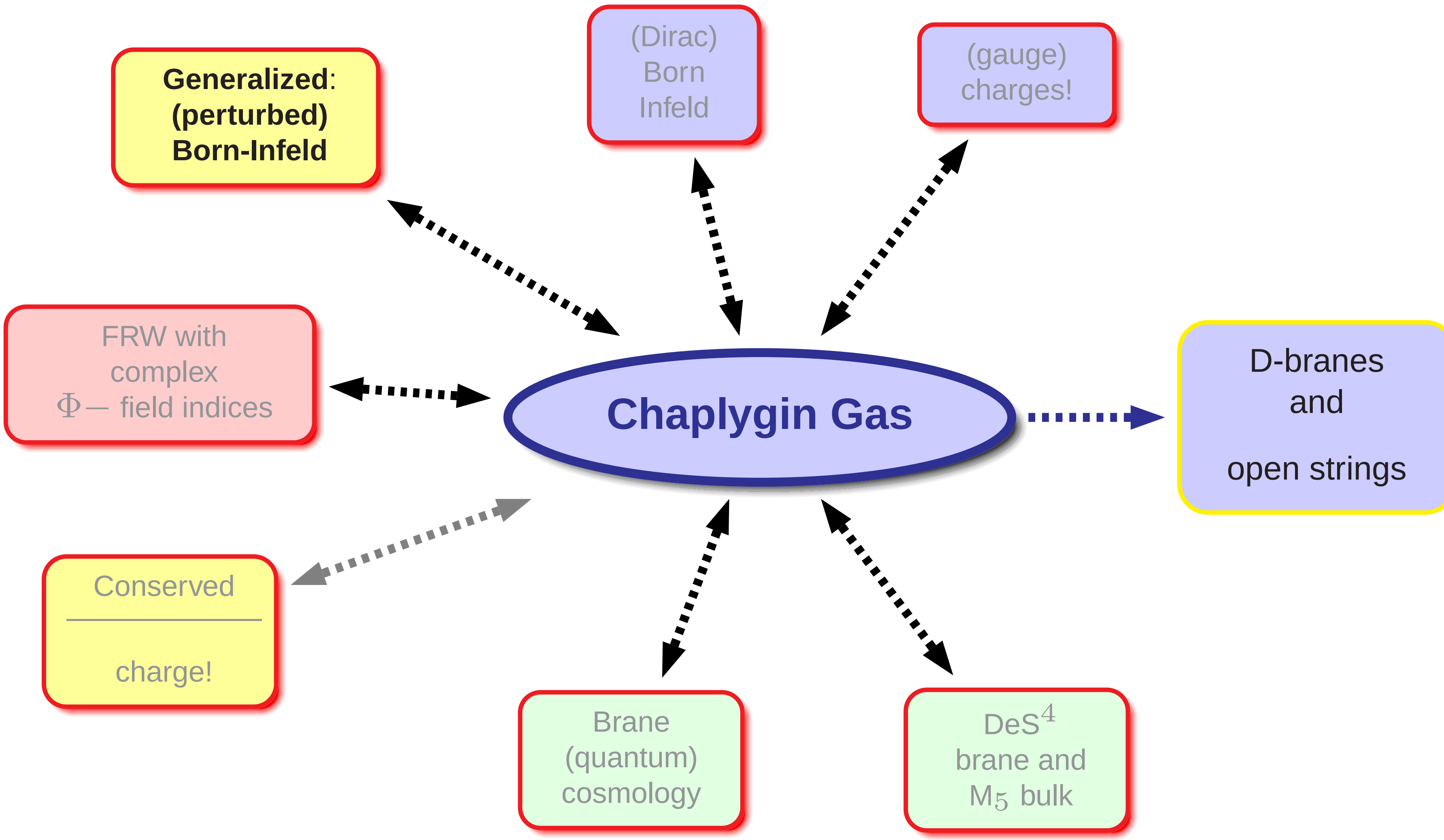

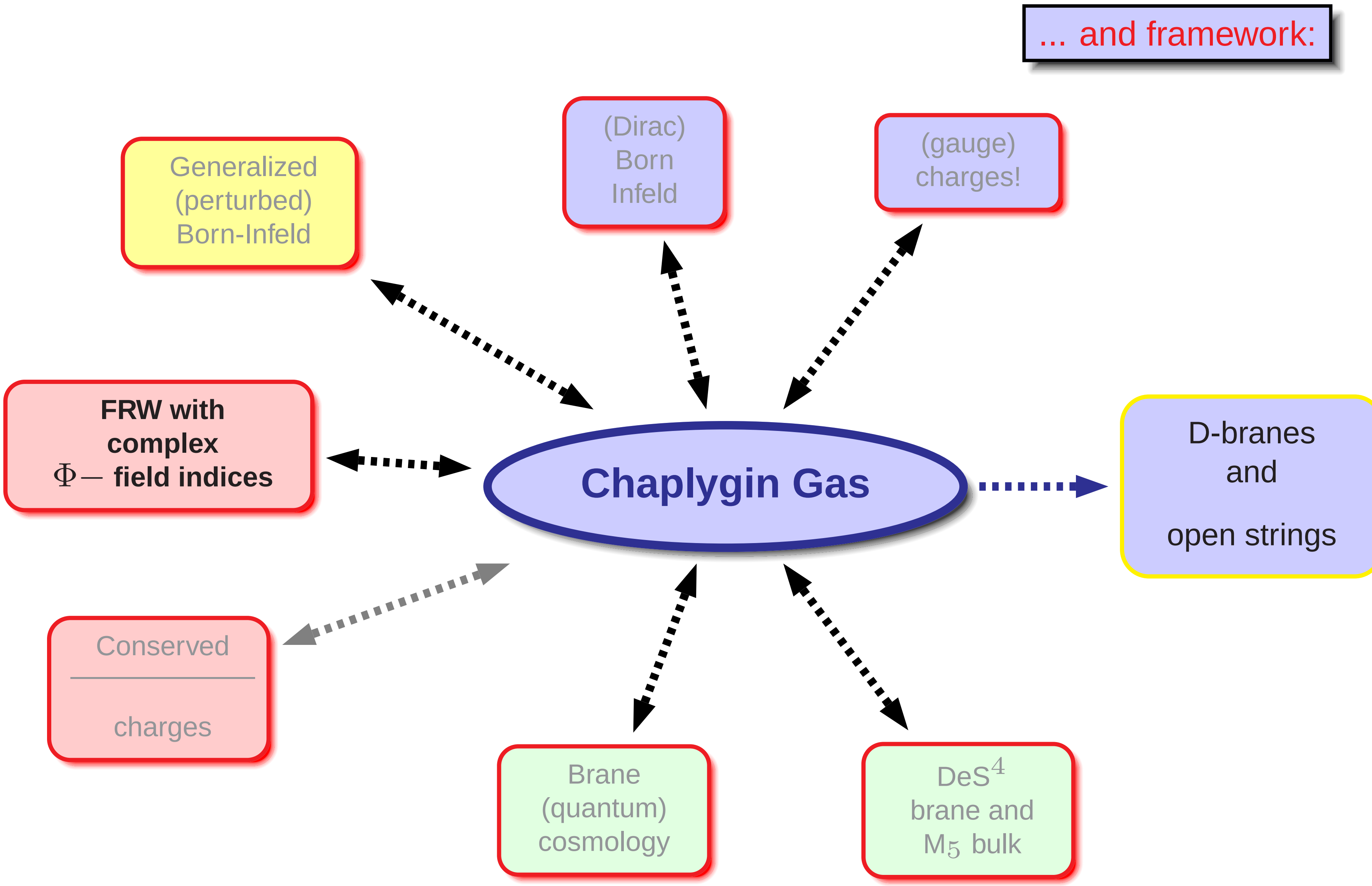
... and framework:
Generalized
(perturbed)
Born-Infeld
(Dirac)
Born
Infeld
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charges!
FRW with
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Chaplygin Gas
D-branes
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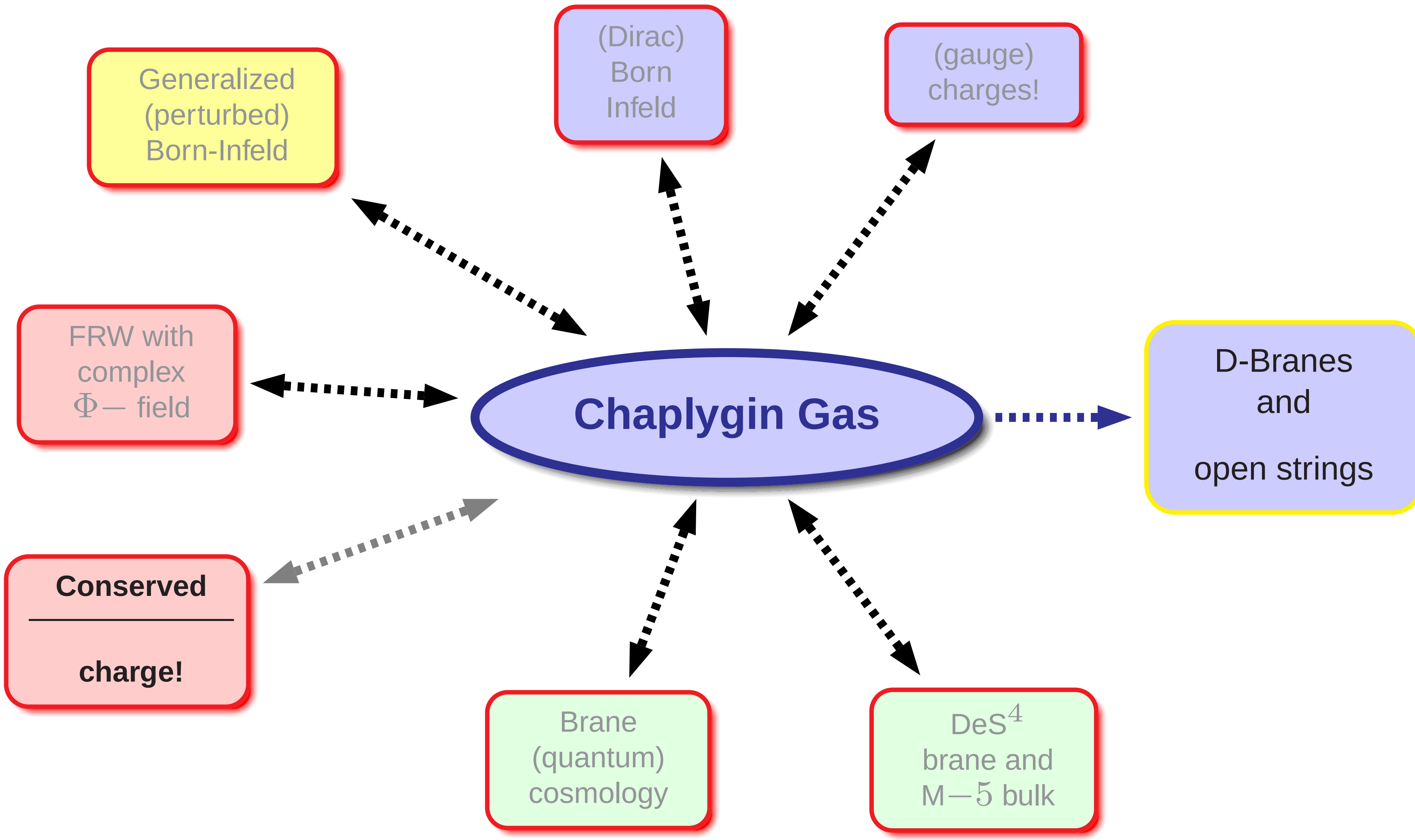

