Phenomenological Classification of Inflationary Potentials

(or, Inflation: What's the Worst that Could Happen?)

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Inflationary Pragmatism

Inflation works...

- Offers solution to horizon problem and flatness problem
- General predictions upheld (flat universe, gaussian and adiabatic metric fluctuations, nearly scale-independent spectrum)

But....

- It may not be the only thing that works
- There is no theoretical consensus on how it works

Back Door to Inflation

- Use phenomenology to seek description of dynamics without favoring a particular theoretical framework

- Create models that can uphold inflation's successes without imposing extra model-dependent constraints

Slow-Roll Parameterization

Use slow-roll parameters to reconstruct a potential through the flow equations

Requirements:

- Slow(ish) roll ($\varepsilon < 1$)
- Sufficient e-foldings to solve horizon problem

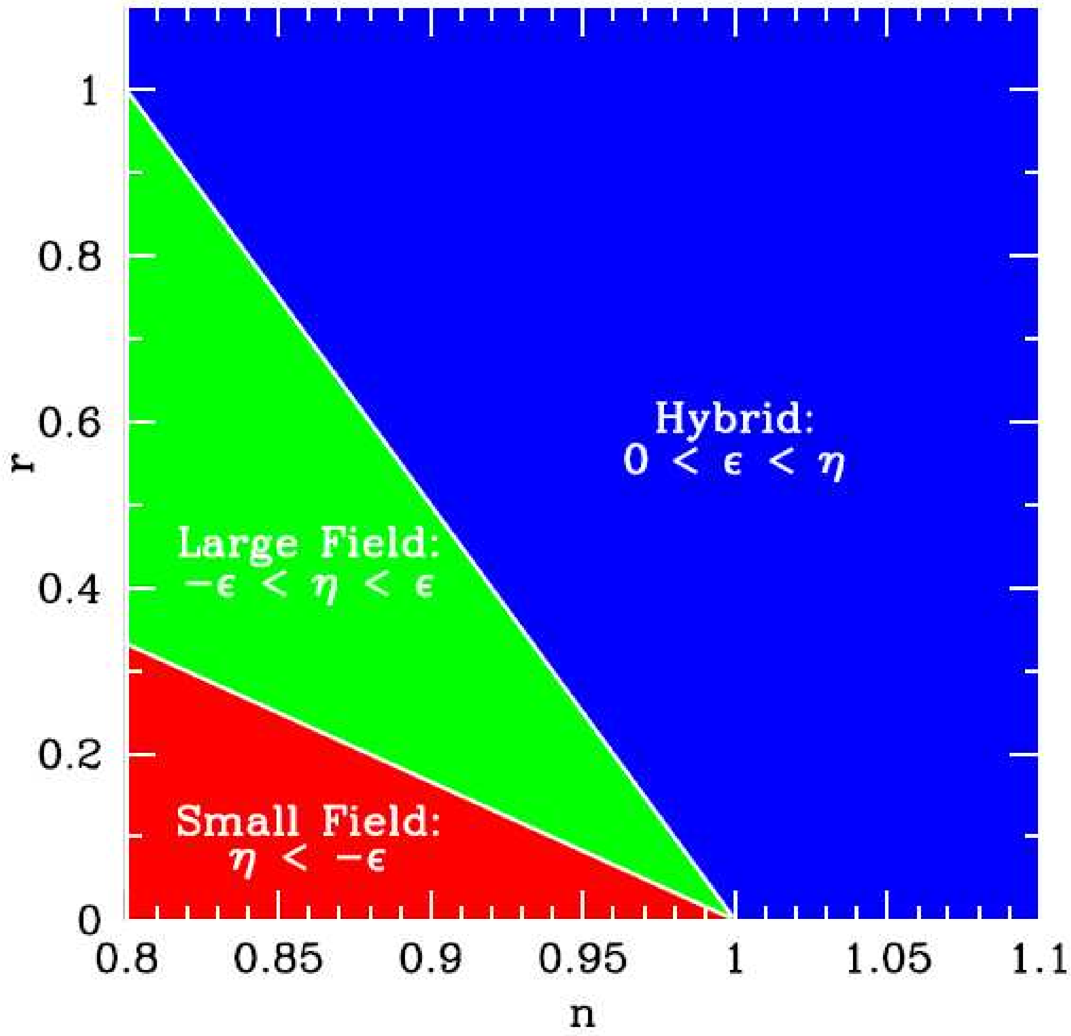
Goal:

- Find a phenomenological classification that can constrain inflationary dynamics

Energy Scale of Inflation

One option: Classify in terms of energy scale
(from V(φ))

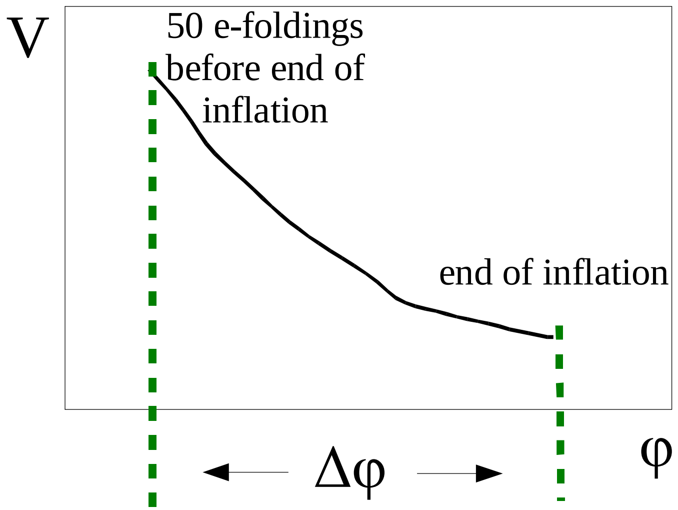
...but a better constraint can be obtained by also including LSS information (found in tensor-scalar ratio)



(Kinney et al., 2003)

Classification by r and Δφ

- Tensor-scalar ratio has two components:
  - tensor amplitude gives energy scale
  - scalar amplitude linked to primordial density perturbations (and hence large scale structure)
- Link with inflationary dynamics by relating to Δφ



$$\Delta\phi \equiv \phi_{end} - \phi_i$$

$$N = number\ of\ e-\ folds\ before\ end\ of\ inflation$$

$$\phi_{end} = \phi\ at\ end\ of\ inflation\ (N=0)$$

$$\phi_i = \phi\ at\ 50\ e-\ folds\ before\ end\ of\ inflation\ (N=50)$$

r vs. $\Delta\varphi$

- Lyth (1996) suggests rough relation:
  - $\Delta\varphi/m_{pl} \sim 0.5(r/0.07)^{1/2}$
  - Approximation based on flow equations
- General expectation: large r => large $\Delta\varphi$
- Small r allows large $\Delta\varphi$ – this just the case of very low energy potential

Going with the Flow

- Can use flow equations to evolve inflationary potentials and to calculate observables
- Using Hamilton-Jacobi Formulation, following Monte Carlo Reconstruction method of Easther & Kinney (2003) and Peiris et al. (2003)

Inflationary Flow Equations

Equations of Motion:

$$\dot{\phi} = -2M_{\rm Pl}^2 H'(\phi)$$

$$[H'(\phi)]^2 - \frac{3}{2M_{\rm Pl}^2} H^2(\phi) = -\frac{1}{2M_{\rm Pl}^4} V(\phi)$$

$$H^2(\phi)\left(1 - \frac{1}{3}\epsilon_H\right) = \frac{1}{3M_{\rm Pl}^2} V(\phi)$$

$$\left(\frac{\ddot{a}}{a}\right) = \frac{1}{3M_{\rm Pl}^2}\left[V(\phi) - \dot{\phi}^2\right]$$
$$= H^2(\phi)[1 - \epsilon_H(\phi)]$$

note:

$$M_{\rm Pl} \equiv (8\pi G)^{-1/2} = m_{\rm Pl}/\sqrt{8\pi}$$

Slow Roll Parameters:

$$\epsilon_H \equiv 2M_{\rm Pl}^2 \left[\frac{H'(\phi)}{H(\phi)}\right]^2$$

$$\eta_H \equiv 2M_{\rm Pl}^2 \left[\frac{H''(\phi)}{H(\phi)}\right]$$

$${}^\ell\lambda_H \equiv (2M_{\rm Pl}^2)^\ell \frac{(H')^{\ell-1}}{H^\ell} \frac{d^{(\ell+1)}H}{d\phi^{(\ell+1)}}$$

Condition for inflation:

$$(\ddot{a}/a) > 0$$

Satisfied when:

$$\epsilon_H < 1$$

Inflationary Flow Equations

Evolution Equations:

$$N \equiv \int_t^{t_e} H\, dt = \int_\phi^{\phi_e} \frac{H}{\dot{\phi}} d\phi = \frac{1}{\sqrt{2} M_{\rm Pl}} \int_{\phi_e}^{\phi} \frac{d\phi}{\sqrt{\epsilon_H(\phi)}}$$

$$\frac{d}{dN} = \sqrt{2} M_{\rm Pl} \sqrt{\epsilon} \frac{d}{d\phi}$$

$$\frac{d\epsilon_H}{dN} = 2\epsilon_H(\eta_H - \epsilon_H)$$

$$\frac{d({}^\ell\lambda_H)}{dN} = [(\ell - 1)\eta_H - \ell\epsilon_H]\left({}^\ell\lambda_H\right) + {}^{\ell+1}\lambda_H \qquad (\ell > 0)$$

