

Non-Gaussianity in the CMB

Sujata Gupta

Institute of Cosmology
& Gravitation

Measures of non-Gaussianity so far

Non-
Gaussianity
statistics

Possible
advantages of
the 2-pt
correlation
function

Types of non-
Gaussianity

Measurements of non-Gaussianity

WMAP - Komatsu et al. (2003)
 $-58 < f_{NL} < 134$

Magueijo & Medeiros (2003)
consistent with Gaussianity

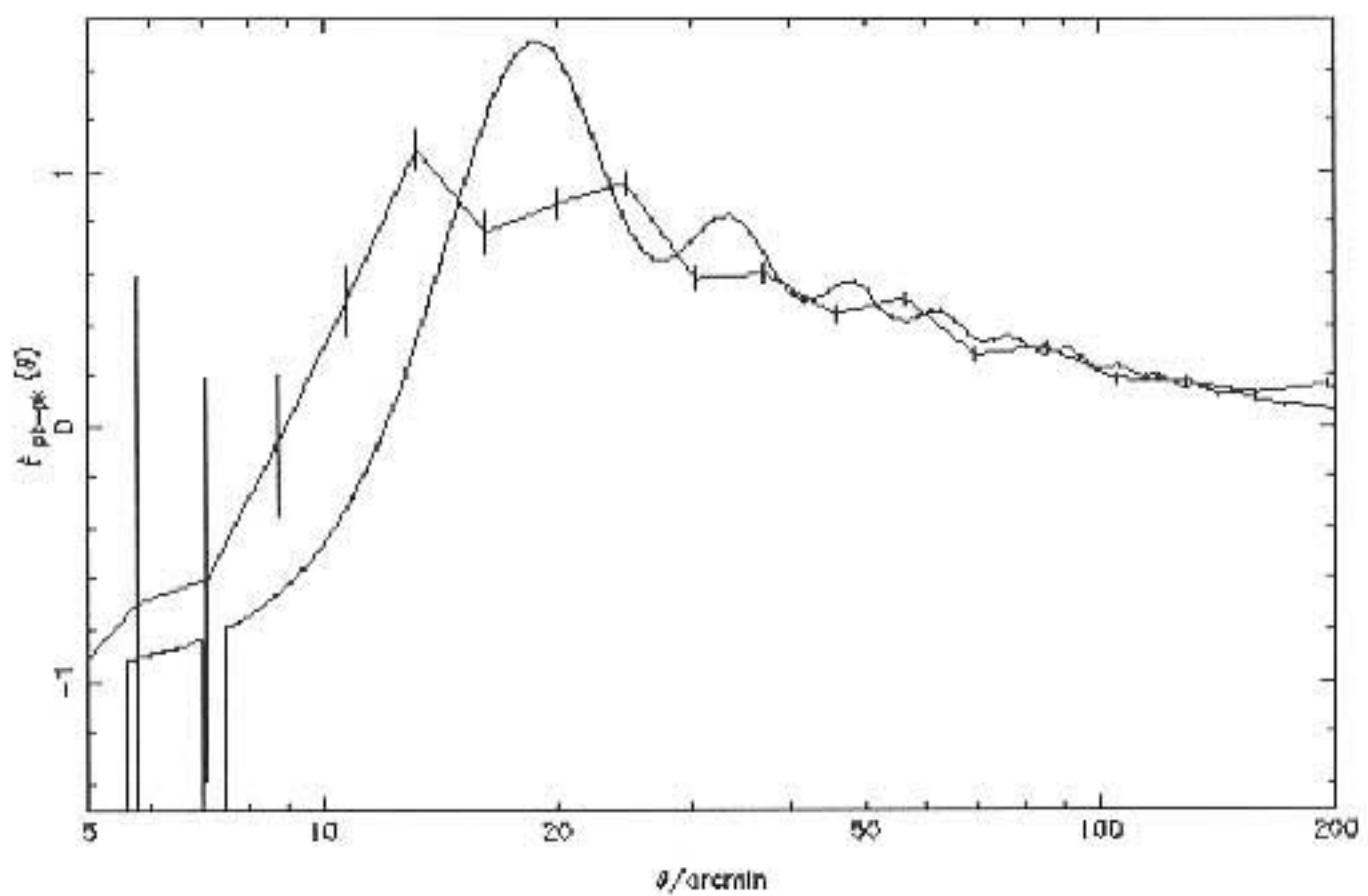
BOOMERANG AND MAXIMA
De Troia et al (2003)
consistent with Gaussianity