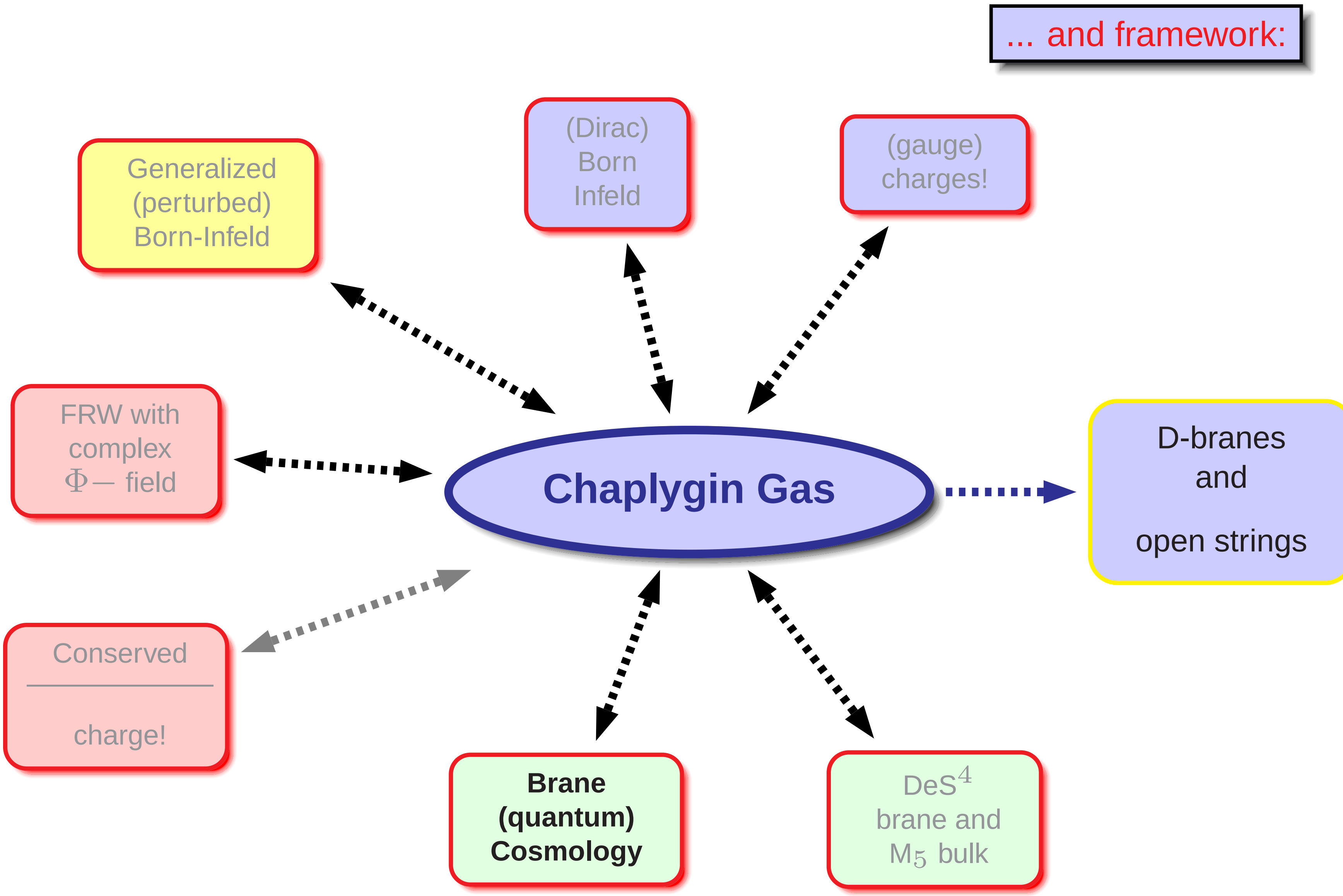
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Generalized (perturbed) Born-Infeld
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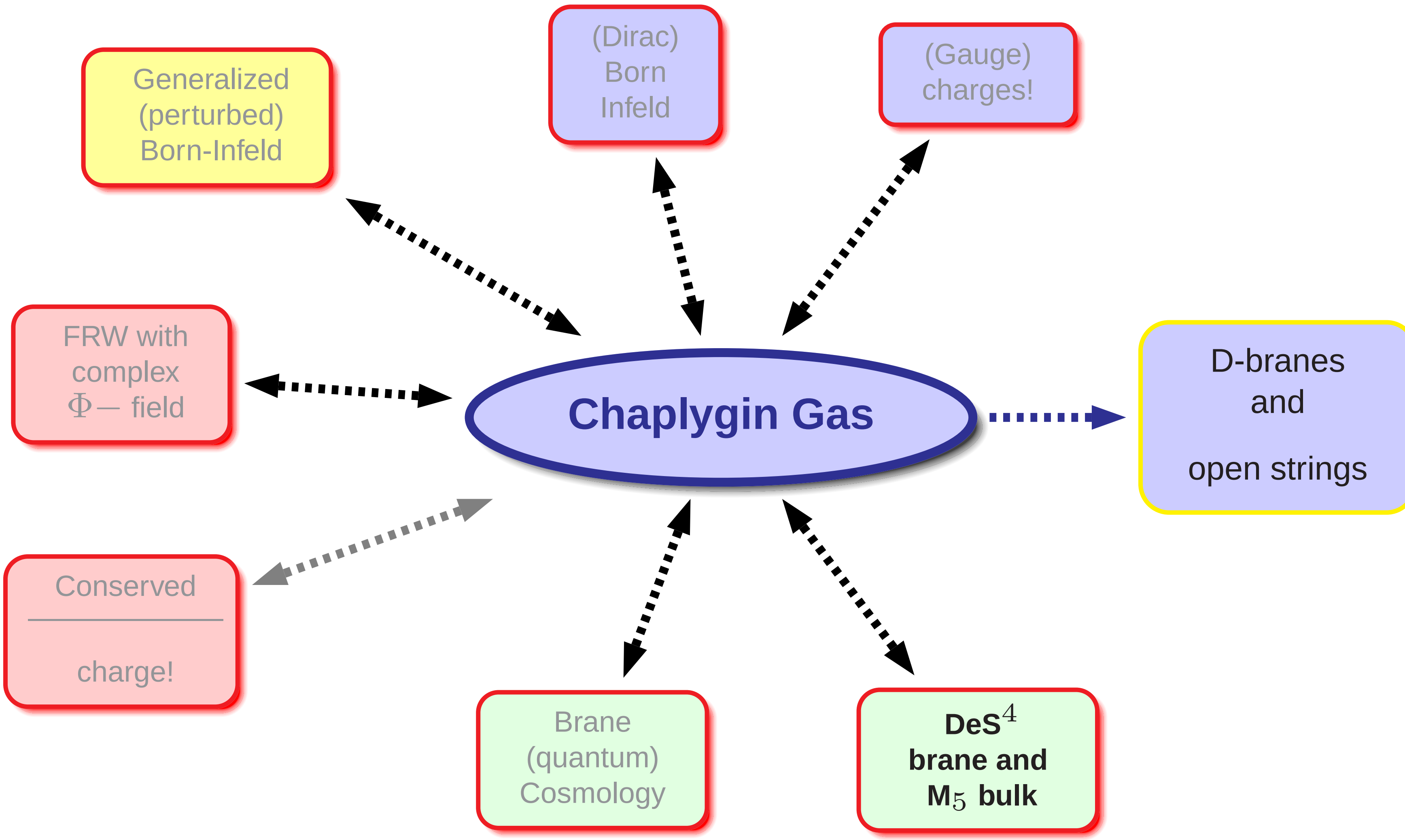

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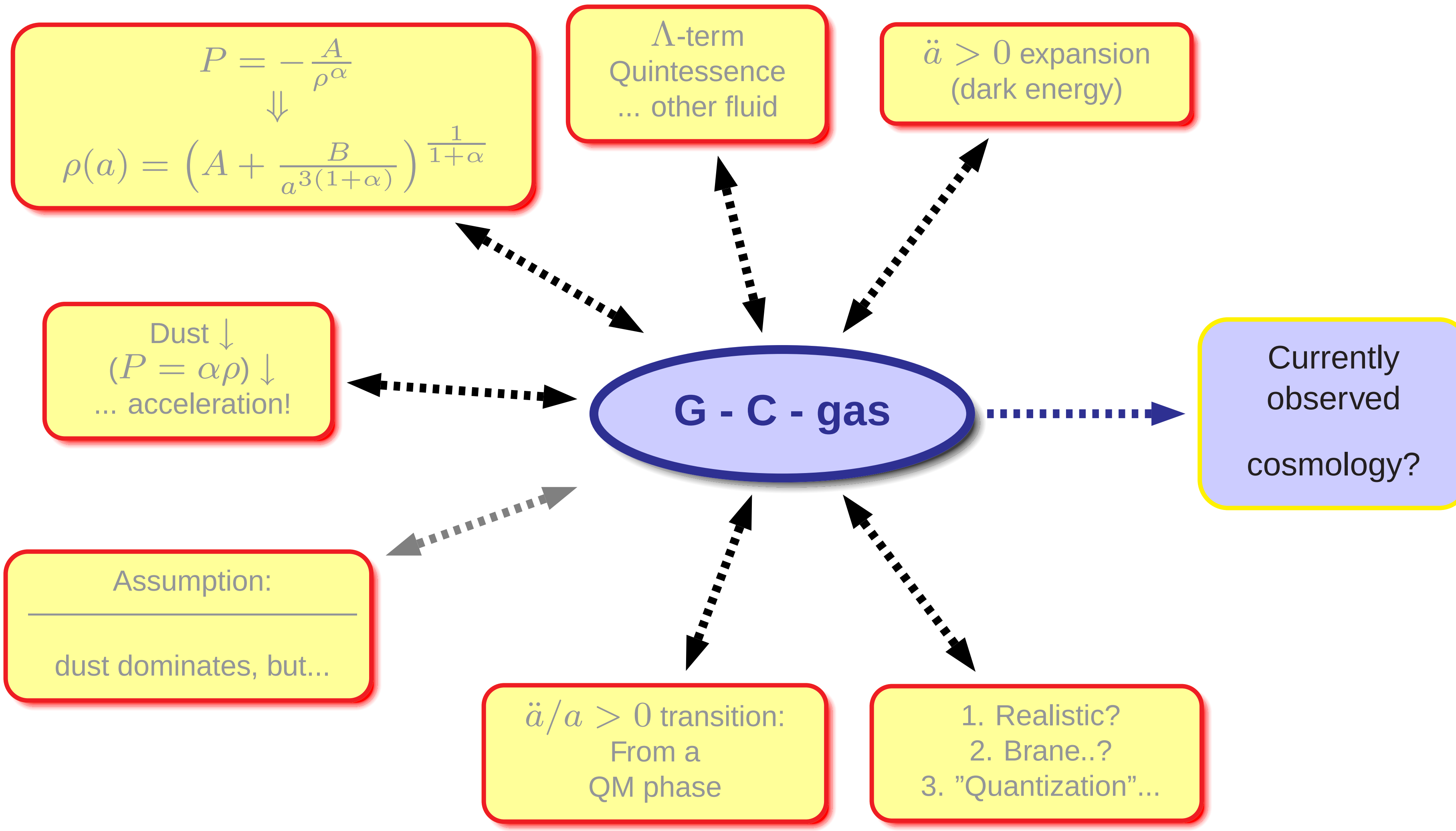

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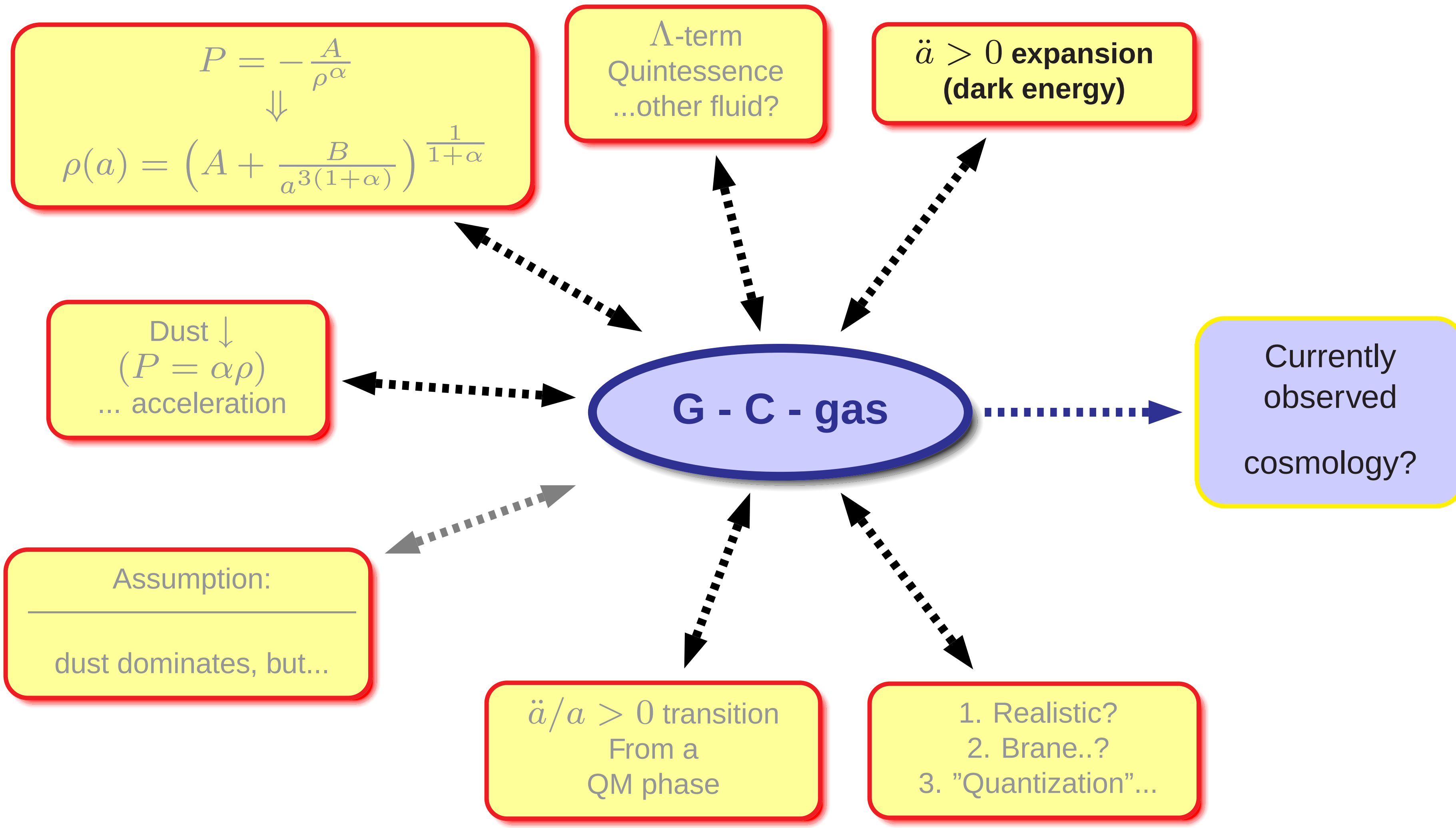