Observables (to second order):

$$r = 16\epsilon_H[1 + 2C(\epsilon_H - \eta_H)]$$

$$n_s - 1 = (2\eta_H - 4\epsilon_H)\left[1 - \tfrac{1}{4}(5 - 3C)\epsilon_H\right] - (3 - 5C)\epsilon_H^2 + \tfrac{1}{2}(3 - C)\xi_H$$

$$\frac{dn_s}{d\ln k} = -\left(\frac{1}{1 - \epsilon_H}\right)\frac{dn_s}{dN}$$

Monte Carlo Reconstruction

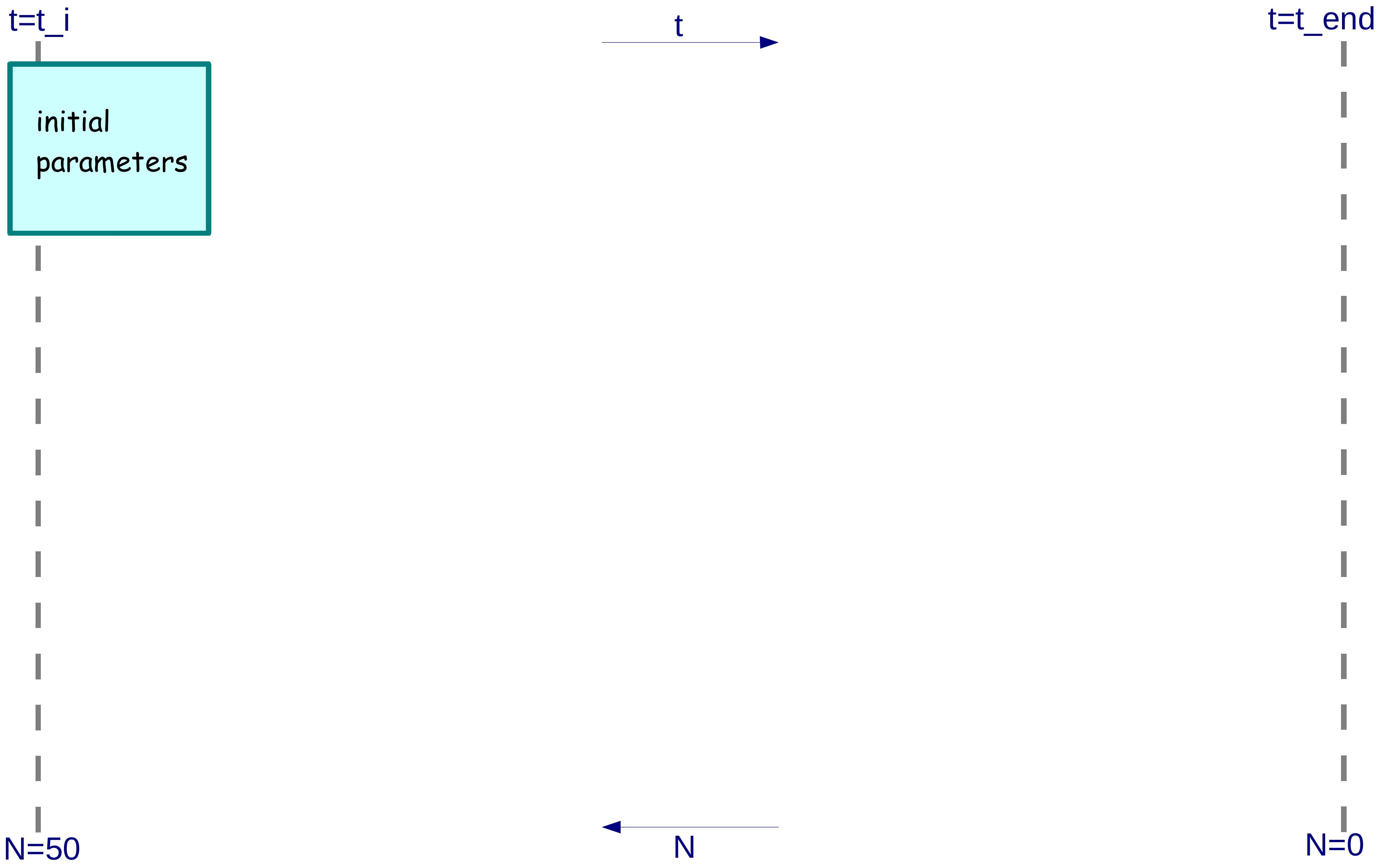
1) Choose initial slow-roll parameters from uniform random distributions: (ϵ= [0, 0.8], η=[-0.1, 0.1], ${}^{\ell}\lambda$=[$-2^{-\ell}/5$, $2^{-\ell}/5$])

2) Evolve forward in time (dN<0) until (A) evolution reaches a late-time fixed point (ϵ=0, ${}^{\ell}\lambda$=constant) or (B) inflation ends (ϵ>1)

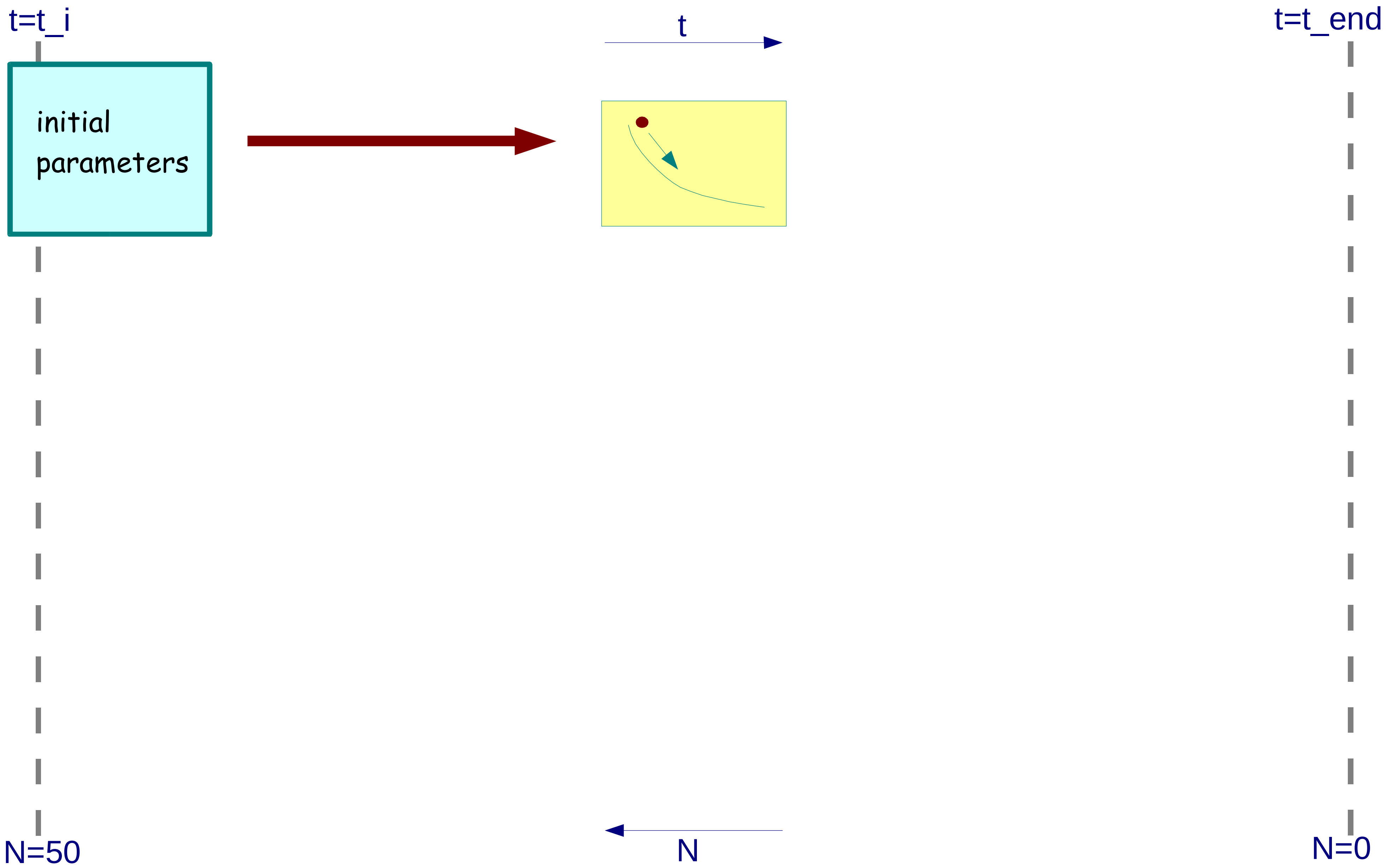
3) If (A), record observables. If (B), evolve back 50 e-foldings (dN>0)

