# Tests of scalar-tensor gravity

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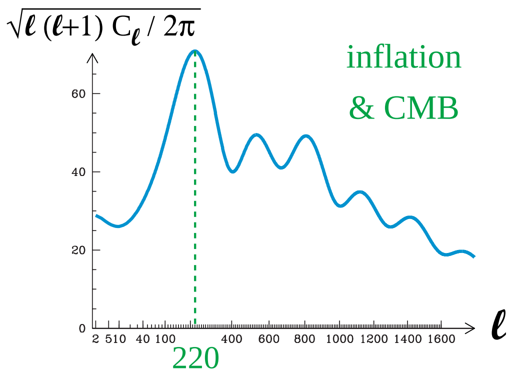
The most natural theories of gravity include
a scalar field $\varphi$ besides the metric $g_{\mu\nu}$

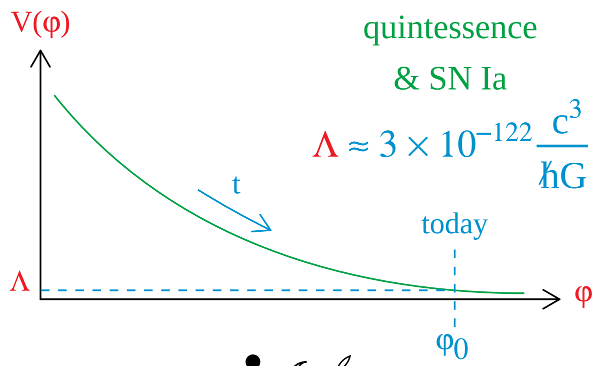
• Mathematically **consistent field theories** (no ghost, no adynamical field)

• **Motivated** by superstrings
– dilaton in the graviton supermultiplet
– moduli after dimensional reduction

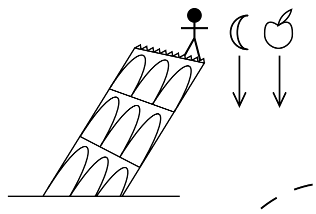
$$g_{mn} = \left(\begin{array}{c|c} g_{\mu\nu} & A_\mu \\ \hline A_\nu & \varphi \end{array}\right)$$

• Scalar fields play a crucial role in modern **cosmology**
(potential $V(\varphi)$ ≈ negative pressure ⇒ accelerated expansion phases of the universe)

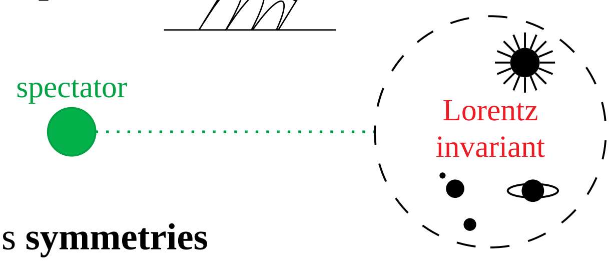



• Only consistent massless field theories able to satisfy the **weak equivalence principle**



• Only known theories satisfying "**extended Lorentz invariance**"




• Preserve most of general relativity's **symmetries**
(explain the key role of $\beta^{\mathrm{PPN}}$ and $\gamma^{\mathrm{PPN}}$)

• Useful as **contrasting alternatives** to general relativity
(simple, but general enough ⇒ many possible deviations)

## Tensor – scalar theories

spin 2 ↓ $R$ , spin 0 ↓ $-\frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi)$

$$S = \frac{c^3}{4\pi G}\int\sqrt{-g}\,\left\{\frac{R}{4} - \frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi)\right\} + S_{\text{matter}}\Big[\text{matter}\,;\ \underbrace{\tilde{g}_{\mu\nu}}_{\text{physical metric}} \equiv A^2(\varphi)\,g_{\mu\nu}\Big]$$

## Solar-system constraints

• "PPN" formalism to study weak-field gravity (order Newton $\times \frac{1}{c^2}$)

[Eddington, Schiff, Baierlin, Nordtvedt, Will]

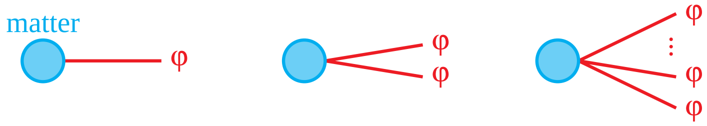
$$\begin{cases} -g_{00} = 1 - 2\,\dfrac{Gm}{rc^2} + 2\,\beta^{PPN}\left(\dfrac{Gm}{rc^2}\right)^2 + \dots \\ g_{ij} = \delta_{ij}\left[1 + 2\,\gamma^{PPN}\,\dfrac{Gm}{rc^2} + \dots\right] \end{cases}$$

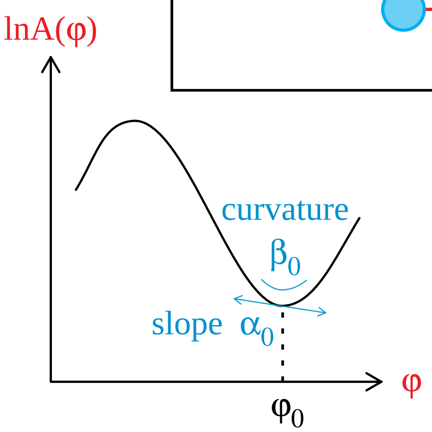
• In scalar-tensor gravity

If $V''(\varphi) = m_\varphi^2 \gg (\text{A.U.})^{-2} \Rightarrow$ $\varphi$ negligible

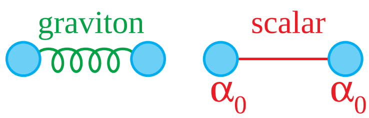
If $V''(\varphi) = m_\varphi^2 \ll (\text{A.U.})^{-2} \Rightarrow$ matter-scalar coupling function $A(\varphi)$ strongly constrained

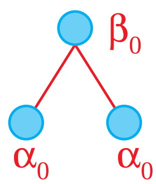
$$\boxed{\ln A(\varphi) = \alpha_0\,(\varphi-\varphi_0) + \frac{1}{2}\beta_0\,(\varphi-\varphi_0)^2 + \dots}$$