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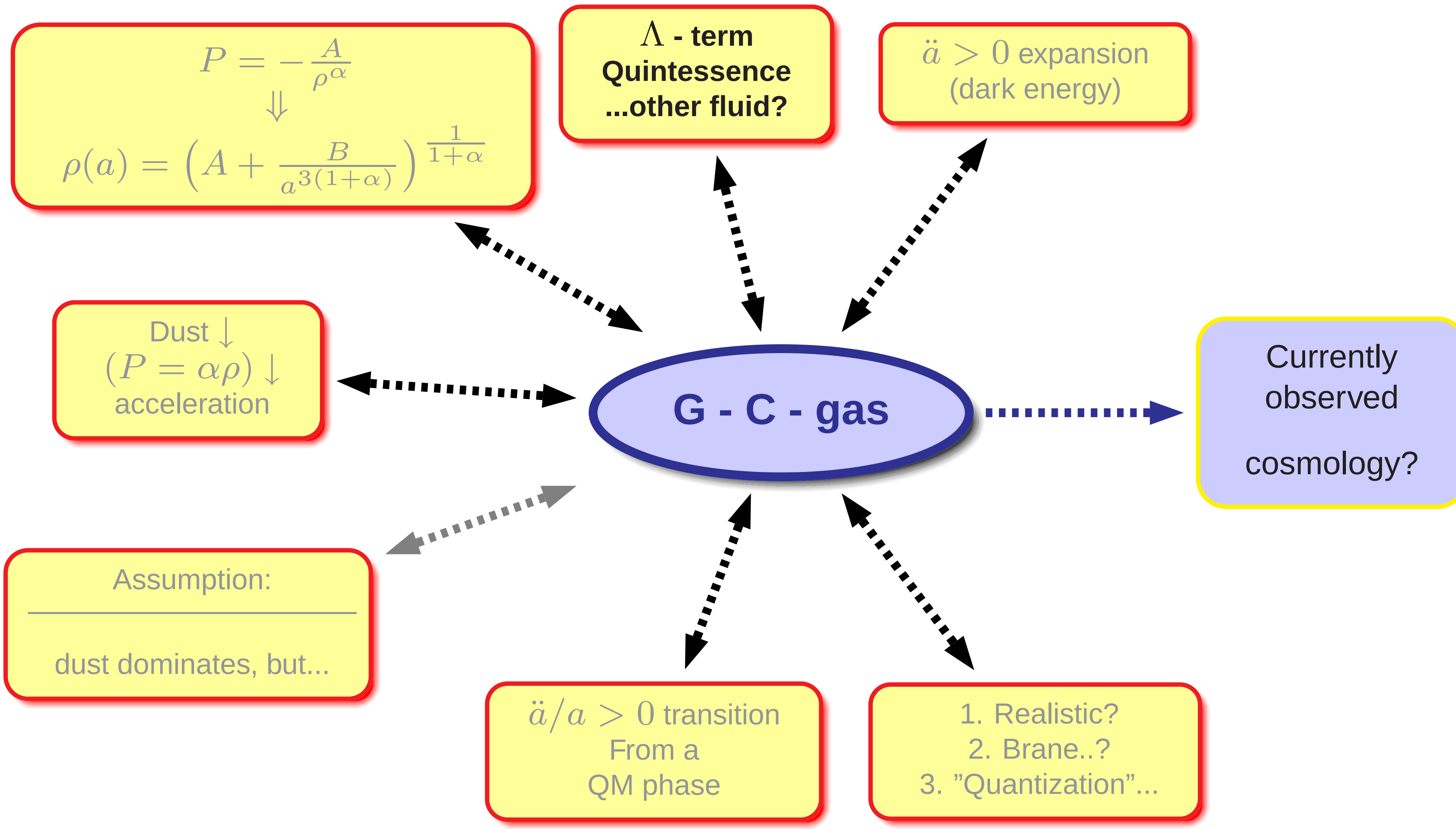


... and Mise en Scene:

$$P = -\frac{A}{\rho^{\alpha}}:$$
$$\Downarrow$$
$$\rho(a) = \left(A + \frac{B}{a^{3(1+\alpha)}}\right)^{\frac{1}{1+\alpha}}$$

Λ - term
Quintessence
...other fluid?

$\ddot{a} > 0$ expansion
(dark energy)

Dust $\downarrow$
$(P = \alpha\rho) \downarrow$
... acceleration

G - C - gas

Currently
observed

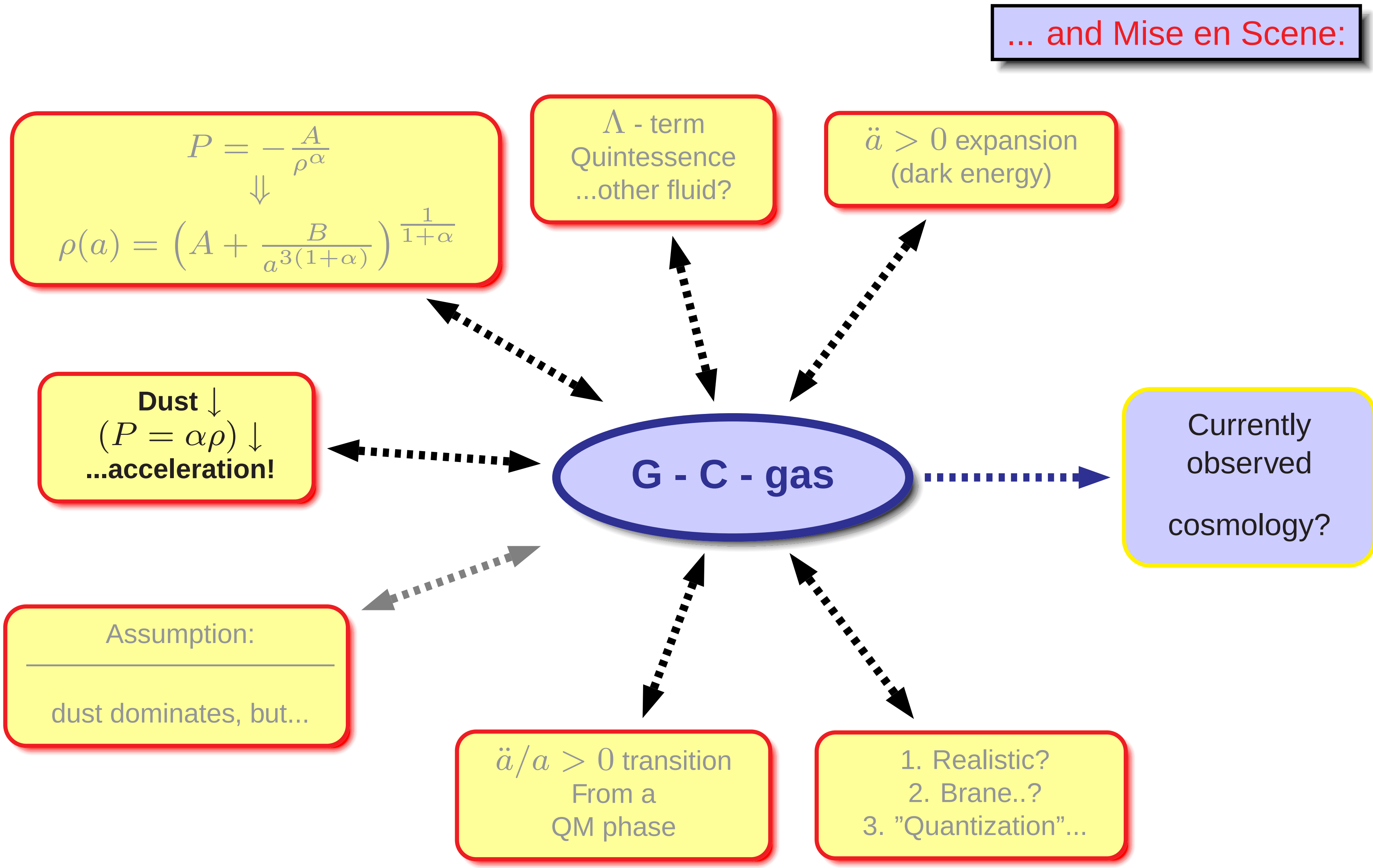
cosmology?

Assumption:

dust dominates, but...