4) Record observables at N=50

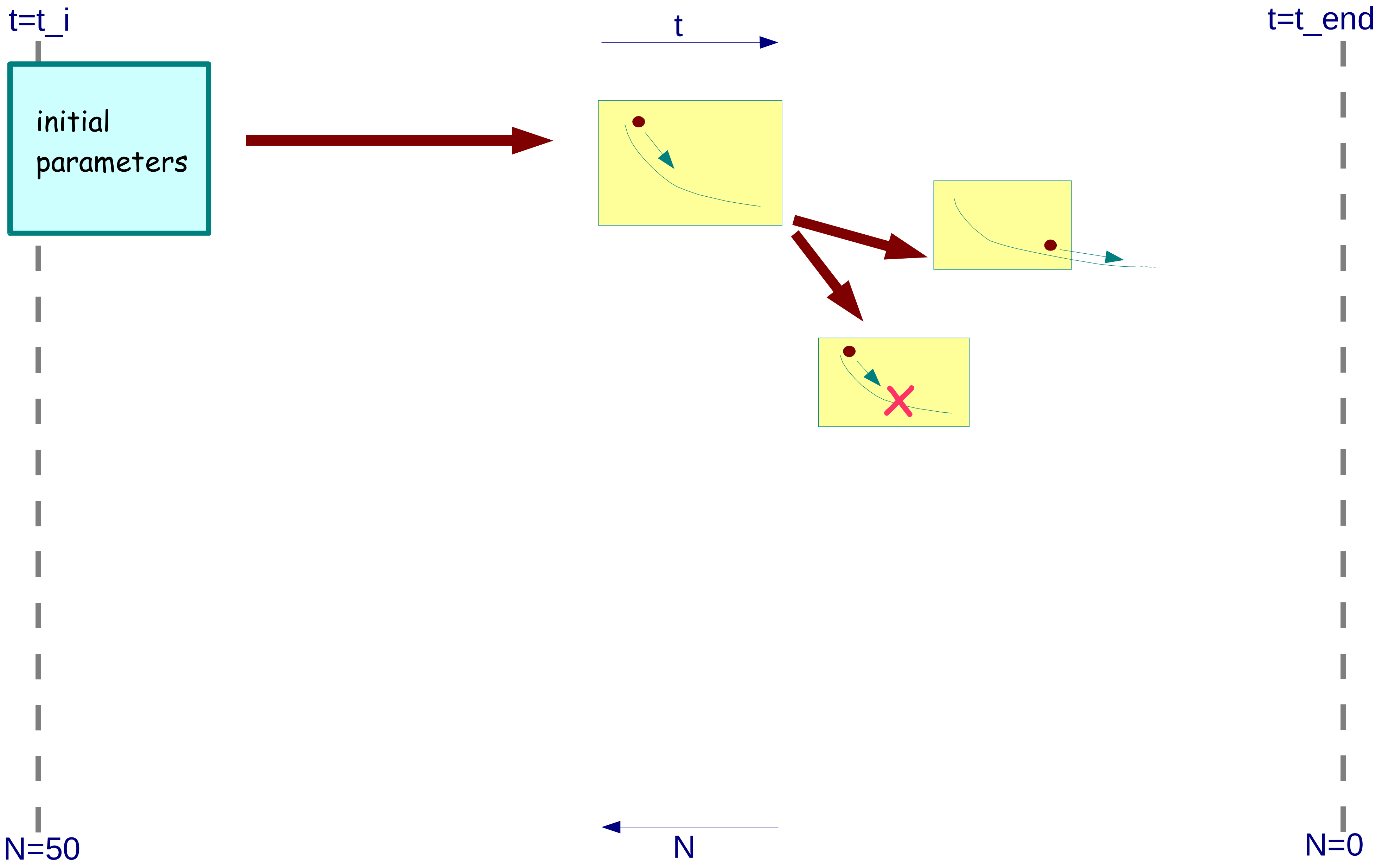
Flow Chart



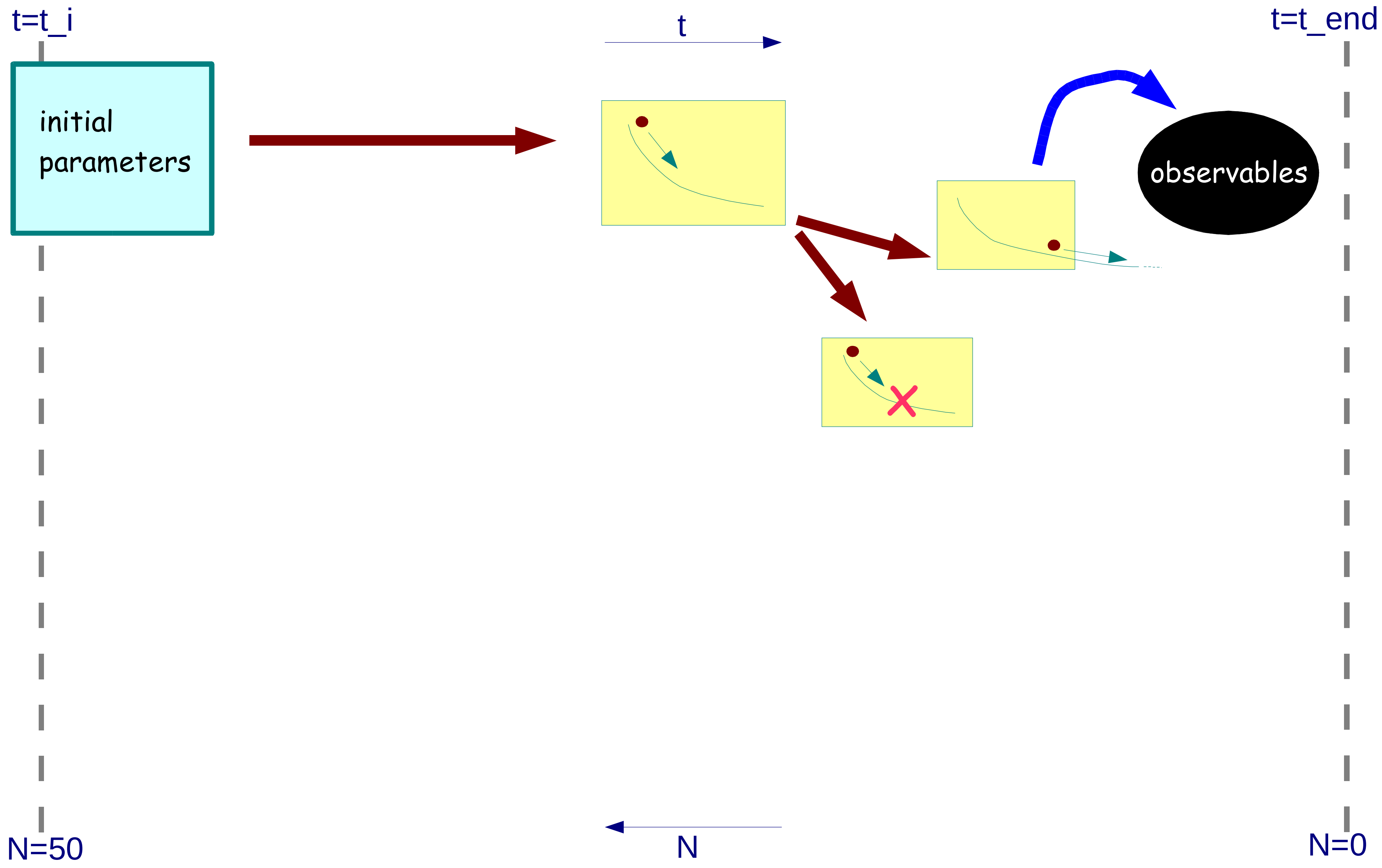
Flow Chart



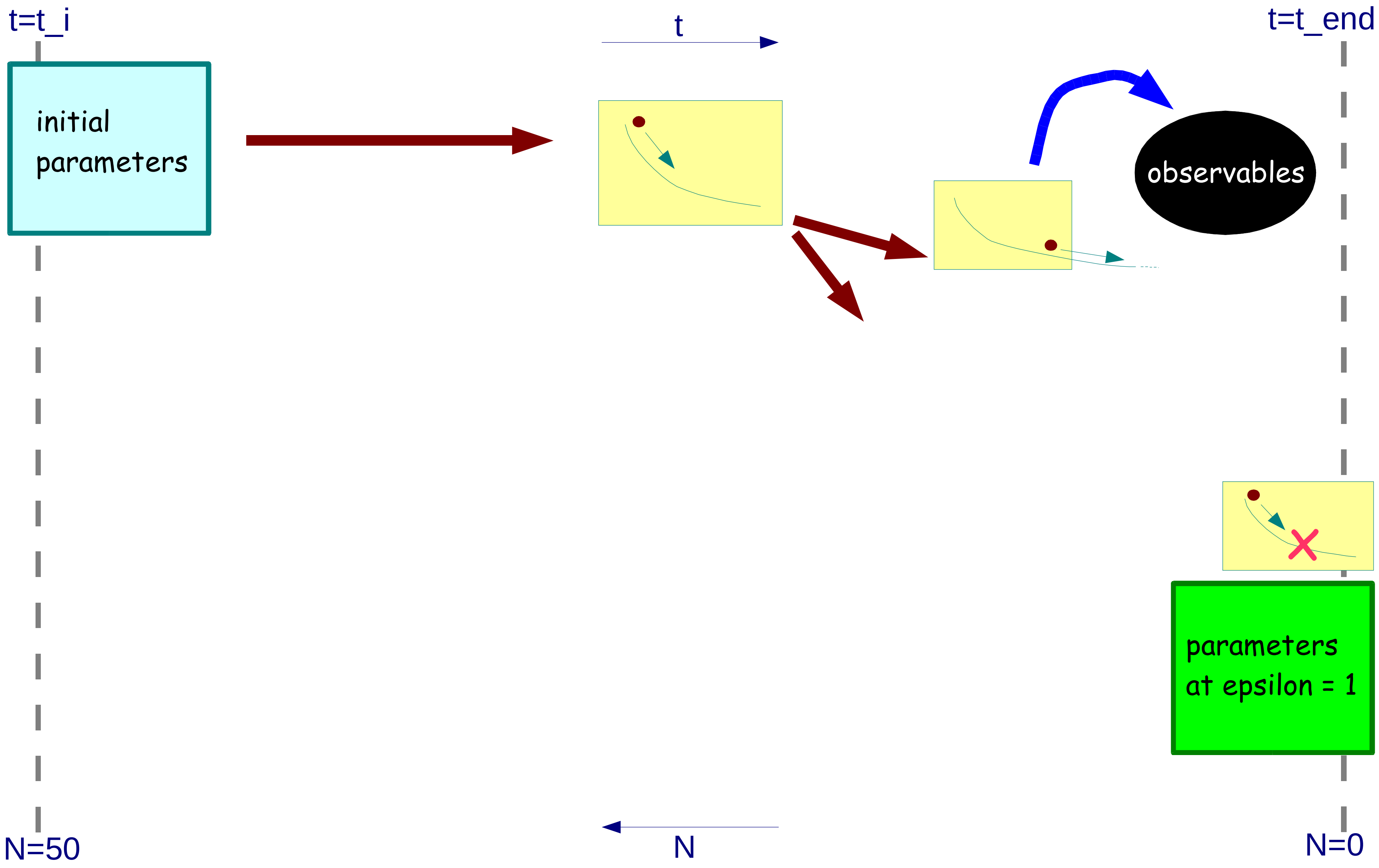
Flow Chart



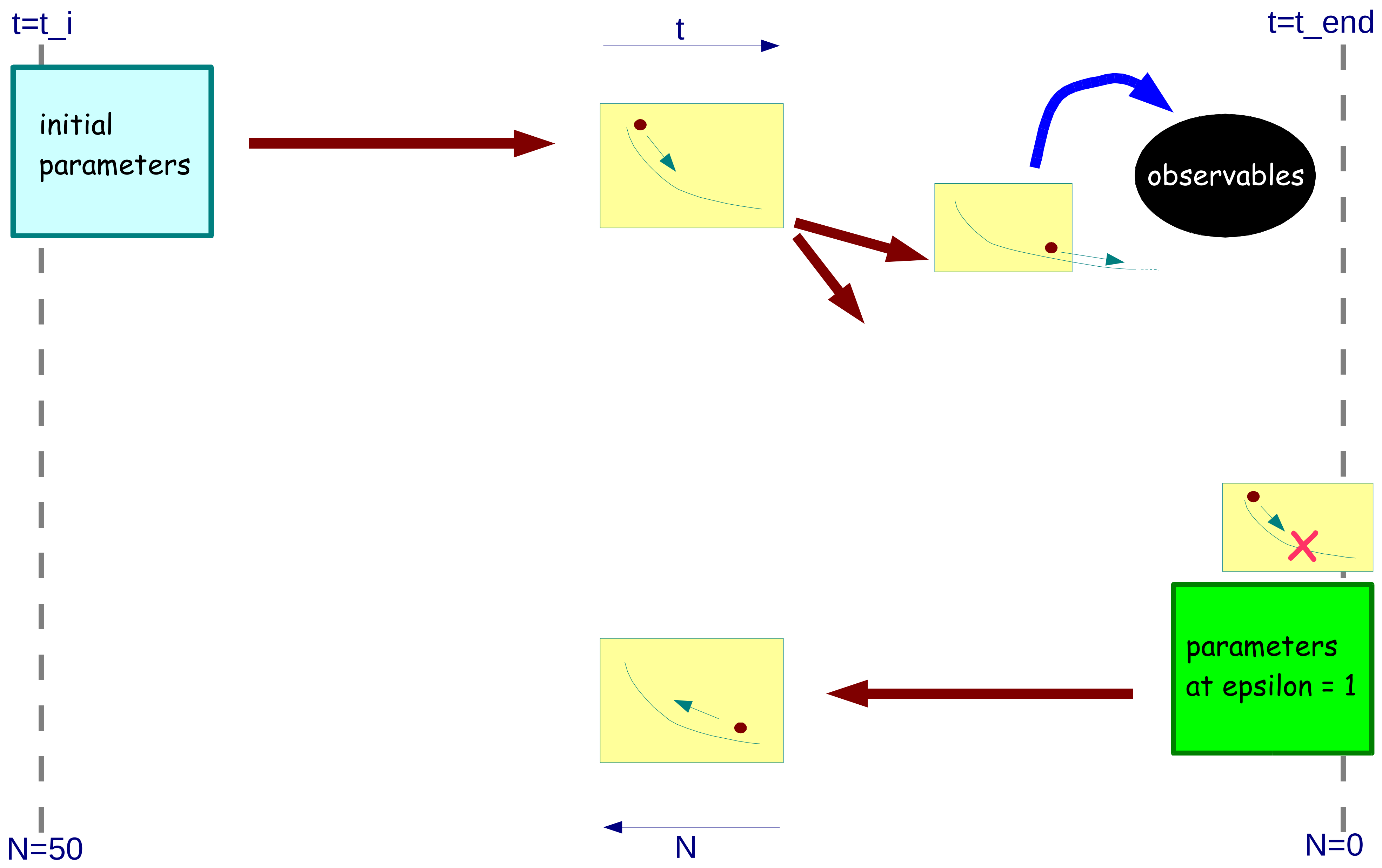