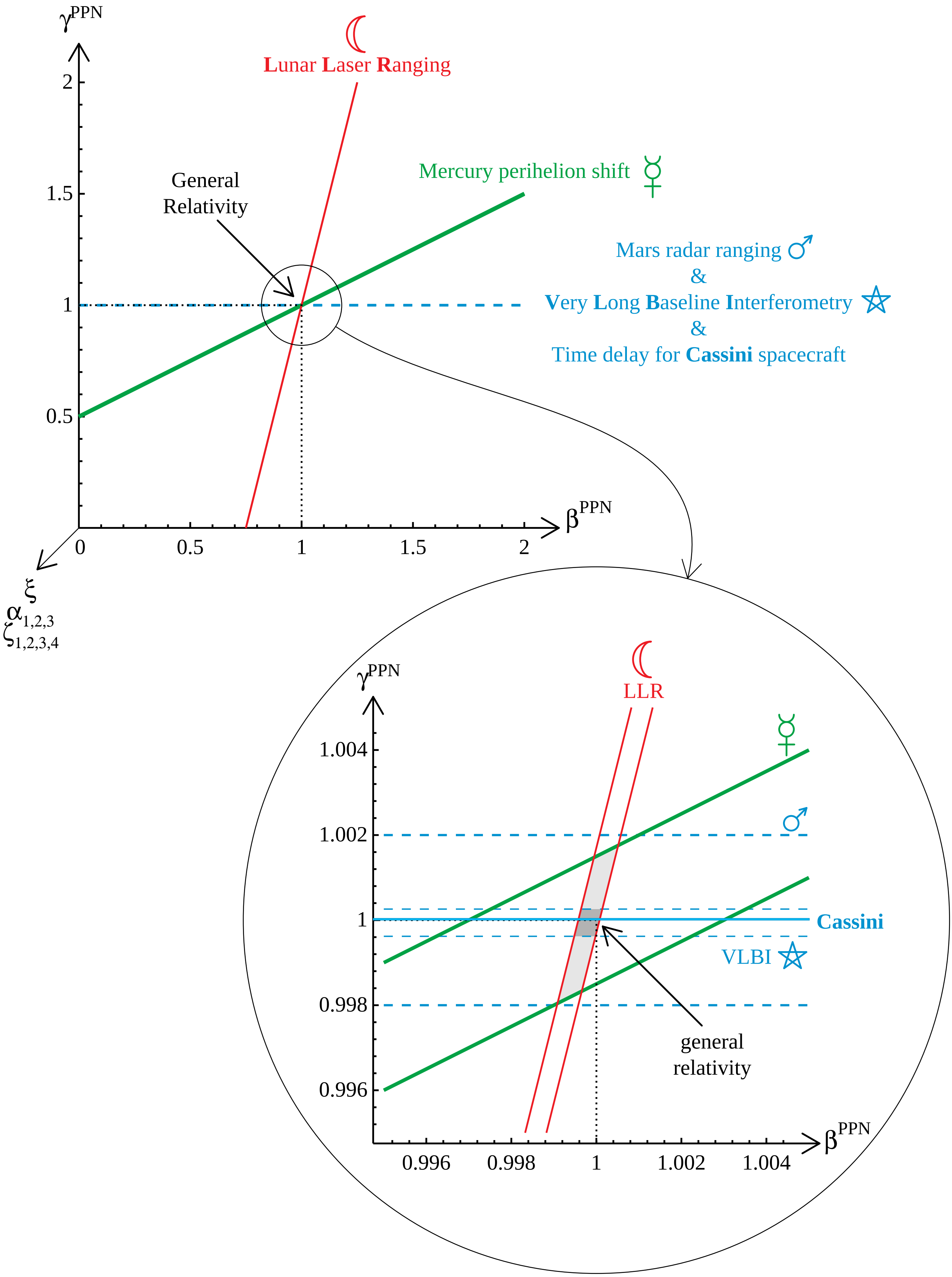




$$\begin{cases} G_{\text{eff}} = G\,(1 + \alpha_0^2) \\ \gamma^{PPN} - 1 \propto \alpha_0^2 \\ \beta^{PPN} - 1 \propto \alpha_0^2\,\beta_0 \end{cases}$$