$\ddot{a}/a > 0$ transition
From a
QM phase

1. Realistic?
2. Brane..?
3. "Quantization"...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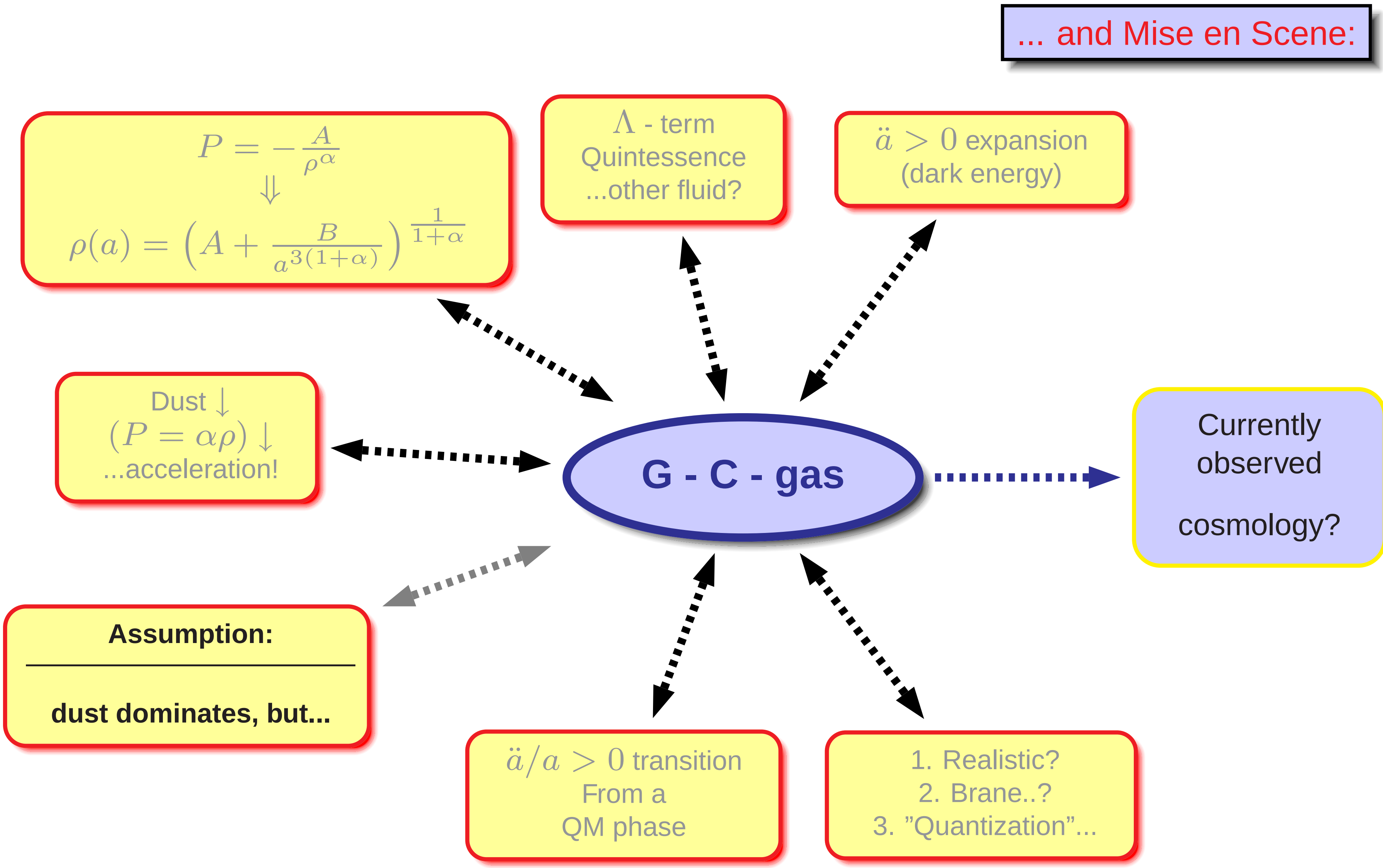
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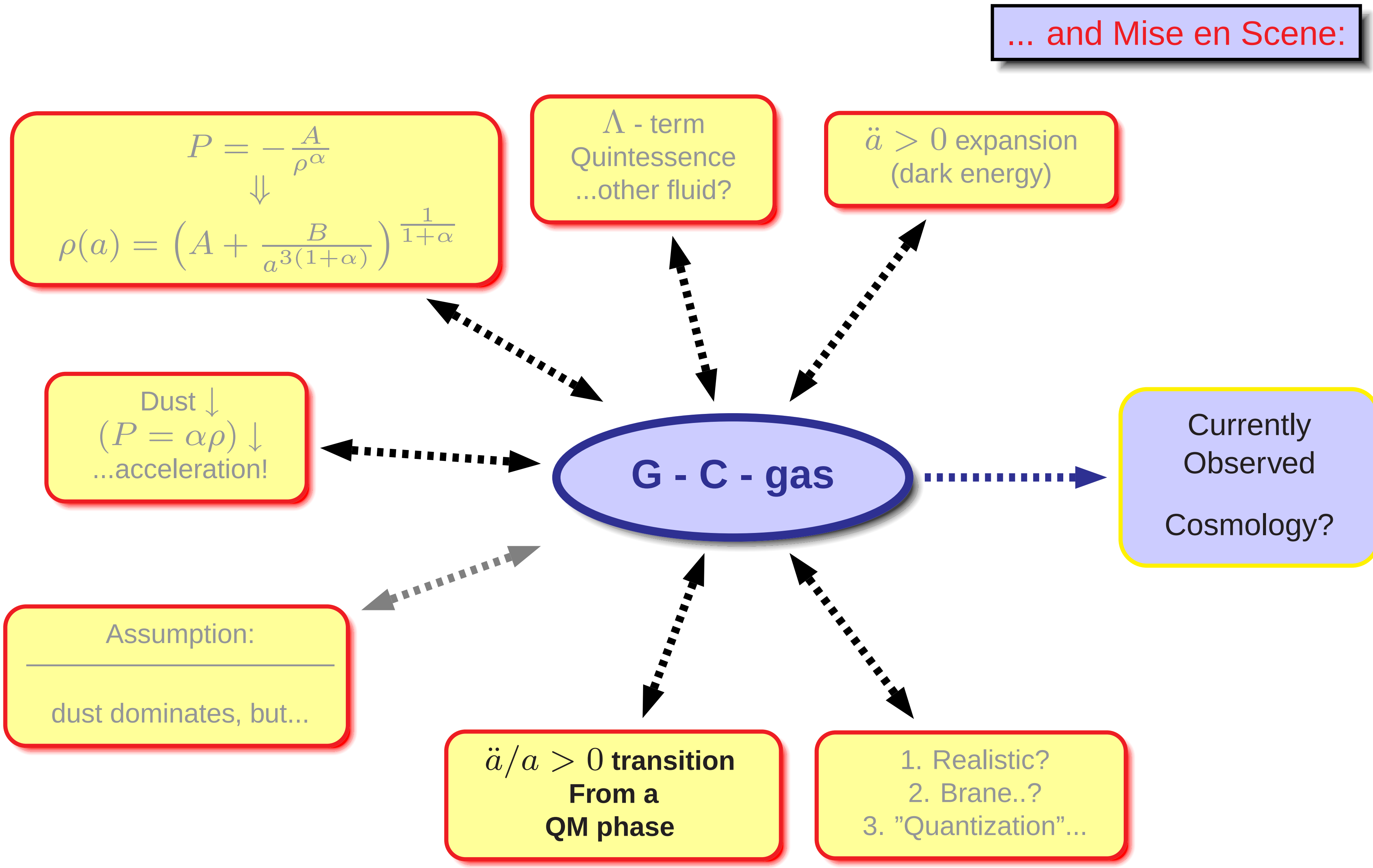
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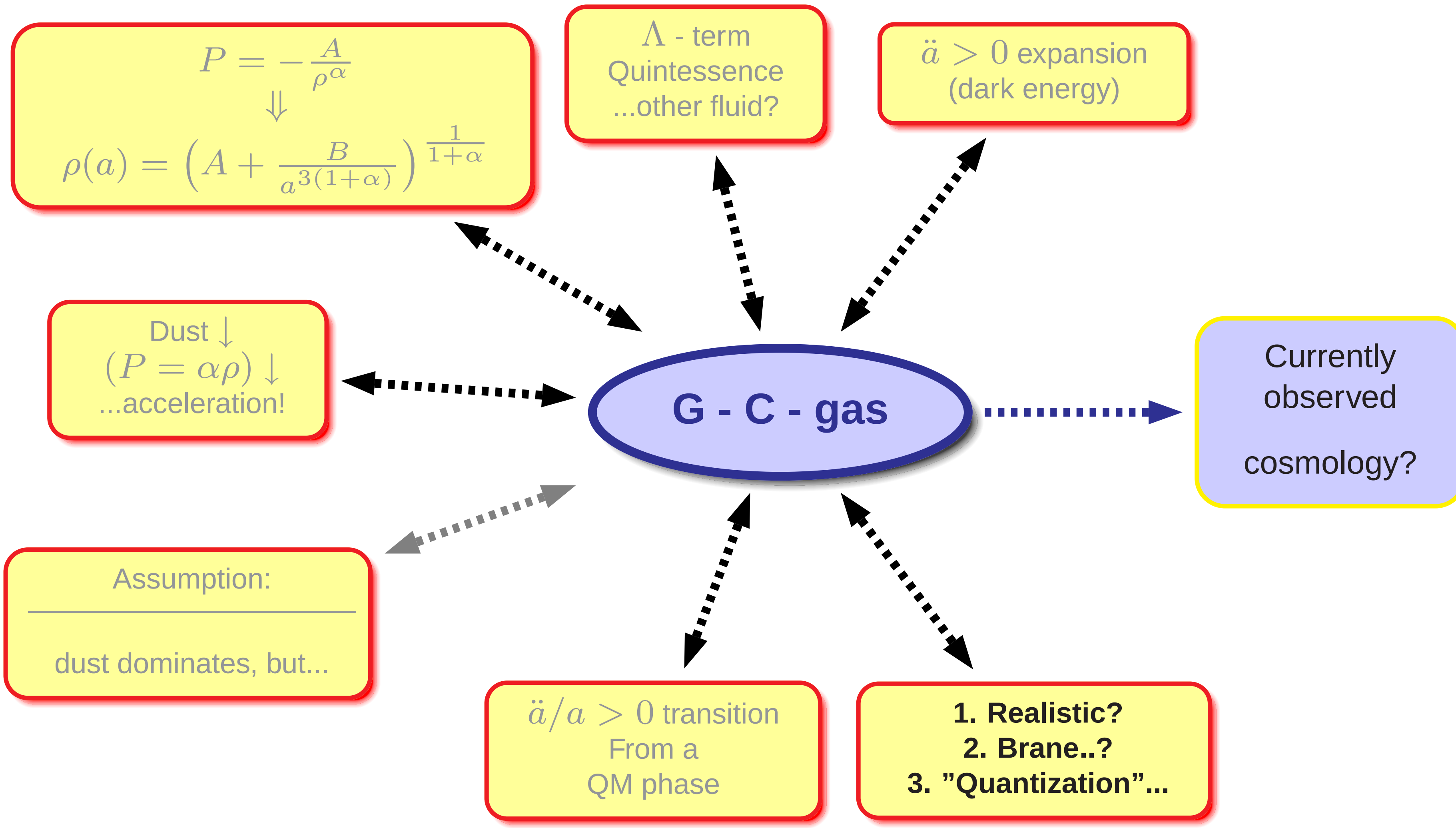

Lagrangian:

$$L = -\frac{3\pi}{4G}\left(\frac{\dot{a}^2 a}{N} - Na\right) - 2\pi a^3 N \frac{\Lambda}{8\pi G} - 2\pi a^3 N \rho$$

Lagrangian:

$$L = -\frac{3\pi}{4G}\left(\frac{\dot{a}^2 a}{N} - Na\right) - 2\pi a^3 N \frac{\Lambda}{8\pi G} - 2\pi a^3 N \rho$$

Wheeler-DeWitt equation:

$$\left[-\frac{G}{3\pi}\frac{d^2}{da^2} + \frac{3\pi}{4G}V_0(a)\right]\Psi(a) = 0$$

$$V_0(a) = a^2 - \lambda a^4 - \frac{8\pi G}{3}\rho(a)a^4$$

Semiclassical approximation $\Psi = c_1\ \psi_1 + c_2\ \psi_2$:

(in/out - going; in/de-creasing modes)

$$\psi_i = \exp\left[-\frac{1}{G}S_0(a)\right], \quad \left(\frac{dS_0(a)}{da}\right)^2 = \frac{9\pi^2}{4}V_0(a)$$

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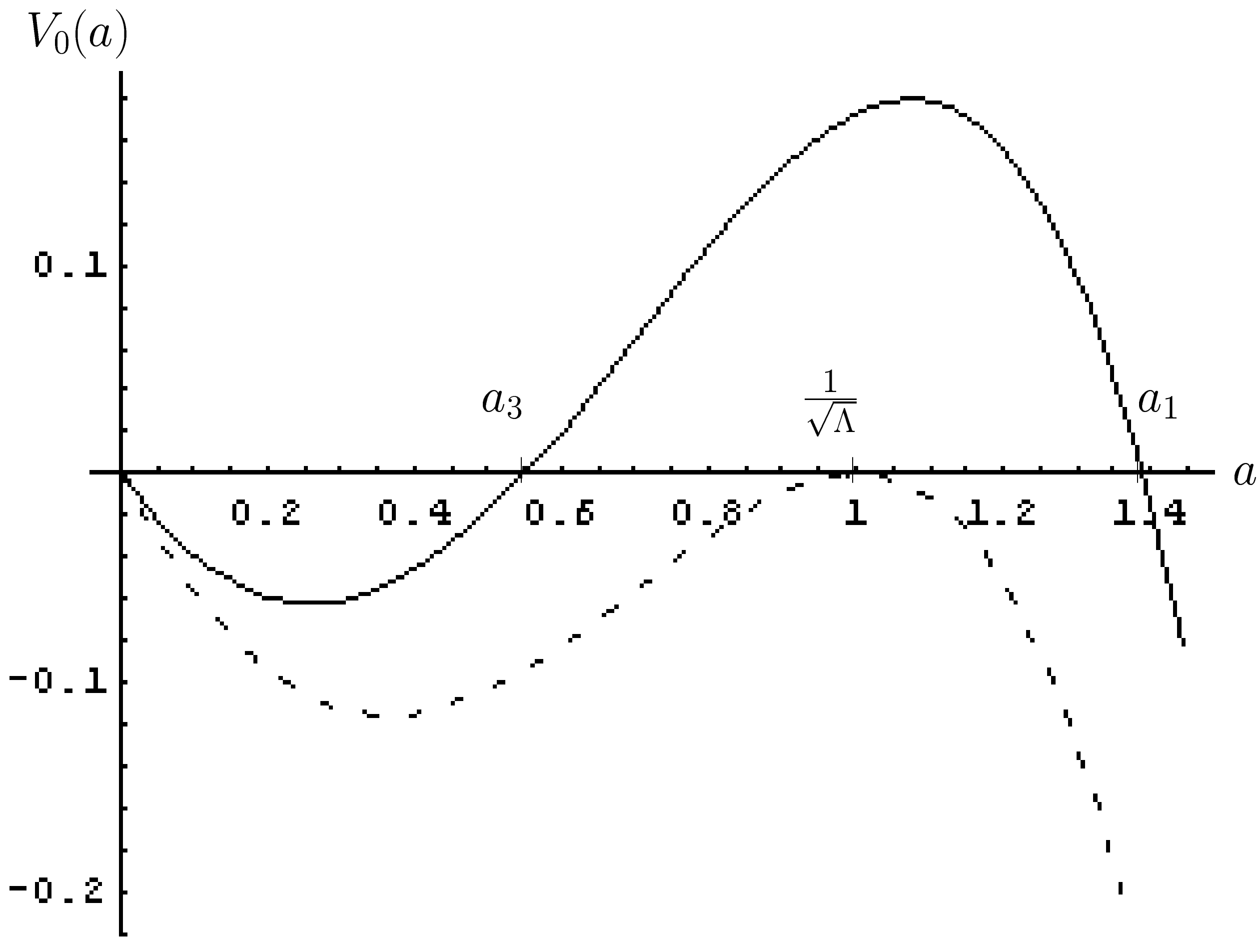
Linear approximation $\longrightarrow$ (saddle points):

$$V_0(a) \simeq a^2 - \lambda a^4 - a B^{\frac{1}{1+\alpha}}, \qquad \Leftarrow a \ll \left[(\alpha+1)\,\frac{B}{A}\right]^{\frac{1}{3(\alpha+1)}}$$



$V_0(a)$
0.1
-0.1
-0.2
0.2
0.4
0.6
0.8
1
1.2
1.4
a_3
$\frac{1}{\sqrt{\Lambda}}$
a_1
a

Turning point: depends on $\Lambda\tilde{B}^2$ where $\tilde{B} \equiv B^{\frac{1}{1+\alpha}}$

- $\Lambda\tilde{B}^2 > 4/9 \Rightarrow$ a classically allowed region.
- $\tilde{B} = \frac{2}{3}\frac{1}{\sqrt{\Lambda}} \Rightarrow$ two classically allowed regions.