Flow Chart



Flow Chart


t=t_i
t
t=t_end
initial parameters
observables
parameters at epsilon = 1
N=50
N
N=0

Flow Chart



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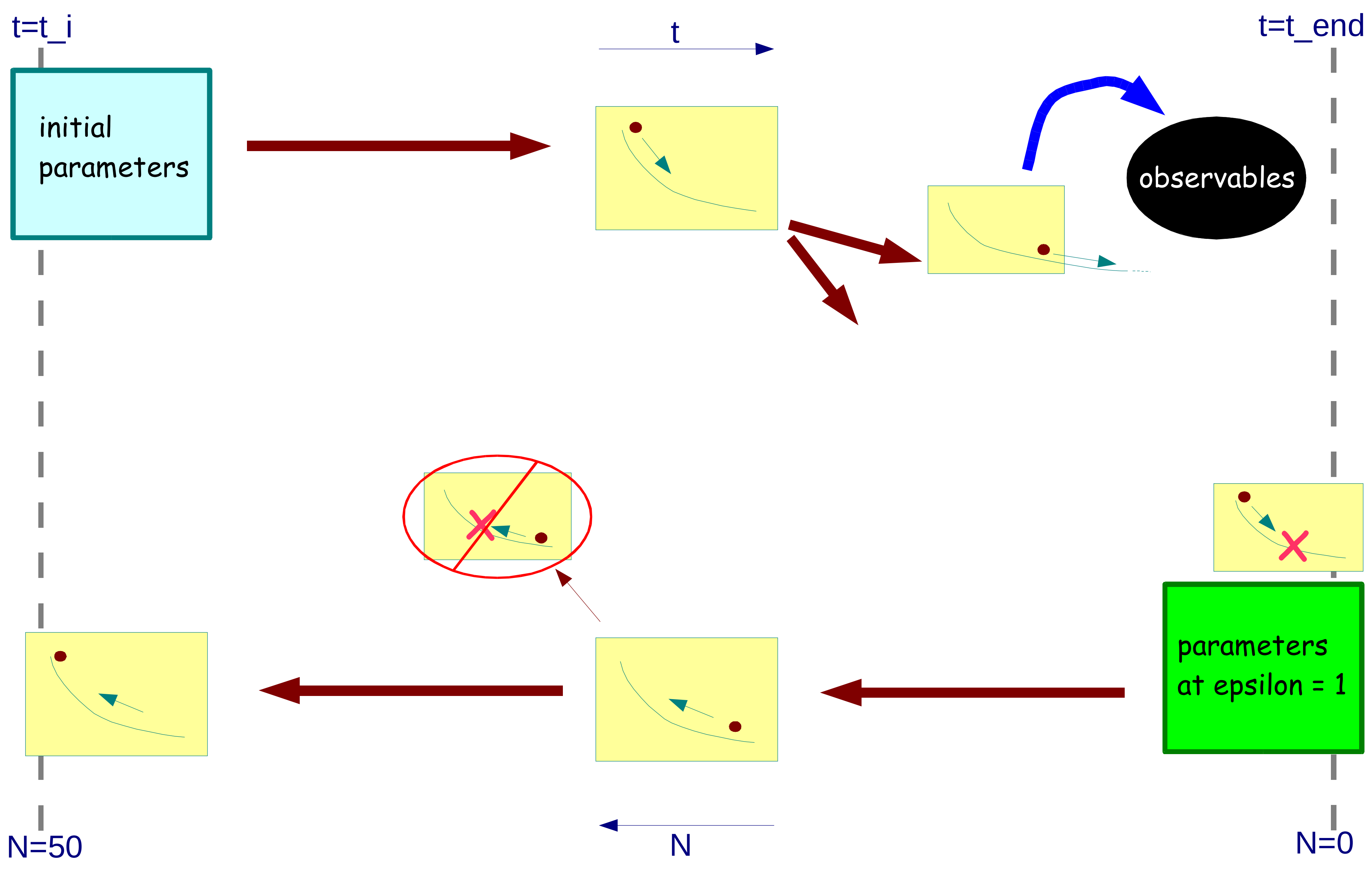
Flow Chart