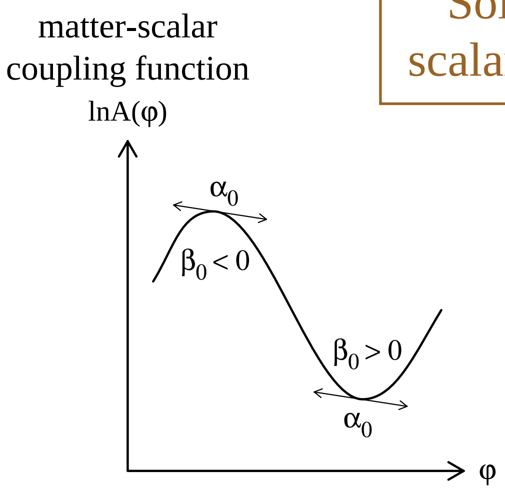


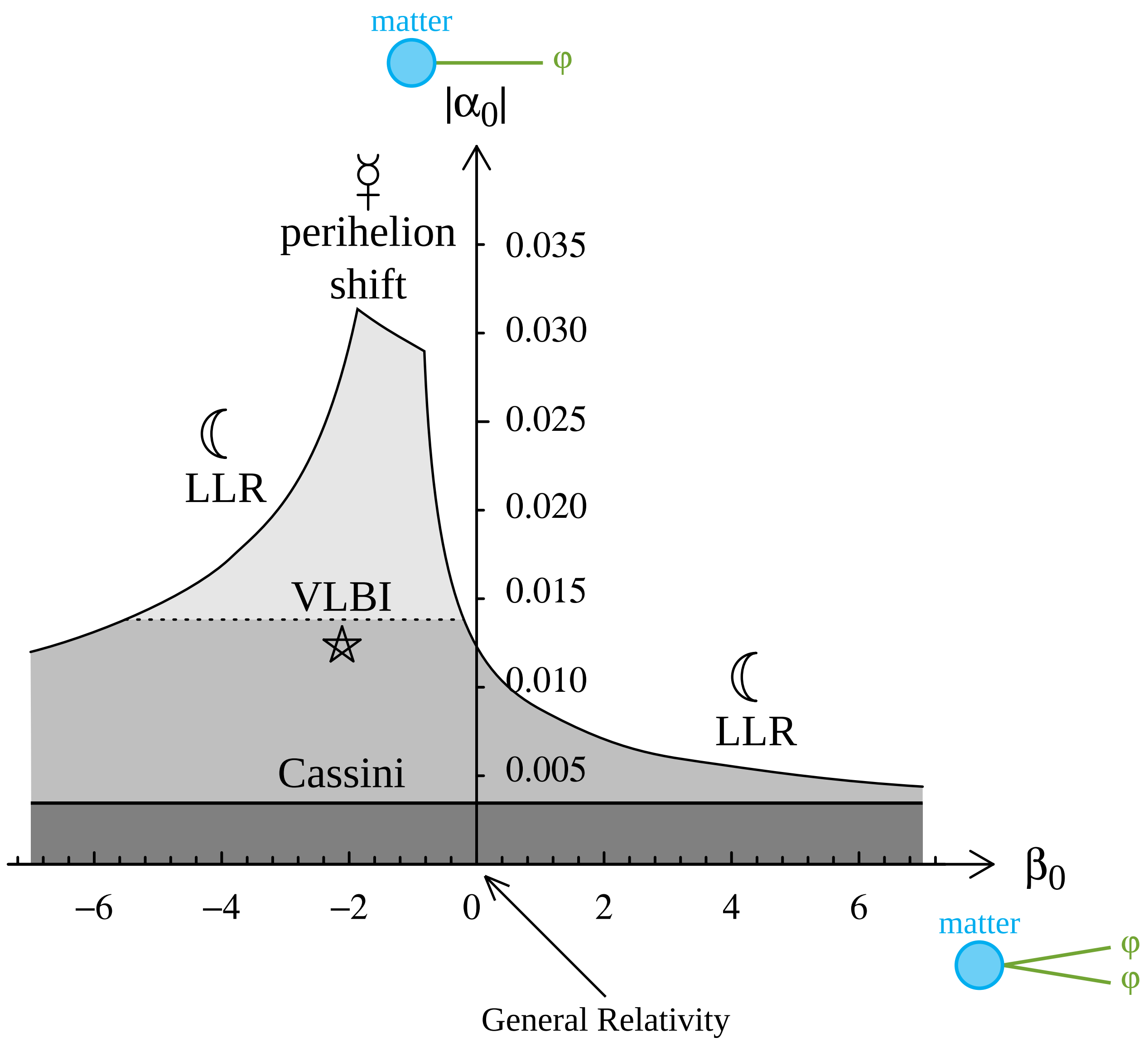



Solar-system experiments
in the **P**arametrized **P**ost-**N**ewtonian formalism



# Solar-system constraints on scalar-tensor theories of gravity



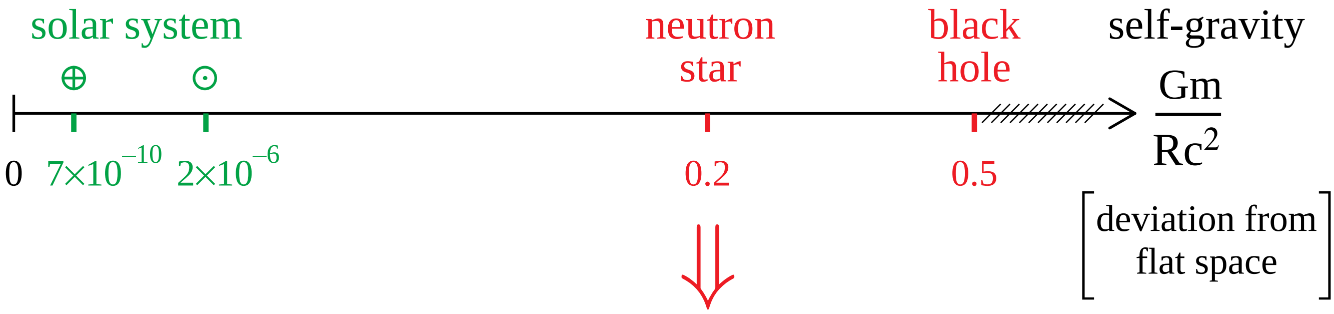

Vertical axis ($\beta_0 = 0$) : Jordan–Fierz–Brans–Dicke theory $\alpha_0^2 = \frac{1}{2\,\omega_{BD} + 3}$

Horizontal axis ($\alpha_0 = 0$) : perturbatively equivalent to G.R.

## Weak-field experiments

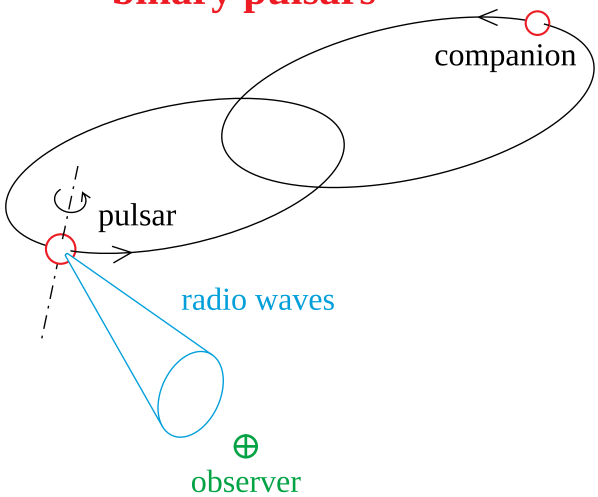
$$\left\{\begin{aligned} -g_{00} &= 1 - 2\,\frac{Gm}{rc^2} + 2\,\beta^{PPN}\left(\frac{Gm}{rc^2}\right)^2 + \dots \\ g_{ij} &= \delta_{ij}\left[\,1 + 2\,\gamma^{PPN}\,\frac{Gm}{rc^2} + \dots\right]\end{aligned}\right.$$

