Scalar field constraints from homogeneous cosmology

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• Physical context: Lagrangian with scalar fields



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- Geometrical context: homogeneous cosmologies



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- Constraining ϕ by requiring Universe isotropisation



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- Supernovae constraints once isotropisation reached
- Conclusion

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Cosmological model

A cosmological model is specified by:

- A Lagrangian describing the Universe content
 ⇒ perfect fluid + scalar field
- A metric describing the Universe geometry
 ⇒ homogeneous but anisotropic metric



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Some reasons to consider a scalar field



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• A spin zero boson



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- Higgs field



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- dark energy



Lagrangian with a minimally coupled and massive ϕ :

 $L = R - \omega \phi^{\mu} \phi_{\mu} \phi^{-1} - U + L_m$



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 - Brans-Dicke function ω
 - Potential U: effective Λ
 - L_m : perfect fluid Lagrangian with $p_m = (\gamma 1)\rho_m$ and $\gamma \in [1, 2]$



Universe geometry: FLRW

• Homogeneous and isotropic FLRW models

$$ds^{2} = -dt^{2} + a(t)^{2} \left(\frac{dr^{2}}{1 - kr^{2}} + r^{2}(d\theta^{2} + sin^{2}\theta d\phi^{2})\right)$$

- expansion is the same everywhere in any direction
- CMB is isotropic



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However:

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 \Longrightarrow Let us remove the isotropy hypothesis



Universe geometry: Bianchi

- Spatially homogeneous models of Bianchi $ds^2 = -dt^2 + a(t)(\omega^1)^2 + b(t)(\omega^2)^2 + c(t)(\omega^3)^2$
 - anisotropic expansion: $a(t) \neq b(t) \neq c(t)$



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 - anisotropic expansion: $a(t) \neq b(t) \neq c(t)$
 - 9 Bianchi models defined by the 1-forms ω^i
 - Example: the flat Bianchi type I model $\omega^1 = dx, \, \omega^2 = dy, \, \omega^3 = dz$



How to constrain ϕ ?

• Looking for ϕ properties implying Universe isotropisation



How to constrain ϕ ?

- Looking for ϕ properties implying Universe isotropisation
- Mathematical tools:
 - The ADM Hamiltonian formalism: it allows getting a first order field equations system(Nariai72, Matzner73)
 - The dynamical system analysis: it allows studying the field equations (WaiEli97)



Defining isotropy

(ColHaw73):

- It occurs asymptotically, during a forever expansion
- The anisotropy parameters Σ_{\pm} vanish

In our models, Σ_{\pm} vanish before the CMB time



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Defining isotropy

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In our models, Σ_{\pm} vanish before the CMB time There are three ways to reach isotropy:

- class 1: Ω_m and Ω_ϕ reach eq with $\Omega_\phi \neq 0$
- class 2: Ω_m and Ω_ϕ reach eq with $\Omega_\phi \to 0$
- class 3: Ω_m and/or Ω_ϕ do not always reach eq

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• class 1 ($\Omega_{\phi} \not\rightarrow 0$)



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- Bianchi class A models.

Let us start with the flat Bianchi type I model and one minimally coupled ϕ



Results for Bianchi I 1/3

(Fay01, Fay01A)

	Constraints on ϕ
	(Isotropisation time)
$\Omega_m = 0$	$\ell^2 \rightarrow \ell_0^2 < 3$: spatial dimension
$\Omega_m \to 0$	$\ell^2 \to \ell_0^2 < \frac{3\gamma}{2} < 3$
$\Omega_m \not\to 0$	$\ell^2 \to \ell_0^2 > \frac{3\gamma}{2}$

with $\ell = \frac{U_{\phi}}{U} \sqrt{\frac{\phi}{2\omega}}$ Constraint on the scalar field mass



Results for Bianchi I 2/3

	Asymptotical behaviours
	(Late times behaviours)
$\Omega_m = 0$	si $\ell_0^2 \not\rightarrow 0$, $a \rightarrow t^{\ell_0^{-2}}$ and $U \rightarrow t^{-2}$
	si $\ell_0^2 \rightarrow 0$, De Sitter
$\Omega_m \to 0$	idem
$\Omega_m \not\to 0$	$a \to t^{\frac{2}{3\gamma}} \text{ and } U \to t^{-2}$