t=t_i
t
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initial parameters
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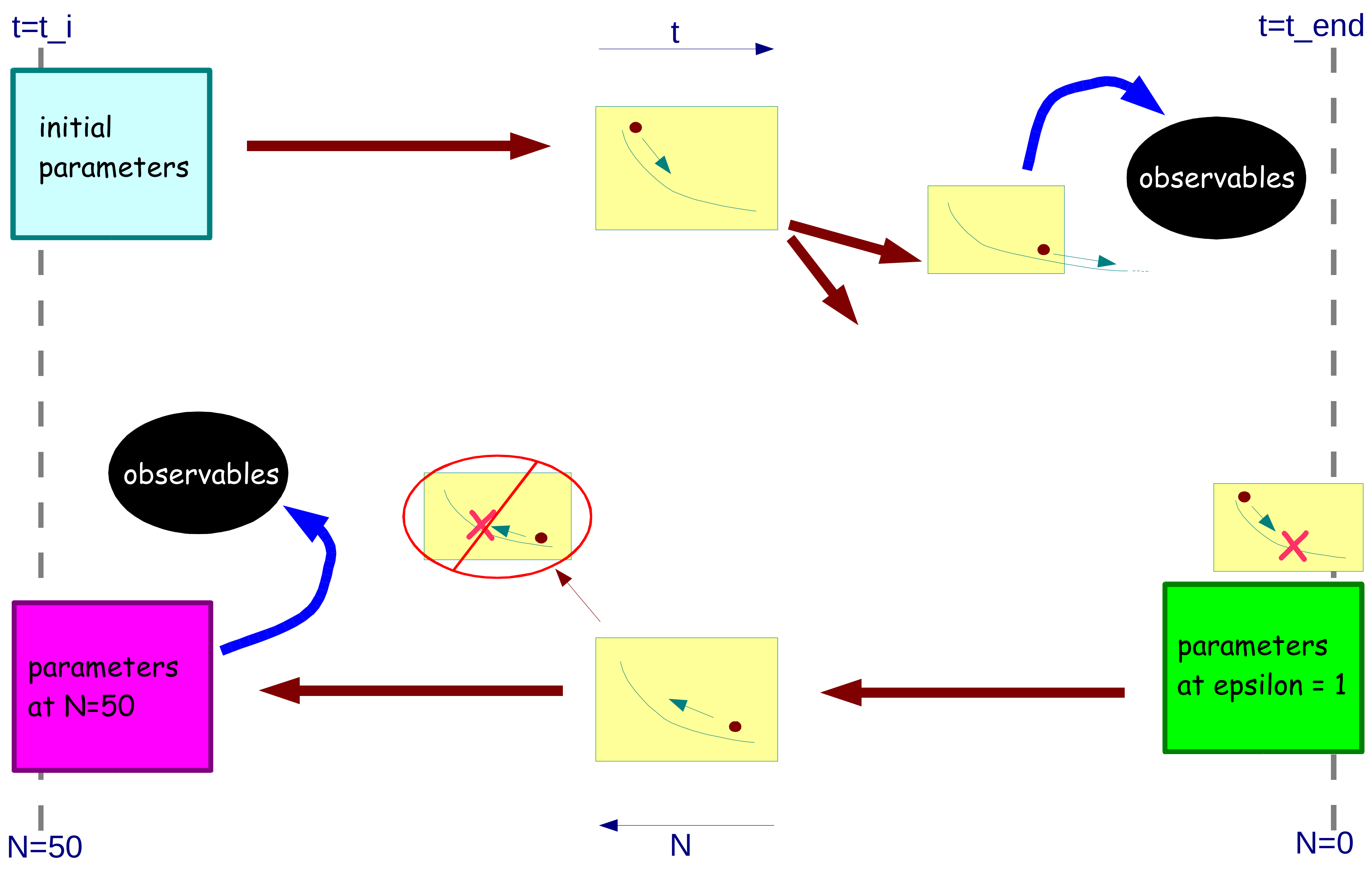
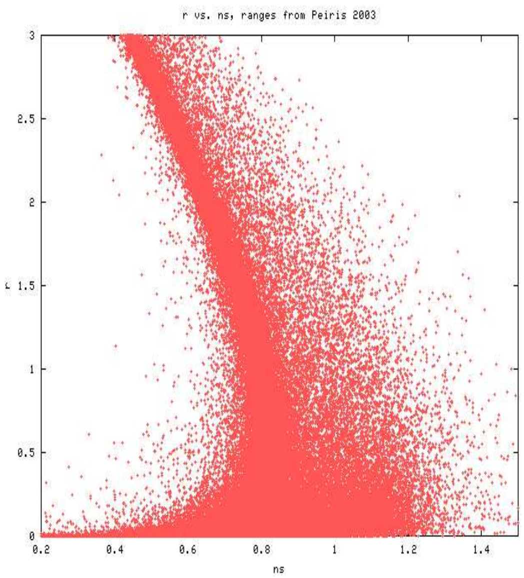
Flow Chart

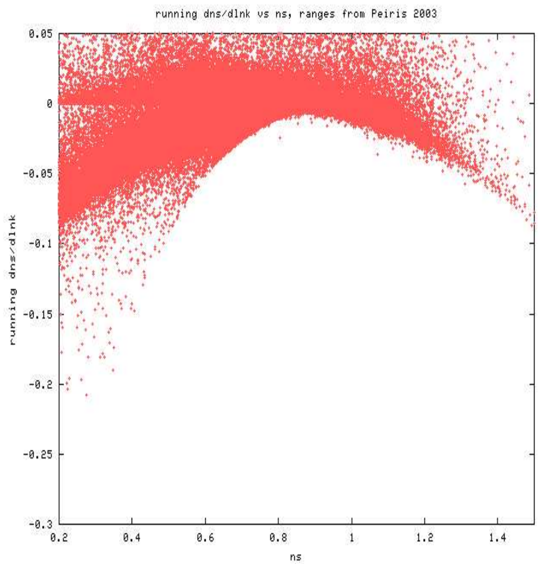

t=t_i
t
t=t_end
initial parameters
observables
observables
parameters at N=50
parameters at epsilon = 1
N=50
N
N=0