## Strong-field tests ?

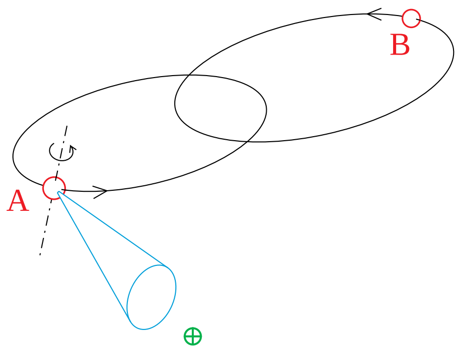



**binary pulsars**

**moving** clock
giving information
about this strong-
gravity region

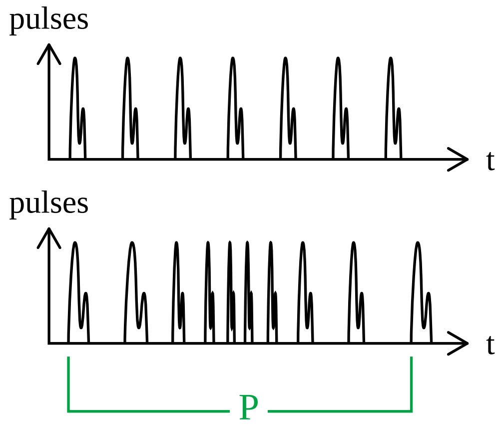


# Binary-pulsar tests




pulsar = (very stable) clock

binary pulsar = moving clock




- Time of flight across orbit $\propto \dfrac{\text{size of orbit}}{c}$ (“Roemer time delay”)

– orbital period $P$
– eccentricity $e$
– periastron angular position $\omega$
– projected semimajor axis $x$
– ...

} “Keplerian” parameters

- Redshift $\propto \dfrac{G\, m_B}{r_{AB}\, c^2}$ + second order Doppler effect $\propto \dfrac{\vec{v}_A^{\,2}}{2\, c^2}$ (“Einstein time delay”)

– parameter $\gamma_{\text{Timing}}$

- Time evolution of Keplerian parameters

– periastron advance $\dot{\omega}$ (order $\frac{1}{c^2}$)

– gravitational radiation damping $\dot{P}$ (order $\frac{1}{c^5}$)

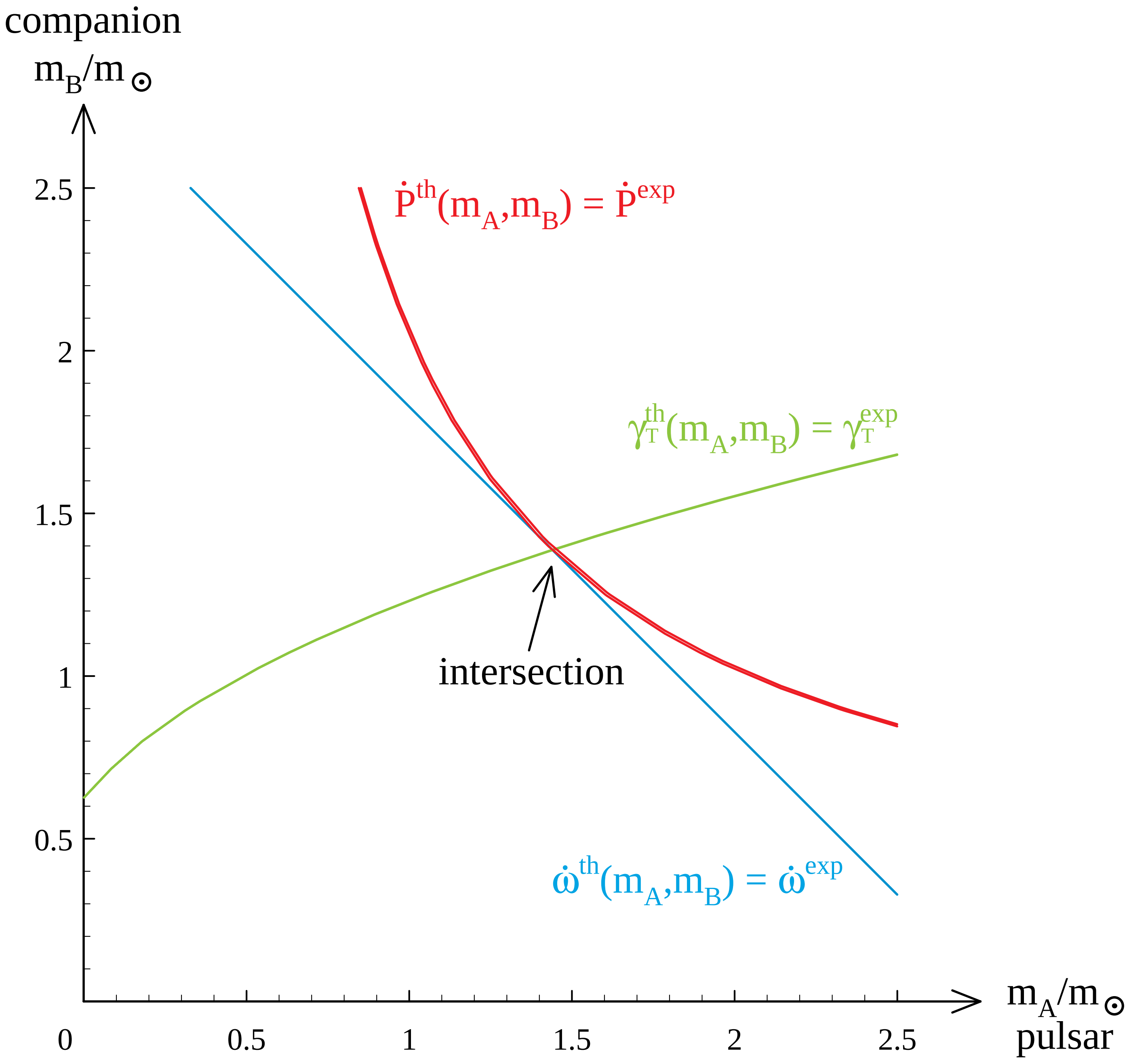
} “post-Keplerian” observables
[PSR B1913+16 • Hulse & Taylor 1974]

| 3 observables | – | 2 unknown masses $m_A$, $m_B$ | = | 1 test |
|---|---|---|---|---|