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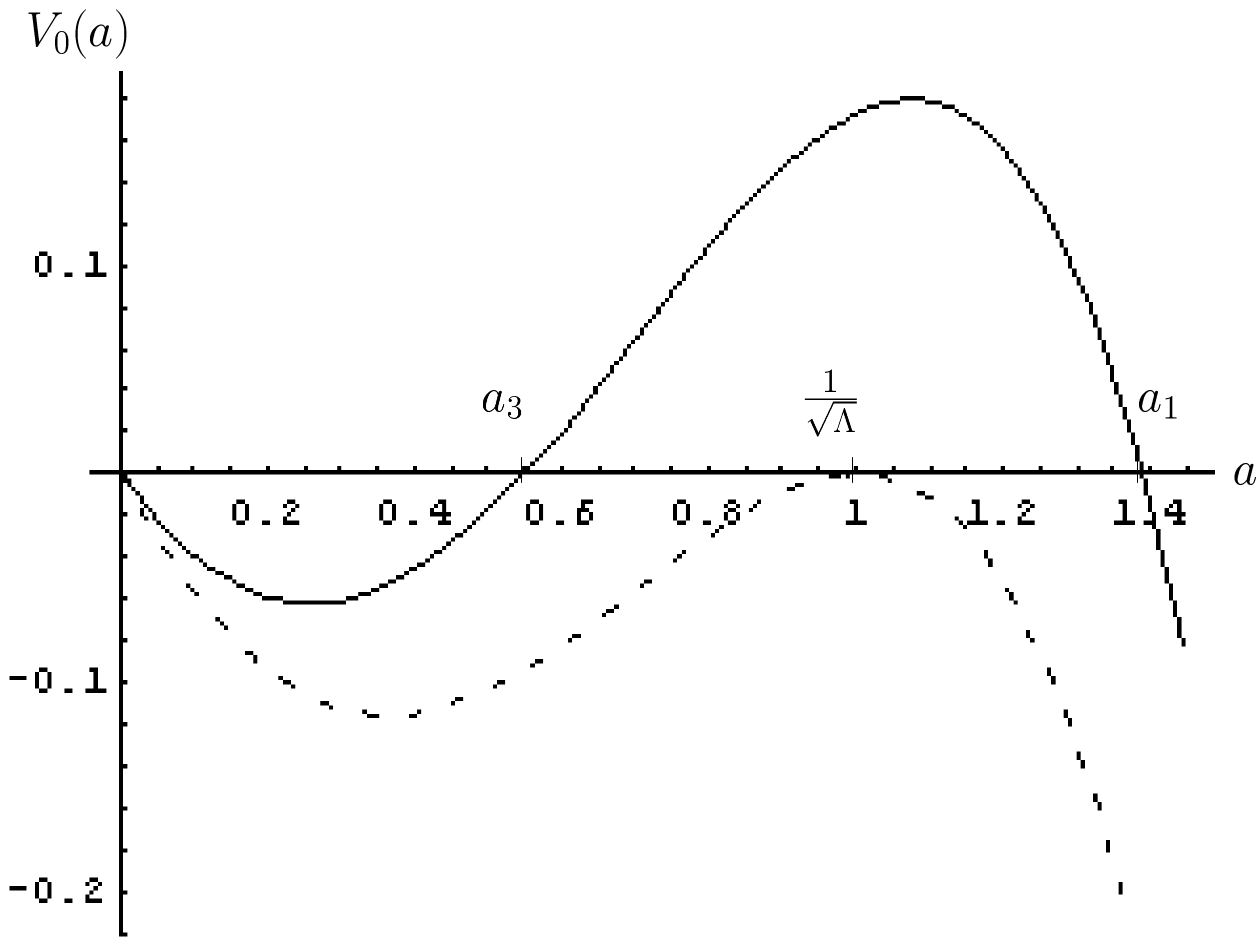
⋆ $\Lambda\tilde{B}^2 < 4/9 \Rightarrow$ two turning points a_3, a_1; two classically allowed regions ($0 < a < a_3$, $a_1 < a$) and one classically forbidden region ($a_3 < a < a_1$).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1.4

- FRW & $\phi = e^{i\theta}\varphi$; $m^2 \sim \varphi^2$; π_θ conserved $\longrightarrow$ Born - Infeld phenomenology
- DeS 3+1 – brane in 4+1 Minkowski – bulk

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Go Quantum:

The wave function in the WBK approximation is related to $S_0(a)$ which depends on $\Lambda\tilde{B}^2$ and the range of values of a; Compute:

$$\int \sqrt{|V_0(a)|}da \equiv \sqrt{\frac{\Lambda}{3}} \int I(a)da$$

$$\Lambda\tilde{B}^2 < 4/9$$

$$\begin{aligned}\int_{a_1}^{a} I(a)da \quad = \quad & \frac{aI}{3} - \beta\frac{a^2(a-a_1)}{I} + \gamma\left\{a_1a_3(a_1^2 - a_3^2)\Pi\left(\upsilon, \frac{2a_1+a_3}{2a_3+a1}, q\right)\right.\\ & + a_1a_3^2(a_1+a_3)F(\upsilon,q) - 6\beta^2F(\upsilon,q) + \beta a_3^2F(\upsilon,q)\\ & \left. + \beta\frac{(a_1+a_3)^2}{2a_3+a_1}\left[(a_3-a_1)\Pi(\upsilon,1,q) + (2a_1+a_3)F(\upsilon,q)\right]\right\}\end{aligned}$$

where $\beta \equiv \frac{\left(a_1^2+a_1a_3+a_3^2\right)}{3}$, $\gamma \equiv \left(\sqrt{a_1(2a_3+a_1)}\right)^{-1}$, $q \equiv \sqrt{\frac{a_3(2a_1+a_3))}{a_1(2a_3+a_1))}}$,

$$\upsilon \equiv \arcsin\sqrt{\frac{(2a_3+a_1)(a-a_1)}{(2a_1+a_3)(a-a_3)}},$$

$$\Lambda \tilde{B}^2 < 4/9$$

$$\int_a^{a_3} I(a)da = -\frac{aI}{3} - \beta\frac{a^2(a_3 - a)}{I} + \gamma\left\{a_1 a_3 (a_1 + a_3)\left[(a_3 - a_1)\Pi\left(\chi, \frac{a_3}{a_1}, q\right.\right.\right.$$

$$-6\beta^2 F(\chi, q) + \beta\gamma a_1^2 F(\chi, q)$$

$$\left.+\beta\frac{(a_1 + a_3)^2}{2a_3 + a_1}\left[(a_1 - a_3)\Pi\left(\chi, q^2, q\right) + (2a_3 + a_1)F(\chi, q)\right]\right\}$$

with $\chi \equiv \arcsin\sqrt{\frac{a_1(a_3-a)}{a_3(a_1-a)}}$

$$\Lambda\tilde{B}^2 < 4/9$$

$$\begin{aligned}\int_{a_3}^{a} I(a)da \quad = \quad & \frac{aI}{3} + \beta\frac{a^2(a-a_3)}{I} + \gamma\left\{-a_1a_3^2\left(a_1+a_3\right)\Pi\left(\delta, \frac{a_1-a_3}{a_1}, r\right)\right. \\ & +6\beta^2F\left(\delta,r\right) - \beta a_1\left[\left(a_1-a_3\right)F\left(\delta,r\right)+a_3\Pi\left(\delta,1,r\right)\right] \\ & \left. +\,\beta\left(a_1+a_3\right)\left[a_3\Pi\left(\delta,r^2,r\right)-\left(2a_3+a_1\right)F\left(\delta,r\right)\right]\right\}\end{aligned}$$

with $\delta \equiv \arcsin\sqrt{\frac{a_1(a-a_3)}{(a_1-a_3)a}}$ and $r \equiv \sqrt{\frac{a_1^2-a_3^2}{(2a_3+a_1)a_1}}$

Hartle-Hawking wave function

$$
\begin{aligned}
\Psi(a) &= C \cos\left[\frac{3\pi}{2G}\int_a^{a_3}\sqrt{-V_0(a)}da + \frac{\pi}{4}\right], \quad (a < a_3) \\
\Psi(a) &= C \exp\left[\frac{3\pi}{2G}\int_{a_3}^{a}\sqrt{V_0(a)}da\right], \quad (a_3 < a < a_1) \\
\Psi(a) &= 2\tilde{C} \cos\left[\frac{3\pi}{2G}\int_{a_1}^{a}\sqrt{-V_0(a)}da - \frac{\pi}{4}\right], \quad (a_1 < a)
\end{aligned}
$$

where $\tilde{C} \equiv C \exp\left[\frac{3\pi}{2G}\int_{a_3}^{a_1}\sqrt{V_0(a)}da\right]$ and C is an arbitrary constant.

Vilenkin wave function

$$\Psi(a) = C\exp\left[-\frac{3\pi}{2G}\int_{a_3}^{a_1}\sqrt{V_0(a)}da\right]\cos\left[\frac{3\pi}{2G}\int_{a}^{a_3}\sqrt{-V_0(a)}da+\frac{\pi}{4}\right]$$
$$+\ i4C\exp\left[\frac{3\pi}{2G}\int_{a_3}^{a_1}\sqrt{V_0(a)}da\right]\cos\left[\frac{3\pi}{2G}\int_{a}^{a_3}\sqrt{-V_0(a)}da-\frac{\pi}{4}\right], \quad (a < a_3)$$

$$\Psi(a) = C\left(\exp\left[-\frac{3\pi}{2G}\int_{a}^{a_1}\sqrt{V_0(a)}da\right]+\right.$$
$$\left.+\ 2i\exp\left[\frac{3\pi}{2G}\int_{a}^{a_1}\sqrt{V_0(a)}da\right]\right), \quad (a_3 < a < a_1)$$