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Results for Bianchi I 3/3

	ϕ state equation	Quintessence
$\Omega_m = 0$	$p_{\phi} = (\frac{2}{3}\ell_0^2 - 1)\rho_{\phi}$	Yes if $\ell_0^2 < 3/2$
$\Omega_m \to 0$	idem	idem
$\Omega_m \not\to 0$	$p_{\phi} = (\gamma - 1)\rho_{\phi}$	No

 $\Omega_m \not\rightarrow 0$: no scalar field effect and thus $a \rightarrow t^{\frac{2}{3\gamma}}$



For the other Bianchi class A models:

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- $\Omega_m \not\rightarrow 0$:
 - Isotropisation is impossible
- \forall the model: coincidence problem ($\Omega_m \propto \Omega_{\phi}$)





(FayLuminet04)

$$L = R - \omega \phi^{,\mu} \phi_{,\mu} \phi^{-1} - \mu \psi^{,\mu} \psi_{,\mu} \psi^{-1} - U + L_m$$



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Assumptions:

- Flat Bianchi type I model
- 2 types of theories
 - 1. $\omega(\phi)$, $\mu(\psi)$ and $U(\phi, \psi)$
 - 2. $\omega(\phi, \psi)$, $\mu(\psi)$ and $U(\psi)$



 $\omega(\phi), \mu(\psi) \text{ and } U(\phi, \psi)$



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 $\omega(\phi), \mu(\psi) \text{ and } U(\phi, \psi)$

• Hybride inflation (Copeland and al 94)



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- Example:

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Conformal transformation \Rightarrow



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- Example:

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Conformal transformation \Rightarrow

 $L = R - \phi^{,\mu}\phi_{,\mu} - \psi^{,\mu}\psi_{,\mu}$ $-U_0 e^{-\sqrt{2/3}k\phi} e^{-5\sqrt{3}/6k\psi} (e^{\sqrt{3}/2\psi} - 1)^m + L_m$



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Results



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With 2 scalar fields 3/6 Results

• Results got with one scalar field are generalised: $\ell^2 \rightarrow \ell^2_{\phi_1} + \ell^2_{\psi_1}$ avec

$$\ell_{\phi_1} = \frac{U_{\phi}}{U} \sqrt{\frac{\phi}{2\omega}}$$
$$\ell_{\psi_1} = \frac{U_{\psi}}{U} \sqrt{\frac{\psi}{2\mu}}$$



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$$\ell_{\psi_1} = \frac{U_{\psi}}{U} \sqrt{\frac{\psi}{2\mu}}$$

• can not be detected observationally



 $\omega(\phi,\psi),\,\mu(\psi)$ and $U(\psi)$

• complexe scalar field



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• complexe scalar field $L = R + g^{\mu\nu}\zeta^*_{,\mu}\zeta_{,\nu} - V(|\zeta|^2) + L_m$



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• complexe scalar field $L = R + g^{\mu\nu}\zeta^*_{,\mu}\zeta_{,\nu} - V(|\zeta|^2) + L_m$ Transformation: $\zeta = \psi(\sqrt{2}m)e^{-im\phi}$



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- Some potential examples
 - $U = \zeta \zeta^* = \psi^2$ (Iorio and al 01,Gu and al01)
 - $U = \lambda/2(\psi^2 \eta^2)^2$ (Kasuya and al 98)(topological defects)



Results



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Results

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If $\Omega_m \to const \Rightarrow a \to t^{\frac{2}{3\gamma}}$: no acceleration



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 \Rightarrow always the coincidence problem



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Now, we assume that Universe has reached isotropy. What are then the physical constraints on ℓ_0 with such a constant scalar field equation of state



Contraints on ℓ_0

2 types of constraints when the Universe has reached isotropy

• WMAP: $l_0^2 < 0.33$

• What about supernovae constraints?


Some data

Daly and al (A.P.J., 597, 2003) 92 supernovae and 20 radiogalaxies





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No assumption on curvature or Ω_m



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No assumption on curvature or Ω_m

• $\Rightarrow \chi^2 \simeq 117.208$ and $\ell_0^2 = 0.03$ with $\ell_0^2 < 0.75$



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• $z_a = 0.6$, acceleration redhsift is too hight: Riess 01





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- ℓ_0^2 bounding by a constant depending on space dimension, curvature, PF
- In the isotropy vicinity, Universe state is completly described by ℓ_0
- In particular, the dark energy equation of state is a constant



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- Two problems: Coincidence and z_a
- One solution: \u03c6 non minimally coupled to the perfect fluid coincidence problem may be solved (Chimento 03) Compatible with isotropisation!