Plot the three curves [strips]

$$\left.\begin{aligned}
\gamma_{\text{Timing}}^{\text{theory}}(m_A, m_B) &= \gamma_{\text{Timing}}^{\text{observed}} \\
\dot{\omega}^{\text{theory}}(m_A, m_B) &= \dot{\omega}^{\text{observed}} \\
\dot{P}^{\text{theory}}(m_A, m_B) &= \dot{P}^{\text{observed}}
\end{aligned}\right\} \quad \text{“}\gamma_T - \dot{\omega} - \dot{P}\ \text{test”}$$

PSR B1913+16
in general relativity




$$\begin{aligned} \dot{\omega} &= 4.22661°/\text{yr} \\ \gamma_T &= 4.294 \text{ ms} \\ \dot{P} &= -2.421 \times 10^{-12} \end{aligned} \quad \overset{\text{GR}}{\Longrightarrow} \quad \begin{aligned} m_A &= 1.4408\, m_\odot \\ m_B &= 1.3873\, m_\odot \end{aligned}$$

## PSR B1534+12

(discovery Wolszczan 1991)

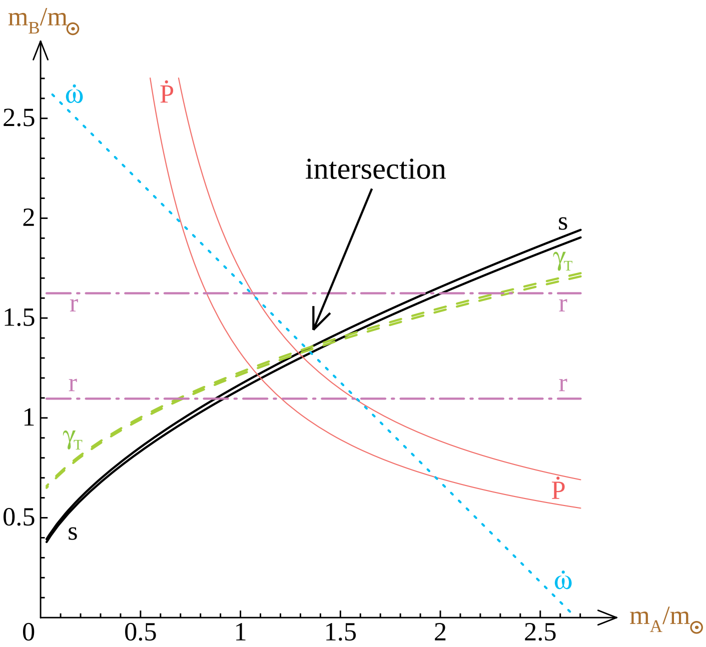
**5** observables – **2** masses = **3** tests

"Galactic" contribution to $\dot{P}$

[Damour–Taylor 1991]

Doppler $\propto$ n.v

$$\Rightarrow \frac{d\,\text{Doppler}}{d\,t} \propto n.a + \frac{v_\perp^2}{d_{\odot PSR}}$$




## PSR J1141–6545

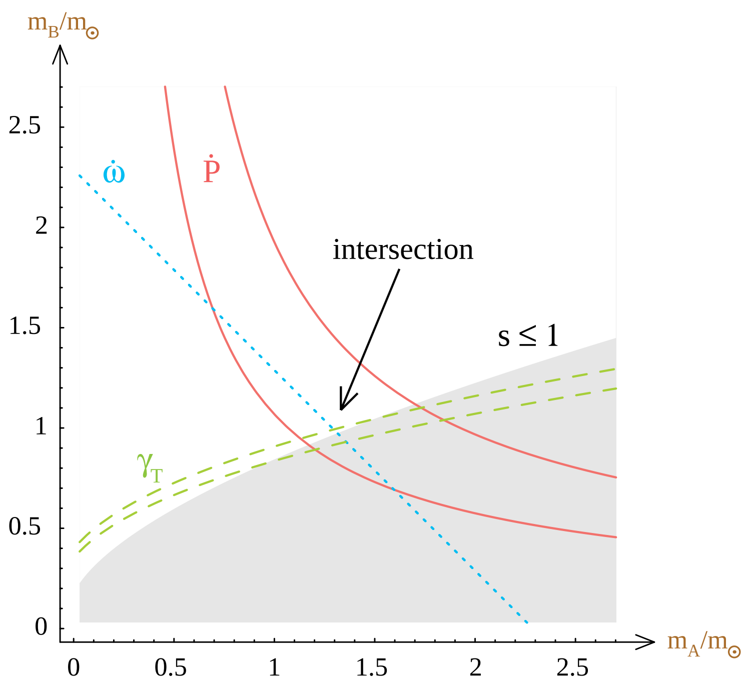
(discovery Kaspi *et al.* 1999,
timing Bailes *et al.* 2003)

Asymmetrical system
neutron star – **white dwarf**

$\dot{P} = -4 \times 10^{-13}$

Mass function

$$\frac{(m_B \sin i)^3}{(m_A + m_B)^2} = \left(\frac{2\pi}{P}\right)^2 \frac{(x\,c)^3}{G}$$