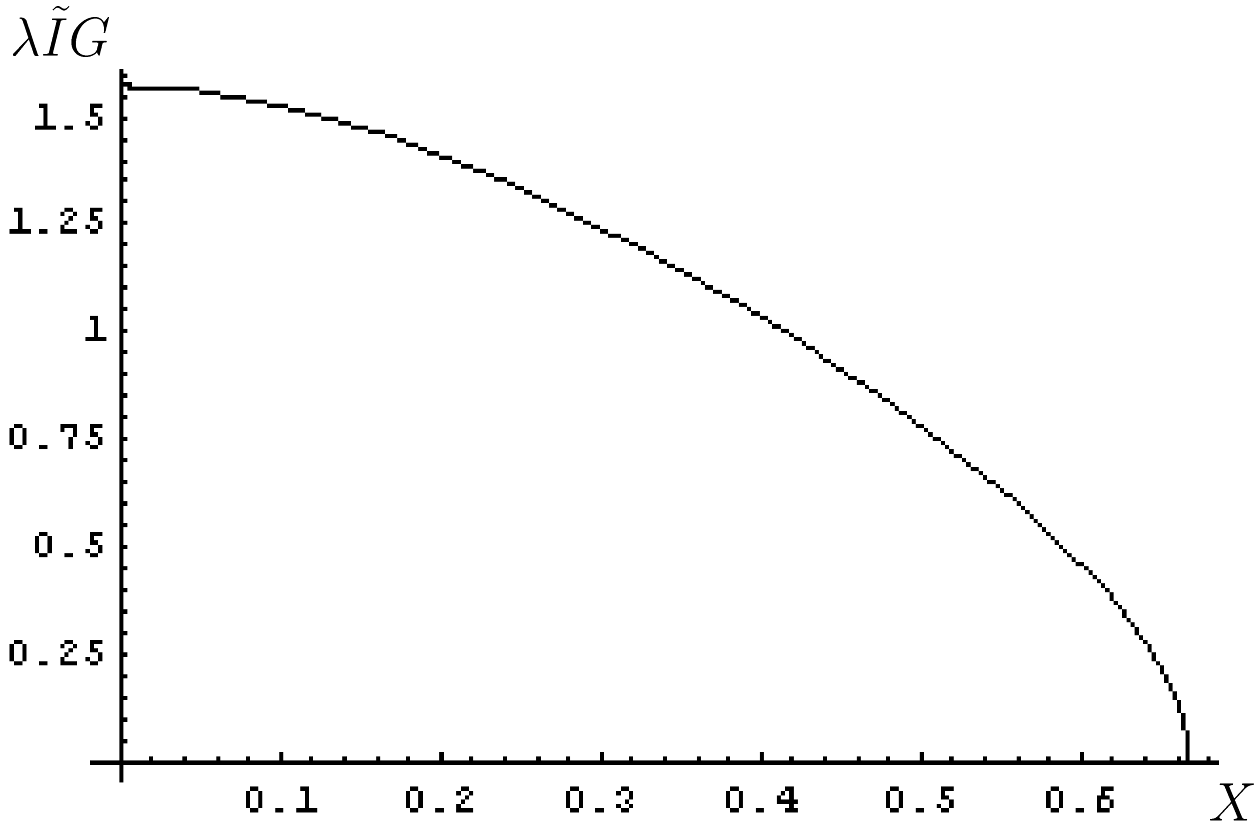
$$\Psi(a) = 2Ce^{i\pi/4}\exp\left[-i\frac{3\pi}{2G}\int_{a_1}^{a}\sqrt{-V_0(a)}da\right], \quad (a_1 < a)$$

Transition amplitude

- Hartle-Hawking wave function: $A = \exp(2\tilde{I})$

Transition amplitude

- Hartle-Hawking wave function: $A = \exp(2\tilde{I})$
- Vilenkin wave function: $A = -\exp(2\tilde{I})$


$\lambda\tilde{I}G$
1.5
1.25
1
0.75
0.5
0.25
0.1
0.2
0.3
0.4
0.5
0.6
X

1. When $X \equiv B^{1/(\alpha+1)}\sqrt{\Lambda}$ increases $\lambda\tilde{I}G$ decreases (for fixed values of Λ).

1a. When X increases $\Rightarrow$ The transition amplitude for the Vilenkin boundary condition increases, while the transition amplitude for the Hartle-Hawking condition decreases (for fixed values of Λ).

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1a. When X increases $\Rightarrow$ The transition amplitude for the Vilenkin boundary condition increases, while the transition amplitude for the Hartle-Hawking condition decreases (for fixed values of Λ).

2. When B increases X increases (for fixed values of Λ and α).

3. When α increases X can increase or decrease (for fixed values of Λ and B).

There is a singularity at $a = 0$ (R diverges)

$$R = 4\left\{\Lambda + 2\pi G\left[A + \frac{B}{a^{3(1+\alpha)}}\right]^{-\frac{\alpha}{\alpha+1}}\left[4A + \frac{B}{a^{3(1+\alpha)}}\right]\right\}.$$

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$$R = 4\left\{\Lambda + 2\pi G\left[A + \frac{B}{a^{3(1+\alpha)}}\right]^{-\frac{\alpha}{\alpha+1}}\left[4A + \frac{B}{a^{3(1+\alpha)}}\right]\right\}.$$

We assume that the wave function vanishes at the origin:

$$\Psi(a = 0) = 0$$

Parameters α and B restricted to a set of quantized values

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For the Hartle-Hawking wave function, e.g.,:

$$\frac{3\pi}{2G}\int_0^{a_3}\sqrt{-V_0(a)}da-\frac{\pi}{4}=n\pi, \quad n\in Z$$

$$\Rightarrow \quad \frac{3\pi}{4G}B^{2/(1+\alpha)}-1\simeq 4n$$

Next order of magnitude - going beyond WKB...