Measurements of non-Gaussianity

COBE -

Ferreira, Magueijo & Gorski (1998)
found significant non-Gaussianity

Banday, Zaroubi & Gorski (2000)
attributed non-Gaussianity to artefacts



Plot of peak-peak correlation function as measured from the simulated map and the predicted peak-peak correlation function for a Gaussian field with the same power spectrum.

Curvaton non-Gaussianity prediction:
 $f_{NL} = 5/4r$ Lyth et al (2002)

Warm Inflation Non-Gaussianity predictions:

f_{NL} (weak dissipation limit) = 3.83×10^{-3}
Gupta (2003)

f_{NL} (strong dissipation limit) = 3.72×10^{-3}
Gupta et al (2002)

**Planck's best measurement
is predicted to be:**

$\Delta f_{NL} \sim 5$ Komatsu and Spergel
(2001)

Wavelets:

- non-Gaussianity found in:
 - Vielva et al (2003b);
 - Cruz et al (2004);
 - McEwan et al (2004).

The 2-Point Correlation of Peaks

$$\delta(\theta, \phi) \equiv T(\theta, \phi)/T - 1,$$

$$\langle a_{\ell m} a_{\ell' m'}^* \rangle = C_\ell \delta_{\ell \ell'}^K \delta_{m m'}^K ,$$

$$\delta(\theta, \phi) = \sum_{m=-\ell, \ell; \ell=0, \infty} a_{\ell m} Y_\ell^m(\theta, \phi)$$

$$\mathbf{v} = (\delta, \delta_\phi, \delta_\theta, \delta_{\phi\phi}, \delta_{\phi\theta}, \delta_{\theta\theta}) ,$$

$$p(\mathbf{v}_1, \mathbf{v}_2) = \frac{1}{(2\pi)^6 \|M\|^{1/2}} \exp\left(-\frac{1}{2} v_i M_{ij}^{-1} v_j\right).$$