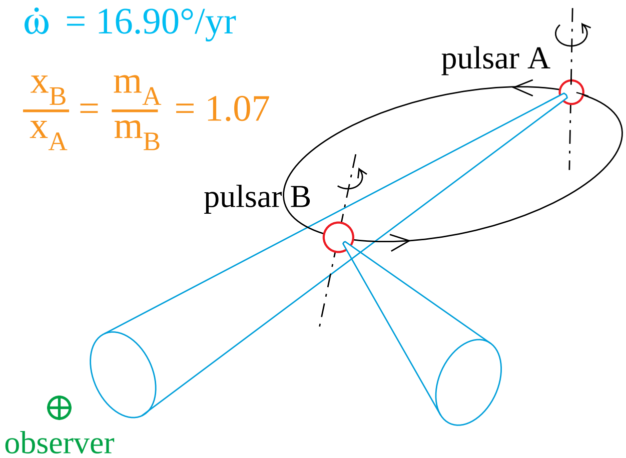

## PSR J0737–3039

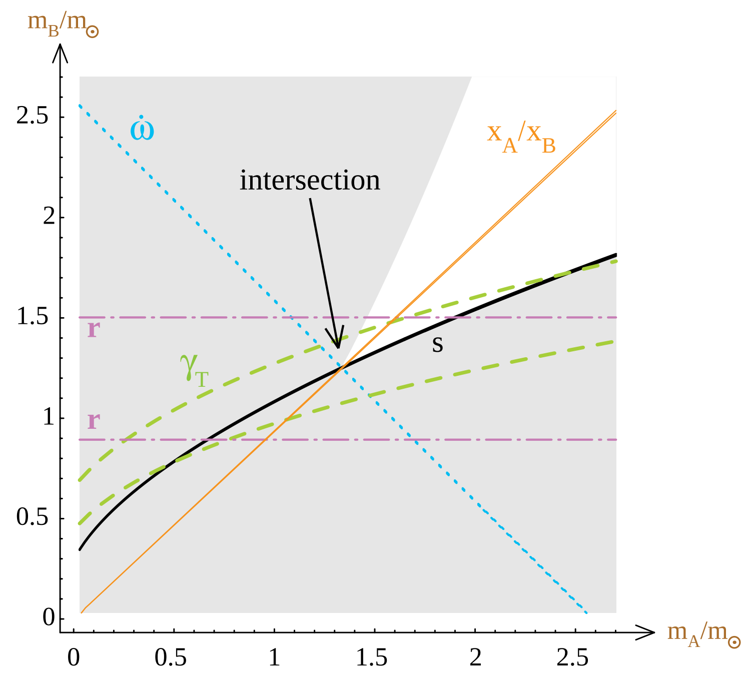
(timing Burgay *et al.* 2003,
**double pulsar** Lyne *et al.* 2004)

$P = 2$ h 27 min 14.5350 s

$\dot{\omega} = 16.90°/\text{yr}$

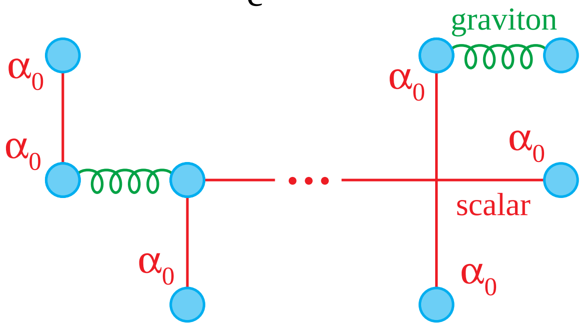
$$\frac{x_B}{x_A} = \frac{m_A}{m_B} = 1.07$$



## Deviations from general relativity due to the scalar field

- At any order in $\frac{1}{c^n}$, the deviations involve at least two $\alpha_0$ factors:




= small deviations!

- But **nonperturbative** strong-field effects may occur:

$$\text{deviations} = \underset{<10^{-5}}{\alpha_0^2} \times \underbrace{\left[ a_0 + a_1 \frac{Gm}{Rc^2} + a_2 \left(\frac{Gm}{Rc^2}\right)^2 + \ \dots \right]}_{\text{LARGE for } \frac{Gm}{Rc^2} \approx 0.2\ ?}$$

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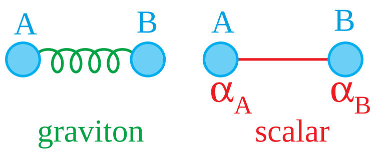
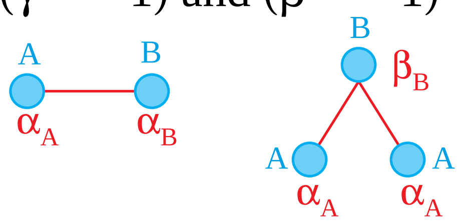
## Strong-field effects