$$\left(A+\frac{B}{a^{3(1+\alpha)}}\right)^{\frac{1}{1+\alpha}} \approx \frac{B^{\frac{1}{1+\alpha}}}{a^3}\left[1+\frac{1}{1+\alpha}\frac{A}{B}a^{3(\alpha+1)}+\frac{1}{2}\frac{1}{1+\alpha}\left(\frac{1}{1+\alpha}-1\right)\frac{A^2}{B^2}a^{6(\alpha+1)}+\ldots\right].$$

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Wheeler-DeWitt equation

$$\left[-\frac{G}{3\pi}\frac{d^2}{da^2}+\frac{3\pi}{4G}V(a)\right]\Psi(a)=0,$$

Wave function

$$\Psi(a)=\exp\left[-\frac{1}{G}\left(S_0(a)+AS_1(a)+A^2S_2(a)+\cdots\right)\right]$$

Hamilton-Jacobi eq and ...

$$\begin{aligned}\left(\frac{dS_0(a)}{da}\right)^2 &= \frac{9\pi^2}{4}V_0(a),\\ \frac{2}{3\pi G}\frac{dS_0(a)}{da}\frac{dS_1(a)}{da} &- \frac{1}{3\pi}\frac{d^2S_1(a)}{da^2}+\frac{3\pi}{4G}V_1(a)=0,\end{aligned}$$

Hamilton-Jacobi eq and ...

$$\left(\frac{dS_0(a)}{da}\right)^2 = \frac{9\pi^2}{4}V_0(a),$$

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Potential terms

$$V_0(a) = a^2 - \lambda a^4 - aB^{\frac{1}{1+\alpha}},$$

$$V_1(a) = -\frac{a^{4+3\alpha}}{2+\alpha}B^{-\frac{\alpha}{1+\alpha}},$$

Compute wave function

$$J = \int_0^{a_3} \frac{V_1(a)}{\sqrt{-V_0(a)}} da,$$

$$J \approx B^{4+3\alpha} \mathbf{B} \left(9/2 + 3\alpha, \frac{1}{2} \right),$$

where $\mathbf{B}$ is the beta function $\downarrow$

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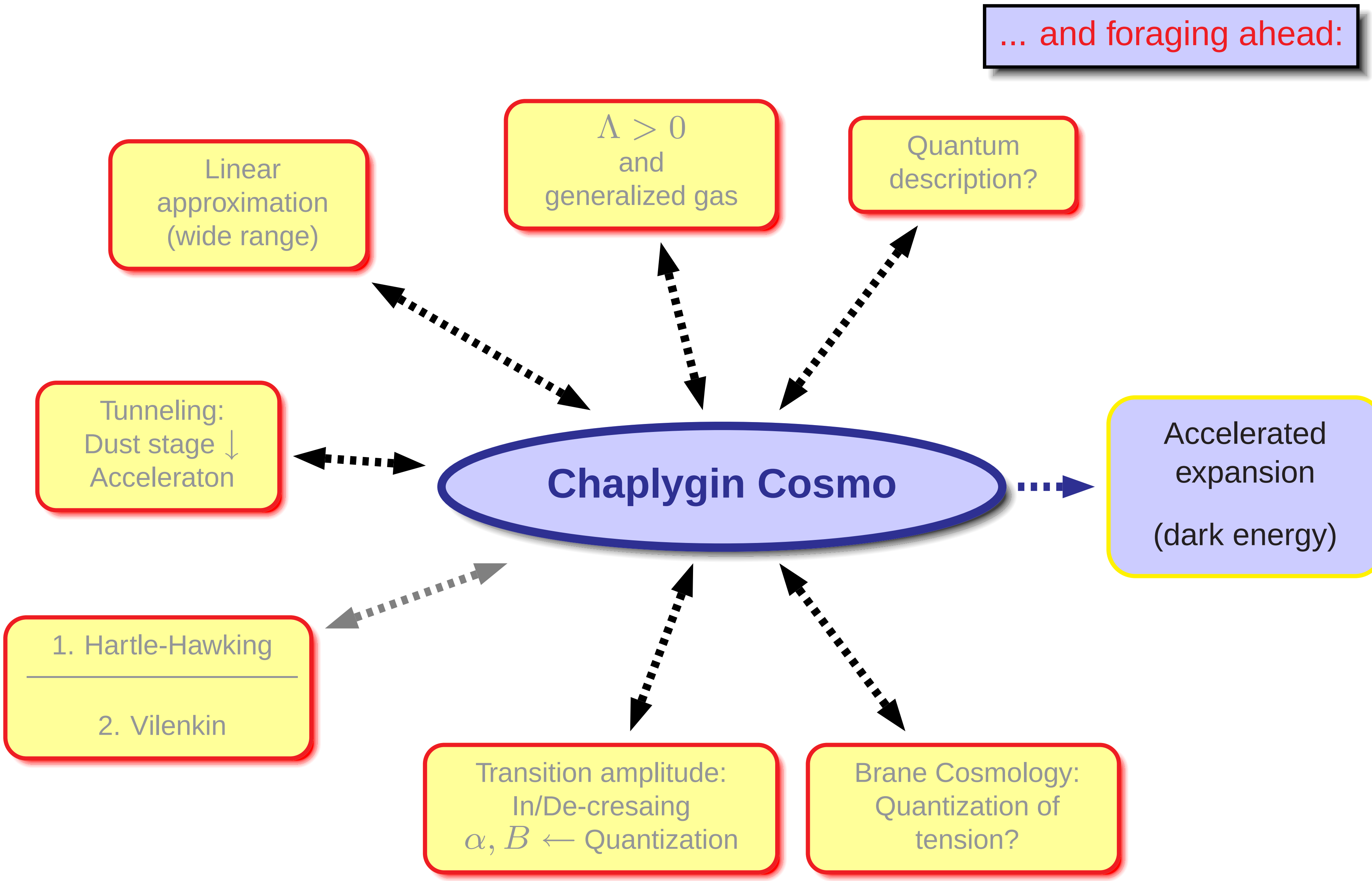
$$\Psi(a) \approx \exp \left[-\frac{1}{G} \left(S_0(a) + A S_1(a) \right). \right]$$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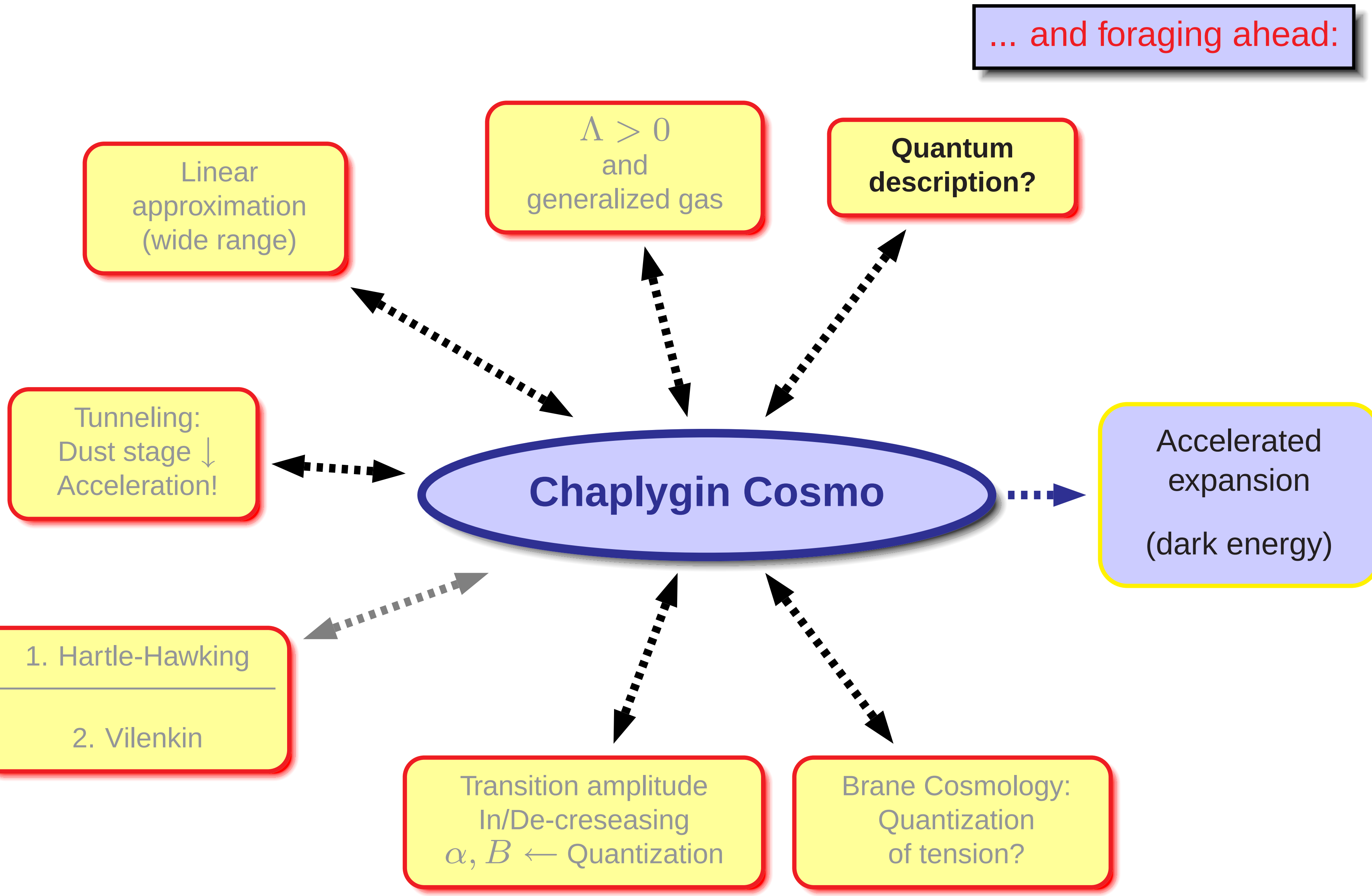
Linear approximation (wide range)
$\Lambda > 0$ and generalized gas
Quantum description?
Tunneling: Dust stage $\downarrow$ Acceleraton
Chaplygin Cosmo
Accelerated expansion
(dark energy)
1. Hartle-Hawking
2. Vilenkin
Transition amplitude: In/De-cresaing $\alpha, B \longleftarrow$ Quantization
Brane Cosmology: Quantization of tension?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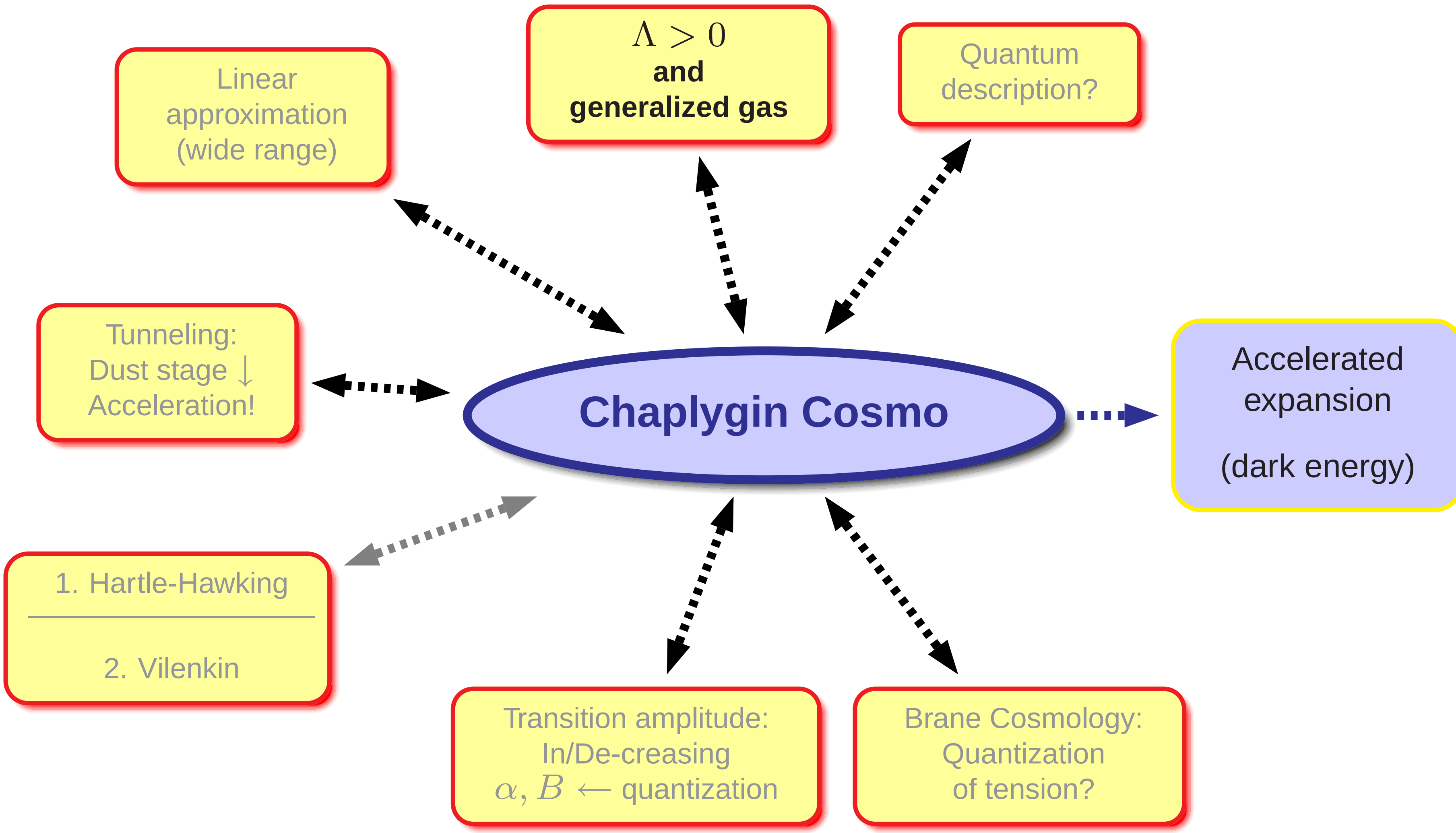