Observables at N=50

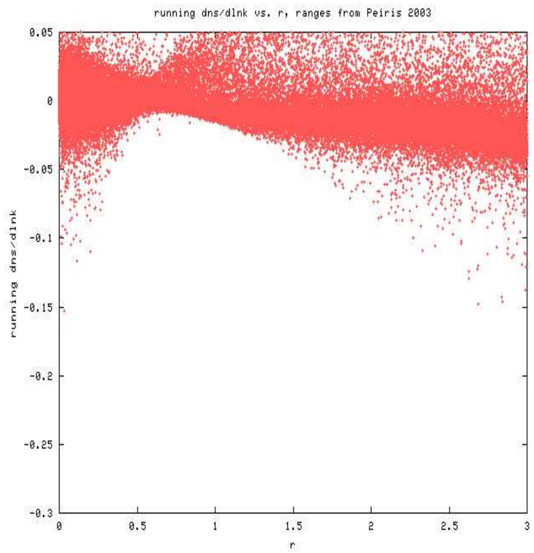
r vs. n_s



running of spectral index vs. n_s



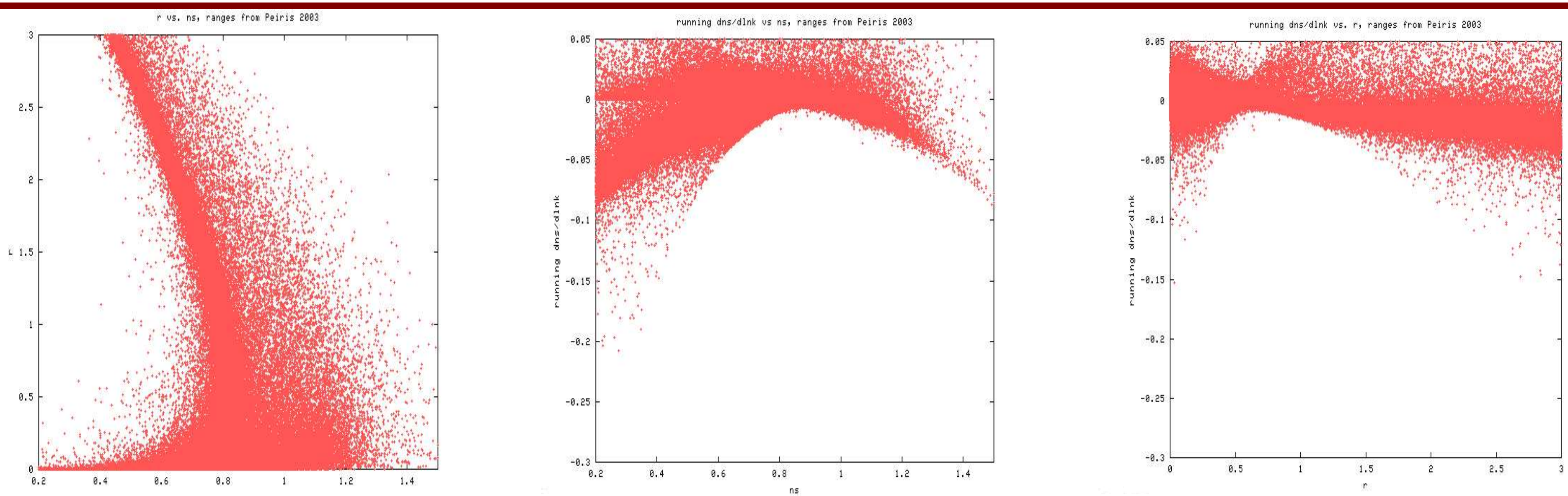
running of spectral index vs. r

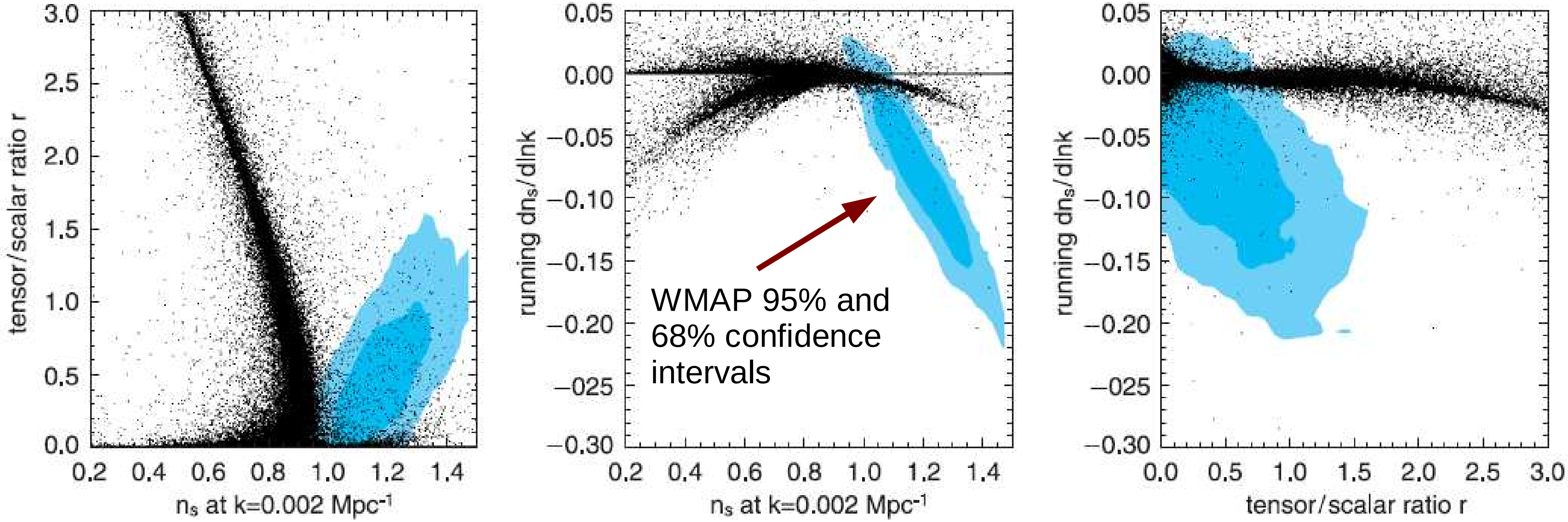


Observable plots show attractors consistent with previous simulations (e.g., Peiris (2003), Kinney (2003))

1,000,000 models simulated
~870,000 working models
(ltfp models have r=0, n>1)

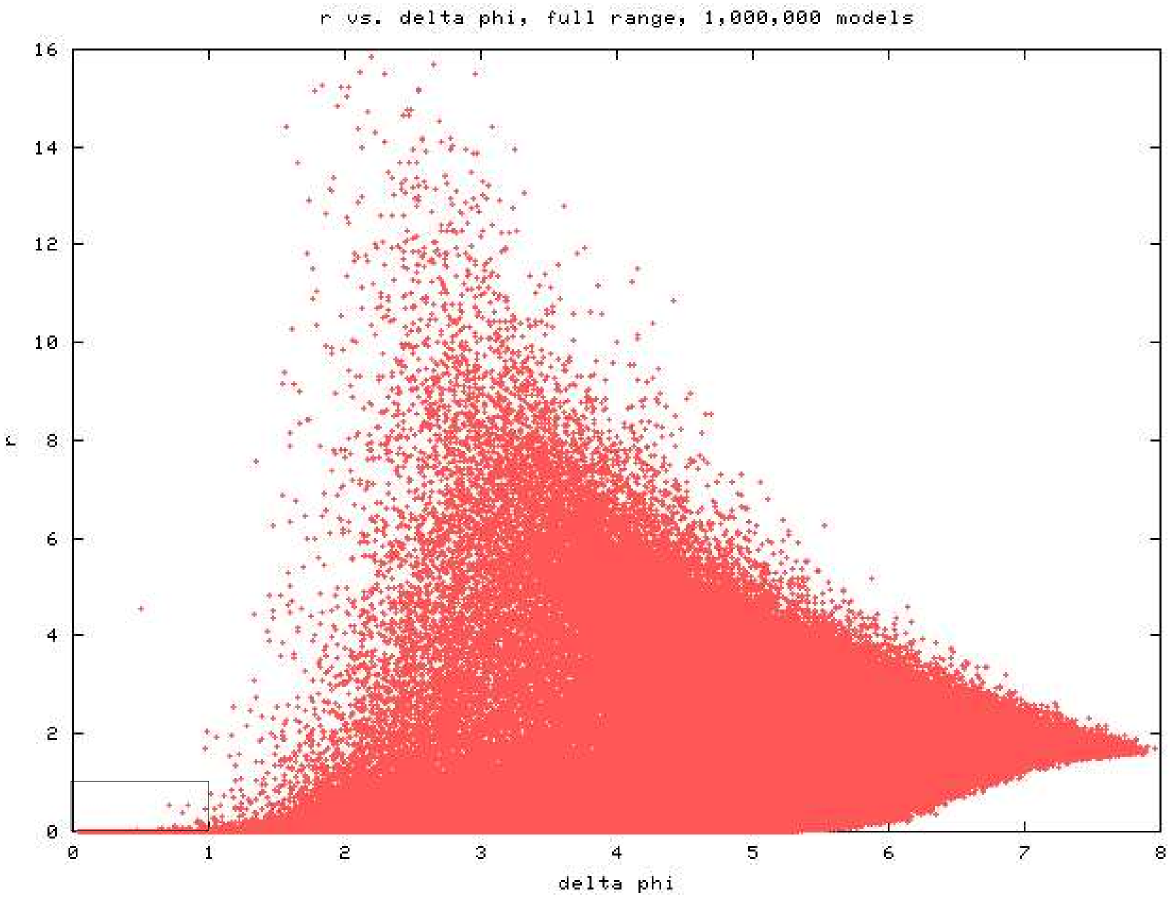
Comparison with Peiris (2003)