$$G_{AB}^{\text{eff}} = G\ (\,1\ + \alpha_A\, \alpha_B\,)$$



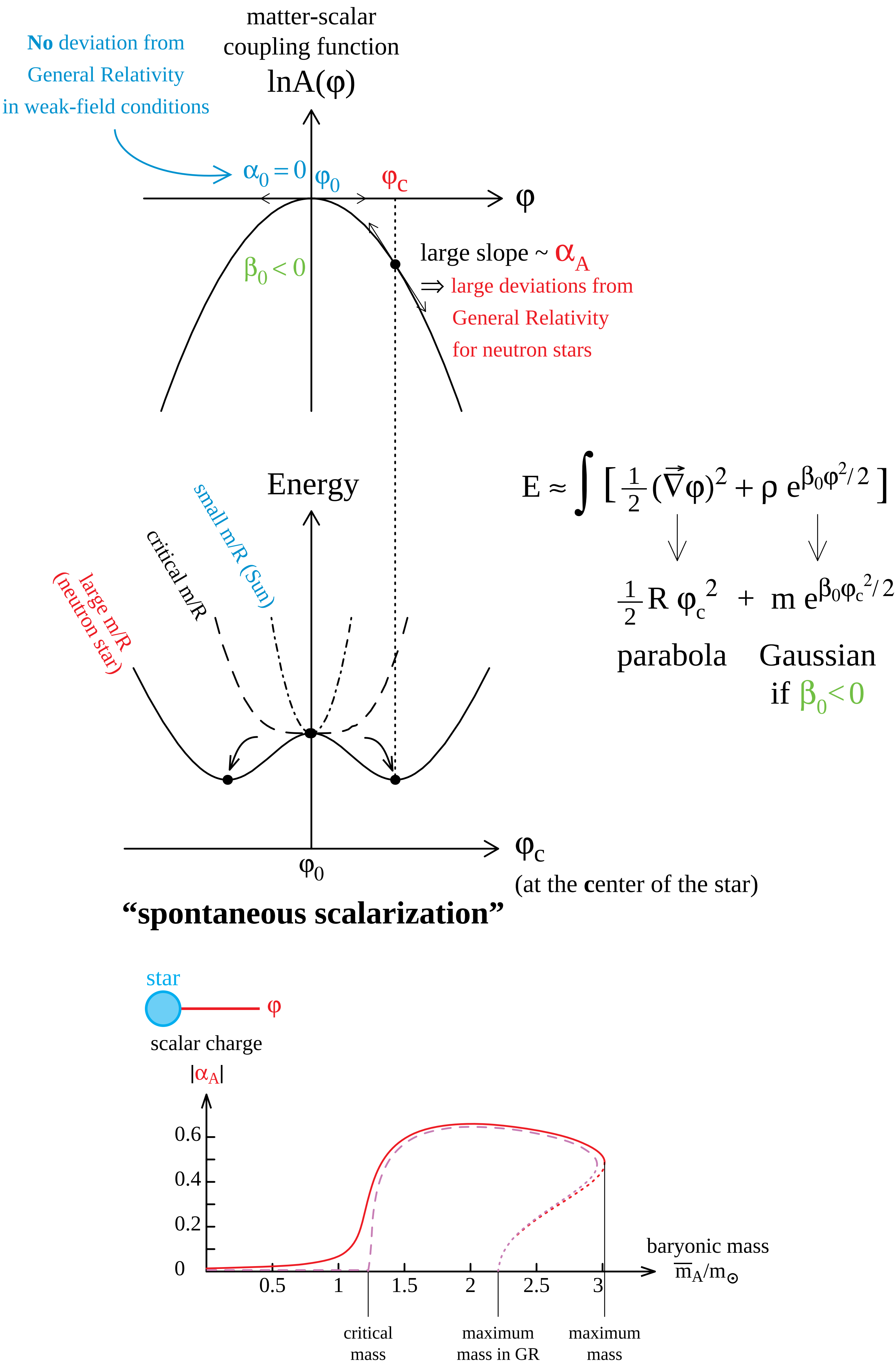

depends on internal structure of bodies A & B

similarly for $(\gamma^{\text{PPN}} - 1)$ and $(\beta^{\text{PPN}} - 1)$ $\Rightarrow$ all post-Newtonian effects



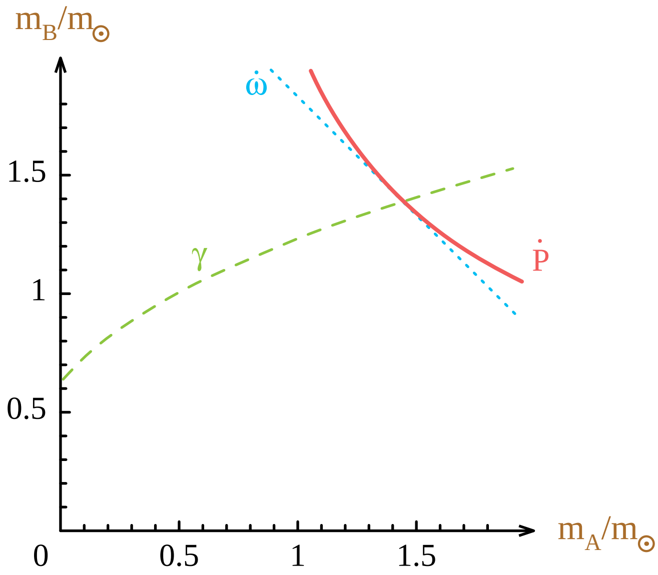

$$\text{Energy flux} = \frac{\text{Quadrupole}}{c^5} + O\left(\frac{1}{c^7}\right) \qquad \text{spin 2}$$

$$+ \frac{\text{Monopole}}{c}\left(0 + \frac{1}{c^2}\right)^2 + \underset{\propto(\alpha_A - \alpha_B)^2}{\underset{\uparrow}{\frac{\text{Dipole}}{c^3}}} + \frac{\text{Quadrupole}}{c^5} + O\left(\frac{1}{c^7}\right) \qquad \text{spin 0}$$

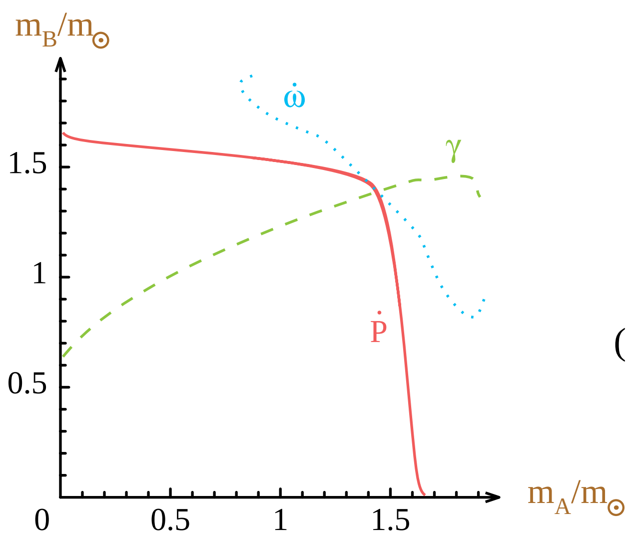


matter-scalar
coupling function
lnA(φ)
**No** deviation from
General Relativity
in weak-field conditions
α0 = 0
φ0
φc
φ
β0 < 0
large slope ~ αA
⇒ large deviations from
General Relativity
for neutron stars
Energy
small m/R (Sun)
critical m/R
large m/R
(neutron star)
E ≈ ∫ [ ½ (∇φ)² + ρ e^(β0φ²/2) ]
½ R φc² + m e^(β0φc²/2)
parabola
Gaussian
if β0<0
φ0
φc
(at the center of the star)
“spontaneous scalarization”
star
φ
scalar charge
|αA|
0.6
0.4
0.2
0
0.5
1
1.5
2
2.5
3
baryonic mass
m̄A/m⊙
critical
mass
maximum
mass in GR
maximum
mass