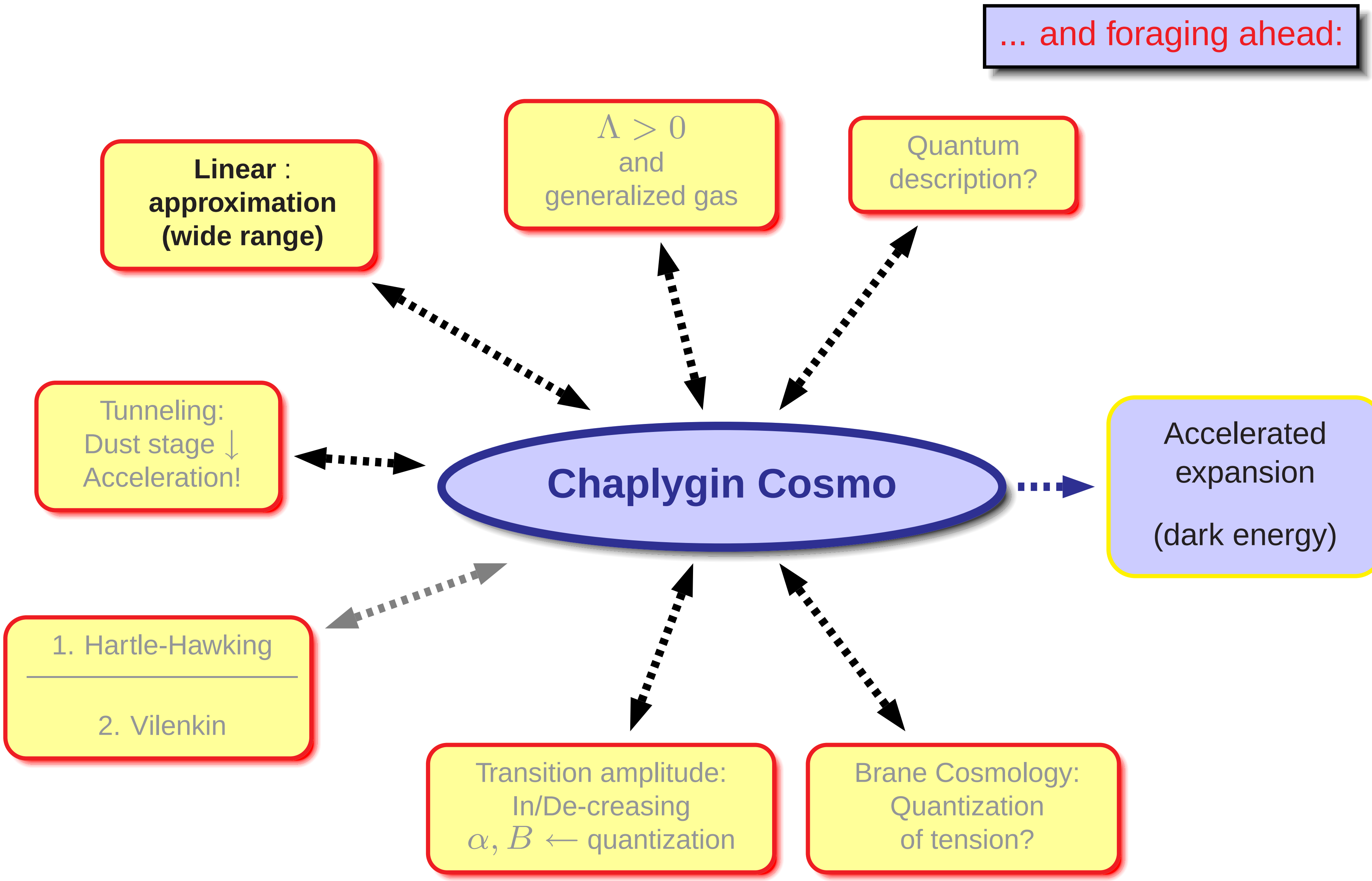
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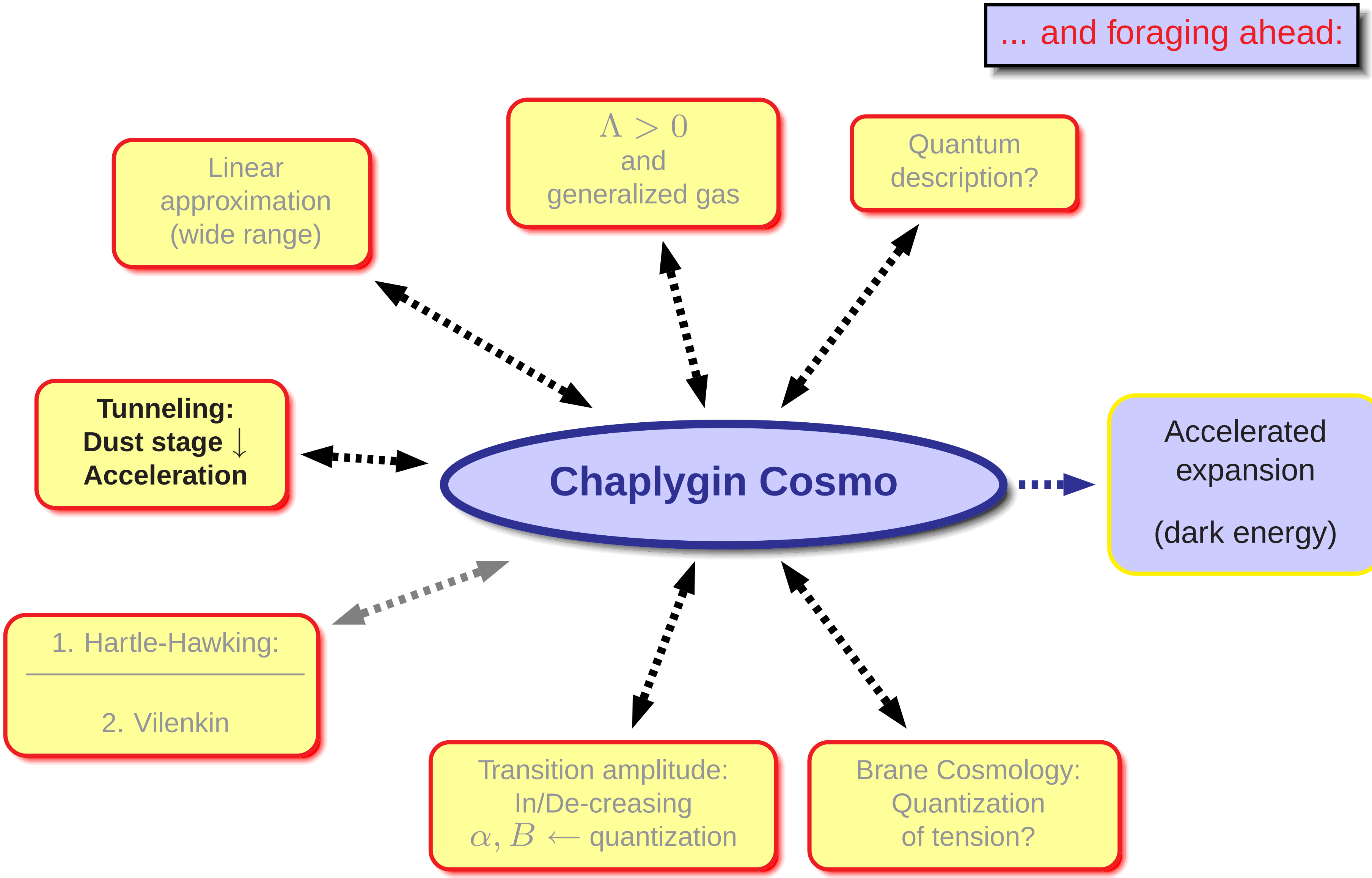
$\Lambda > 0$ and generalized gas
Quantum description?
Tunneling: Dust stage $\downarrow$ Acceleration!
Chaplygin Cosmo
Accelerated expansion
(dark energy)
1. Hartle-Hawking
2. Vilenkin
Transition amplitude: In/De-creasing $\alpha, B \longleftarrow$ quantization
Brane Cosmology: Quantization of tension?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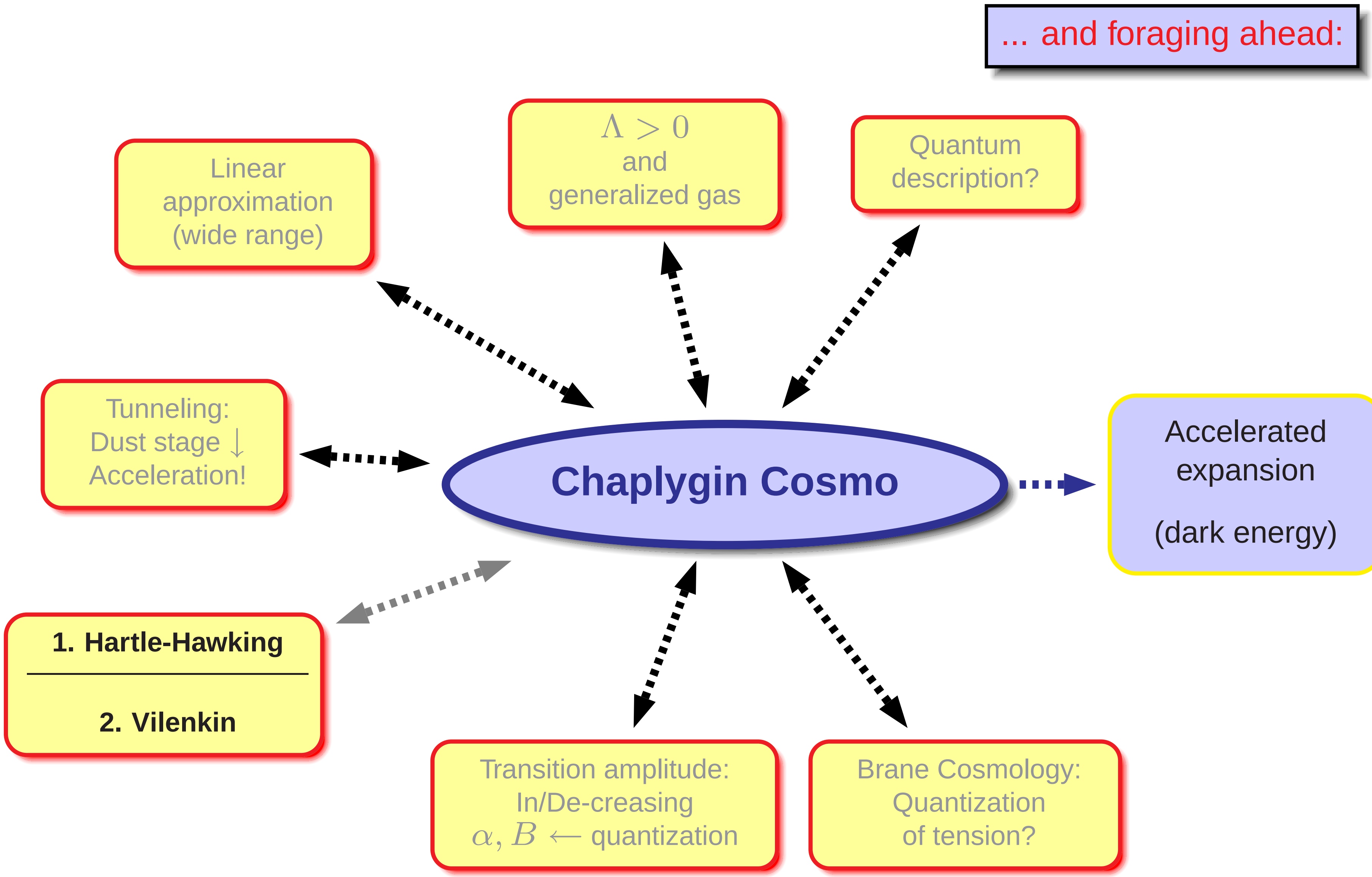
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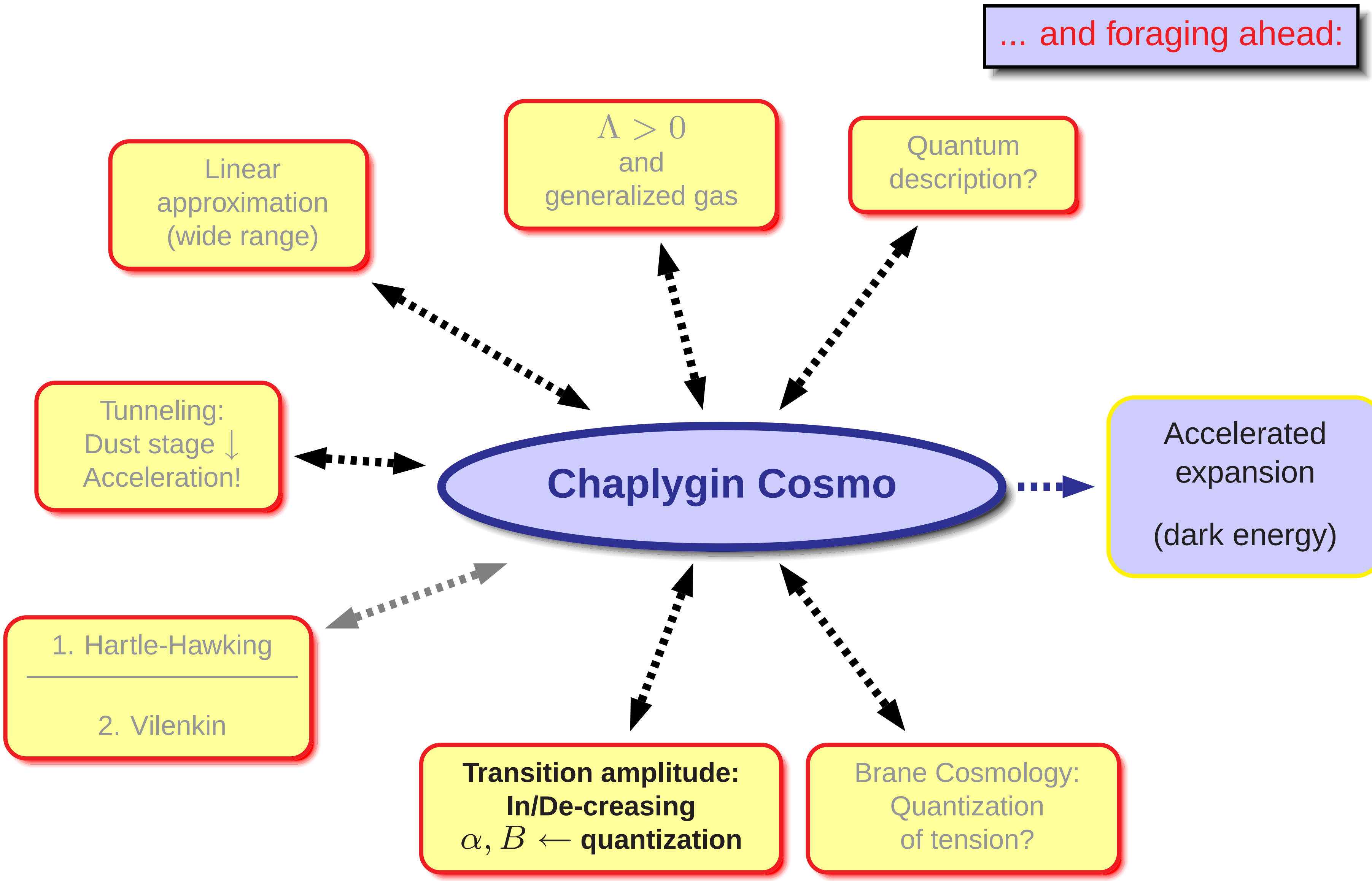
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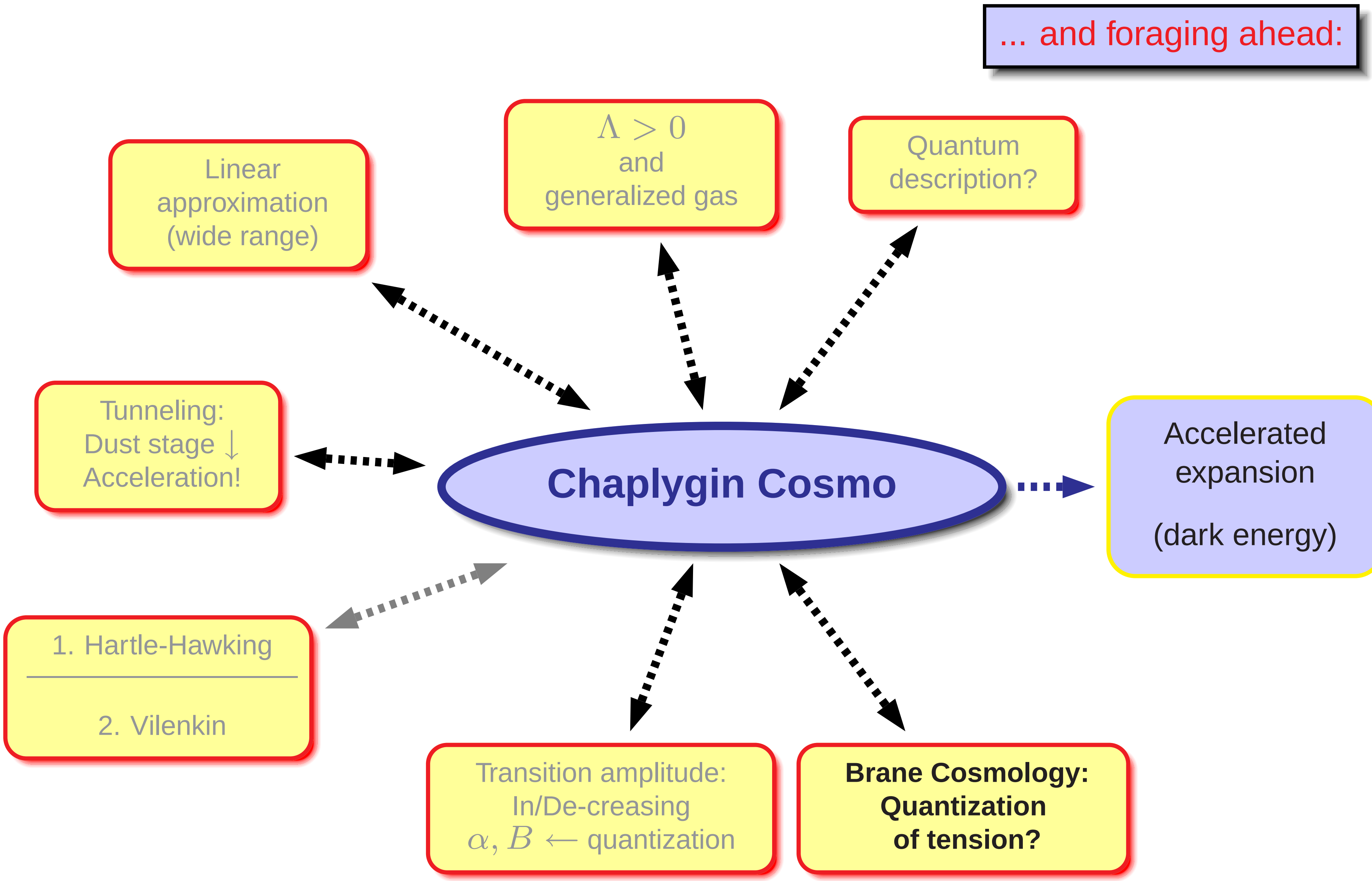
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