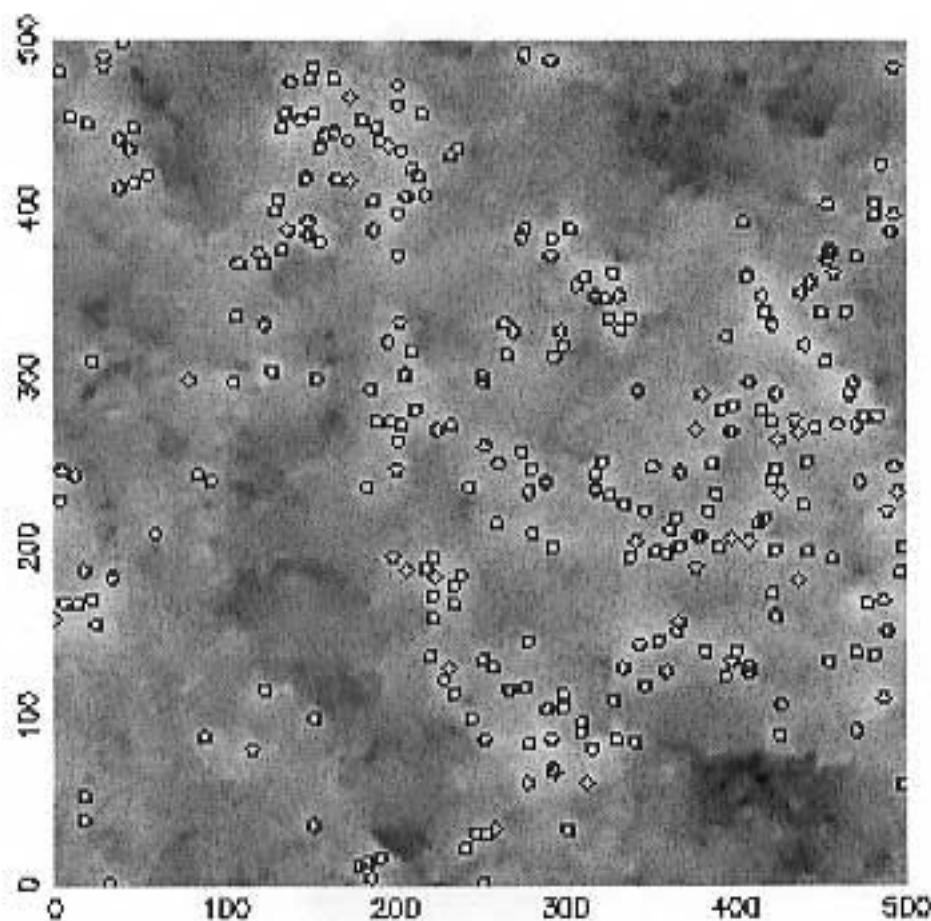
The 2-Point Correlation of Peaks

$$1 + \xi(r|\nu_1, \nu_2)$$

$$\begin{aligned} &= \frac{1}{4\theta^4 n_{ph}(\nu_1) n_{ph}(\nu_2)} \int_{X_1=0}^{\infty} \int_{X_2=0}^{\infty} \int_{Y_1=-X_1}^{X_1} \int_{Y_2=-X_2}^{X_2} \int_{Z_1=-\sqrt{X_1^2 - Y_1^2}}^{\sqrt{X_1^2 - Y_1^2}} \int_{Z_2=-\sqrt{X_2^2 - Y_2^2}}^{\sqrt{X_2^2 - Y_2^2}} dX_1 dX_2 dY_1 dY_2 dZ_1 dZ_2 \\ &\times (X_1^2 - Y_1^2 - Z_1^2) (X_2^2 - Y_2^2 - Z_2^2) p(\nu_1, X_1, Y_1, Z_1, \eta_{\phi,0}^{(1)} = 0, \nu_2, X_2, Y_2, Z_2, \eta_{\phi,0}^{(2)} = 0). \end{aligned}$$

where:

$$\begin{aligned} \nu &= \frac{\delta}{\sigma_0} \\ \eta_\phi &= \frac{\delta_\phi}{\sigma_1} \\ \eta_\theta &= \frac{\delta_\theta}{\sigma_1} \\ X &= -\frac{(\delta_{\phi\phi} + \delta_{\theta\theta})}{\sigma_2} \\ Y &= \frac{(\delta_{\phi\phi} - \delta_{\theta\theta})}{\sigma_2} \\ Z &= \frac{2\delta_{\phi\theta}}{\sigma_2} \end{aligned}$$



12.5' \times 12.5' simulation of
cosmic string structure with
peaks over 1σ circled

The Bispectrum and related quantities

$$\Phi(\mathbf{x}) = \Phi_L(\mathbf{x}) + f_{NL} (\Phi_L^2(\mathbf{x}) - \langle \Phi_L^2(\mathbf{x}) \rangle),$$

$$\langle \Phi_L(\mathbf{k}_1) \Phi_L(\mathbf{k}_2) \Phi_{NL}(\mathbf{k}_3) \rangle = 2(2\pi)^3 \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) f_{NL} P_\Phi(k_1) P_\Phi(k_2),$$

$$\Phi_3 \equiv A_{\text{inf}} \equiv 2f_{NL} \equiv A_{\text{inf}}(2\pi)^6.$$

Conclusions

Using f_{NL} predictions and simulations from defect models, the shape of the peak-peak correlation function can be used to search for the physics behind the non-Gaussianity in the CMB.