r vs. Δφ Results

full range of data



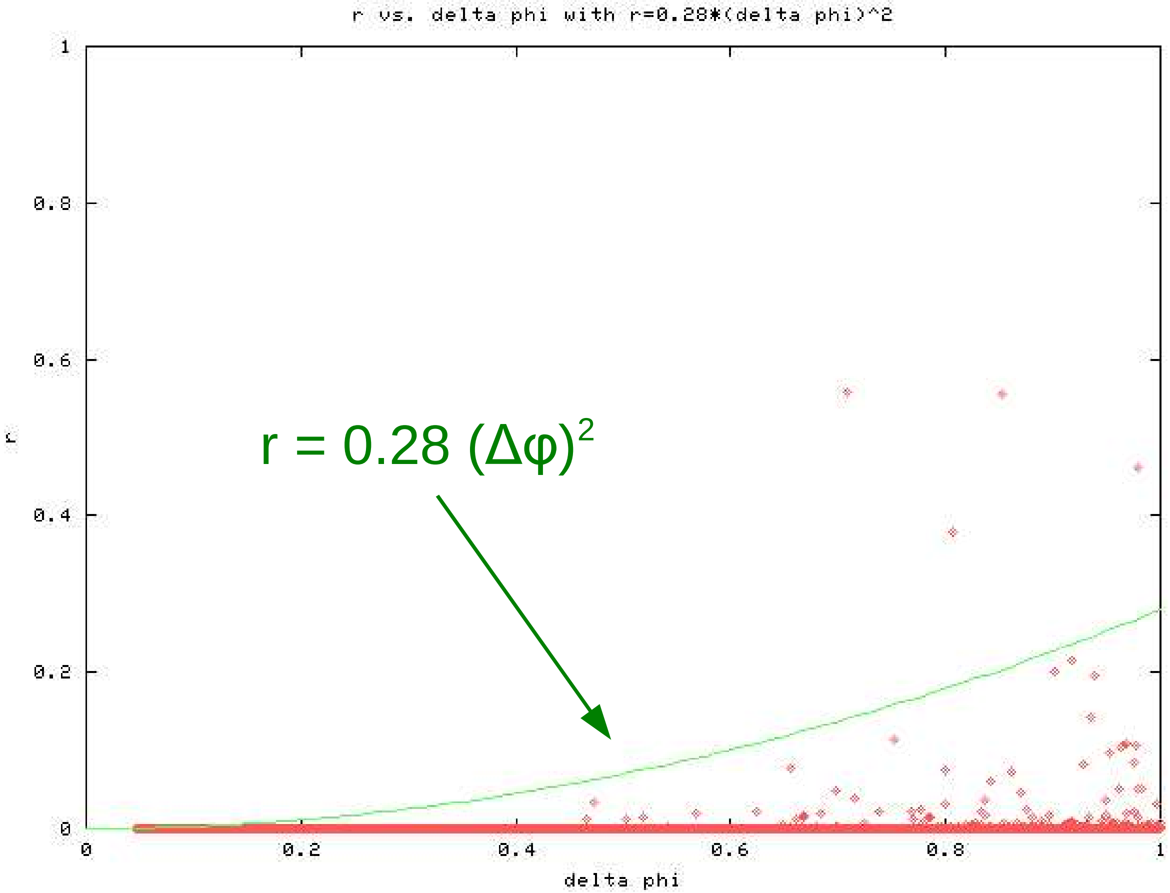
r

Δφ (units of m_{pl})

r vs. Δφ Results

data with Lyth's r, Δφ relationship

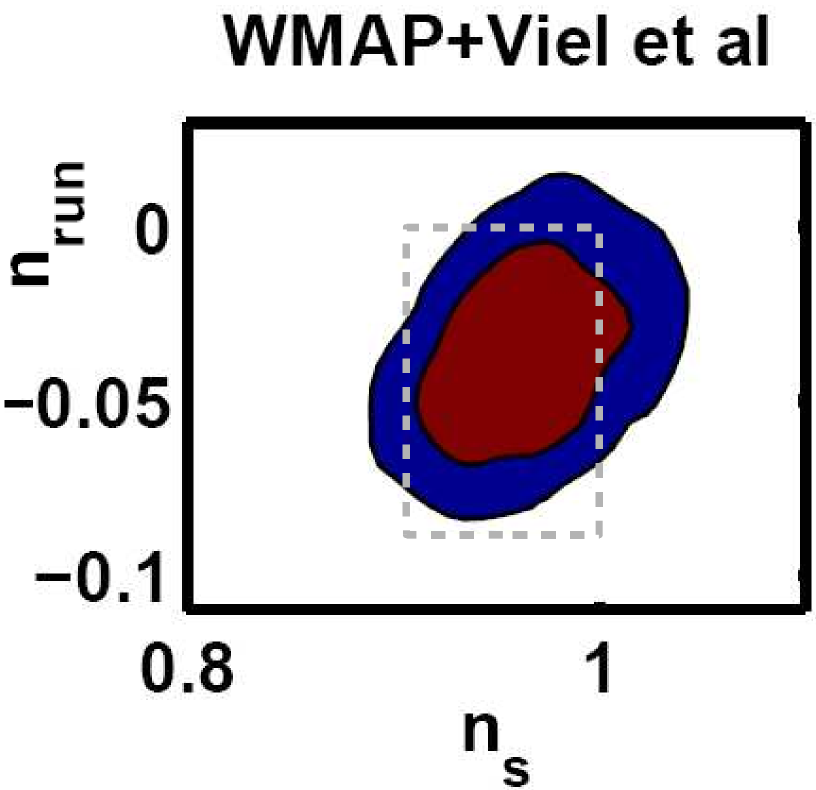
r



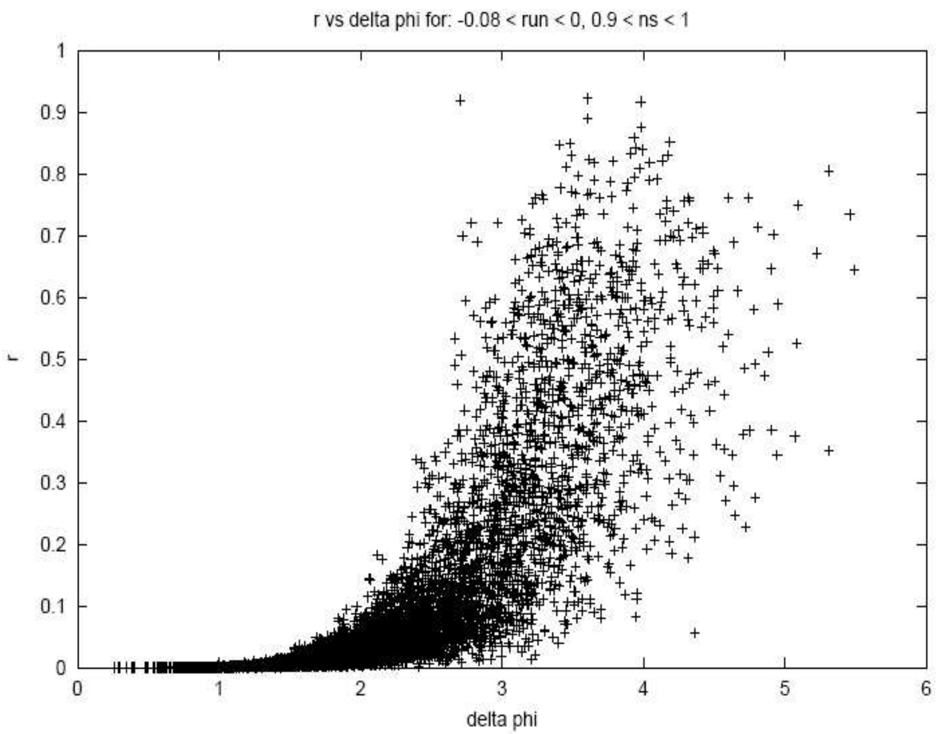
Δφ (units of m_{pl})

Preliminary Observational Cuts

Cuts: $-0.08 < n_{run} < 0$
$0.9 < n_s < 1$



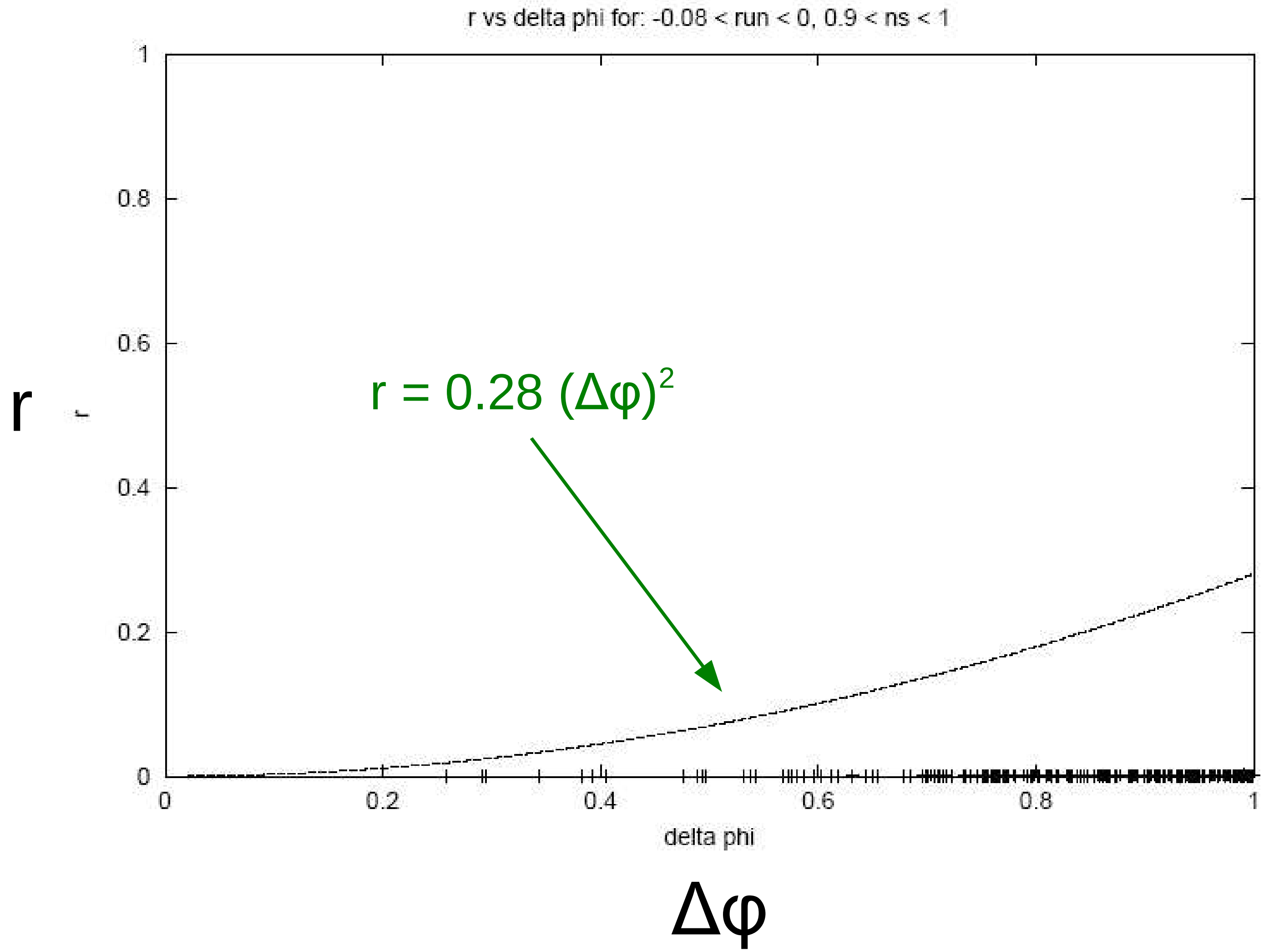
r vs. Δφ with cuts (full range)



Cuts from Lyman-alpha forest (presented at Plumian 300 by Martin Haehnelt) -- work in progress

Preliminary Observational Cuts

cut data with Lyth's r, Δφ relationship



Preliminary Observational Cuts

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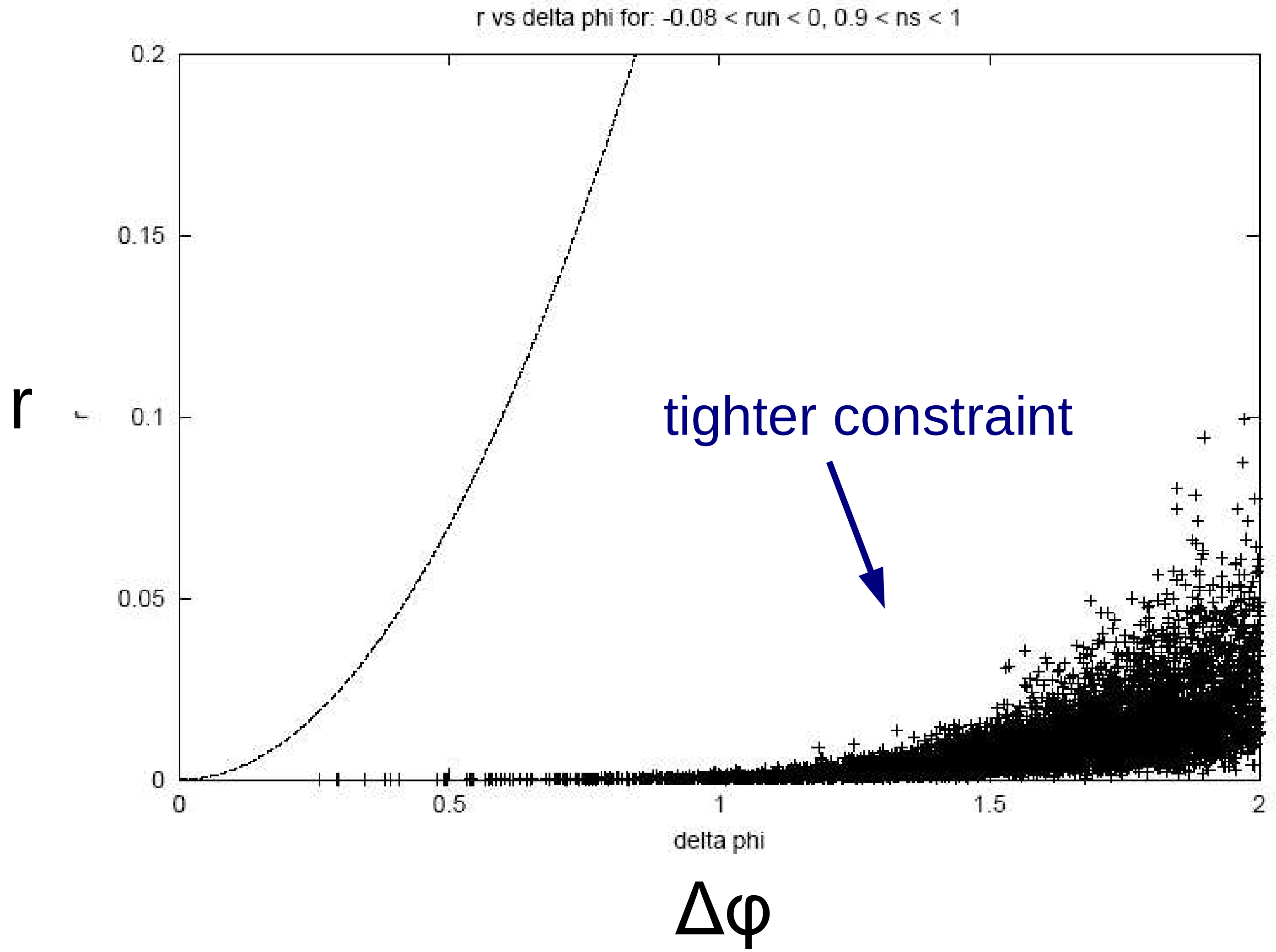


The Fine Print

- Current observational cuts only first-glance, based on work in progress (but still promising method)

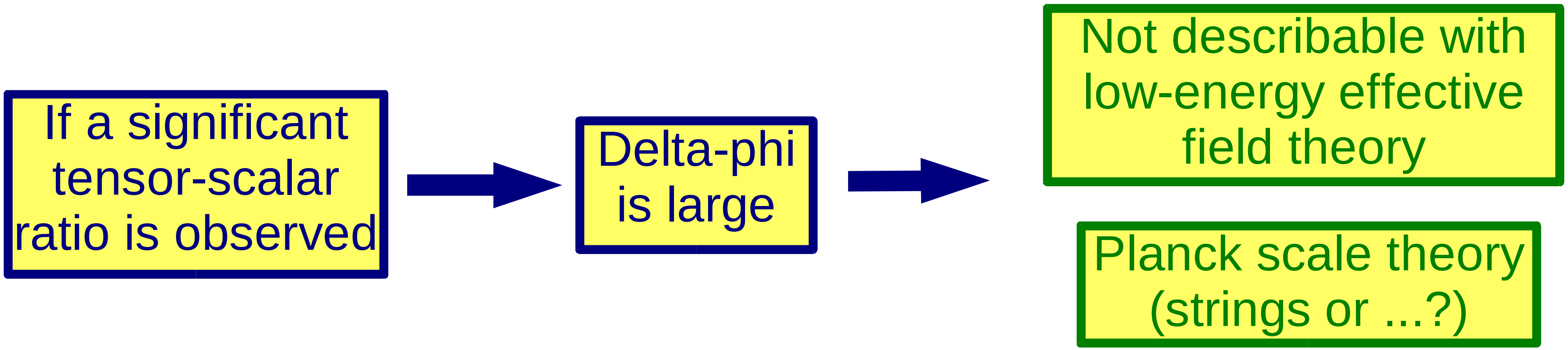
- Some limitations to this approach in general
  - Dependence on initial conditions not yet clear
    - Distribution
    - Ranges
  - Only single-field models
  - No statistical information

Future Work

- Investigate effects of changes to initial distributions and order of expansion
- Refine observable cuts (perhaps weight or categorize models in this way)

Conclusions

- Classification by delta-phi more constraining than classification by energy scale
- Significant improvements possible with observational cuts
- Main conclusion:



Further Reading by Topic

- Slow roll Monte Carlo reconstruction: Easther & Kinney,* astro-ph/0210345
- WMAP & inflation phenomenology: Kinney et al.,* hep-ph/0305130; Peiris et al.,* ApJ 148:213-231 (2003); Kinney,* New Astron. Rev. 47:967-975 (2003)
- The flow equations (and their proper use): Lyth, hep-ph/9606387; Liddle, astro-ph/0307286; Liddle & Lyth, Cosmological Inflation
- This project: Mack & Efstathiou, Phi in the Sky Conference Proceedings, (coming soon)

**Caution: Misprints – check equations carefully*