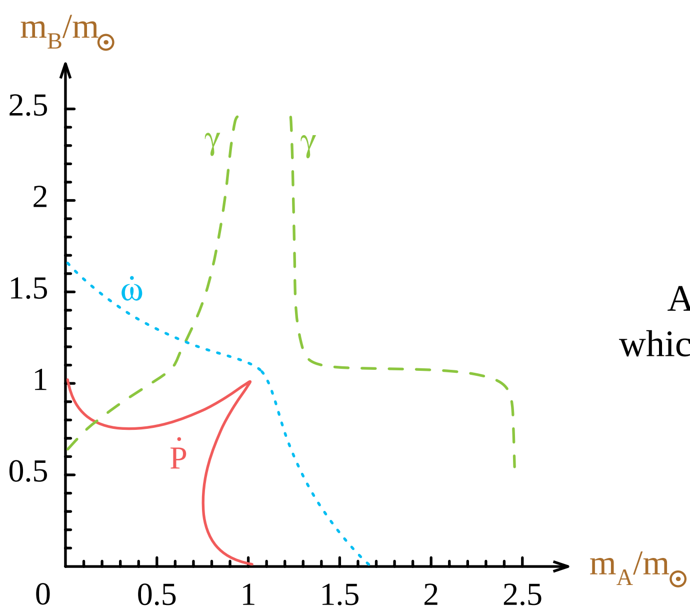
## PSR B1913+16
## in scalar-tensor theories




**General relativity**
passes the test

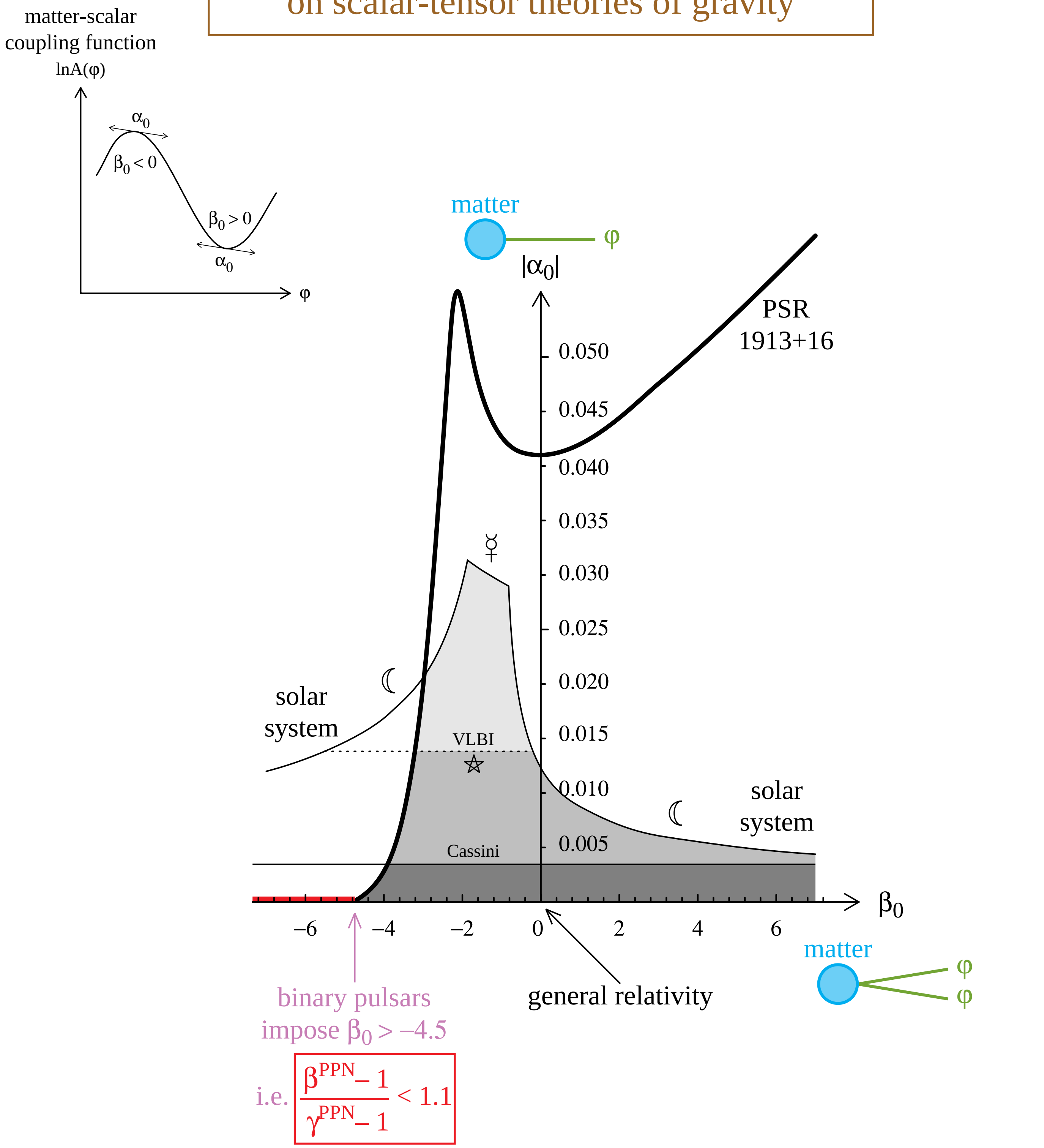



A tensor–scalar theory
which **passes the test**
($\beta_0$ = –4.5, $\alpha_0$ small enough)



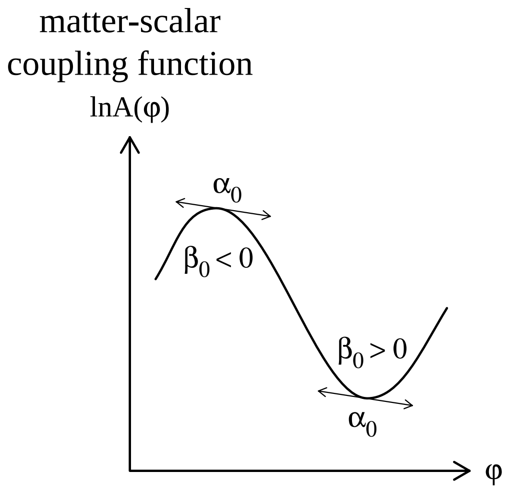

A tensor–scalar theory
which **does not pass the test**
($\beta_0$ = –6, any $\alpha_0$)

## Solar-system & PSR B1913+16 constraints on scalar-tensor theories of gravity

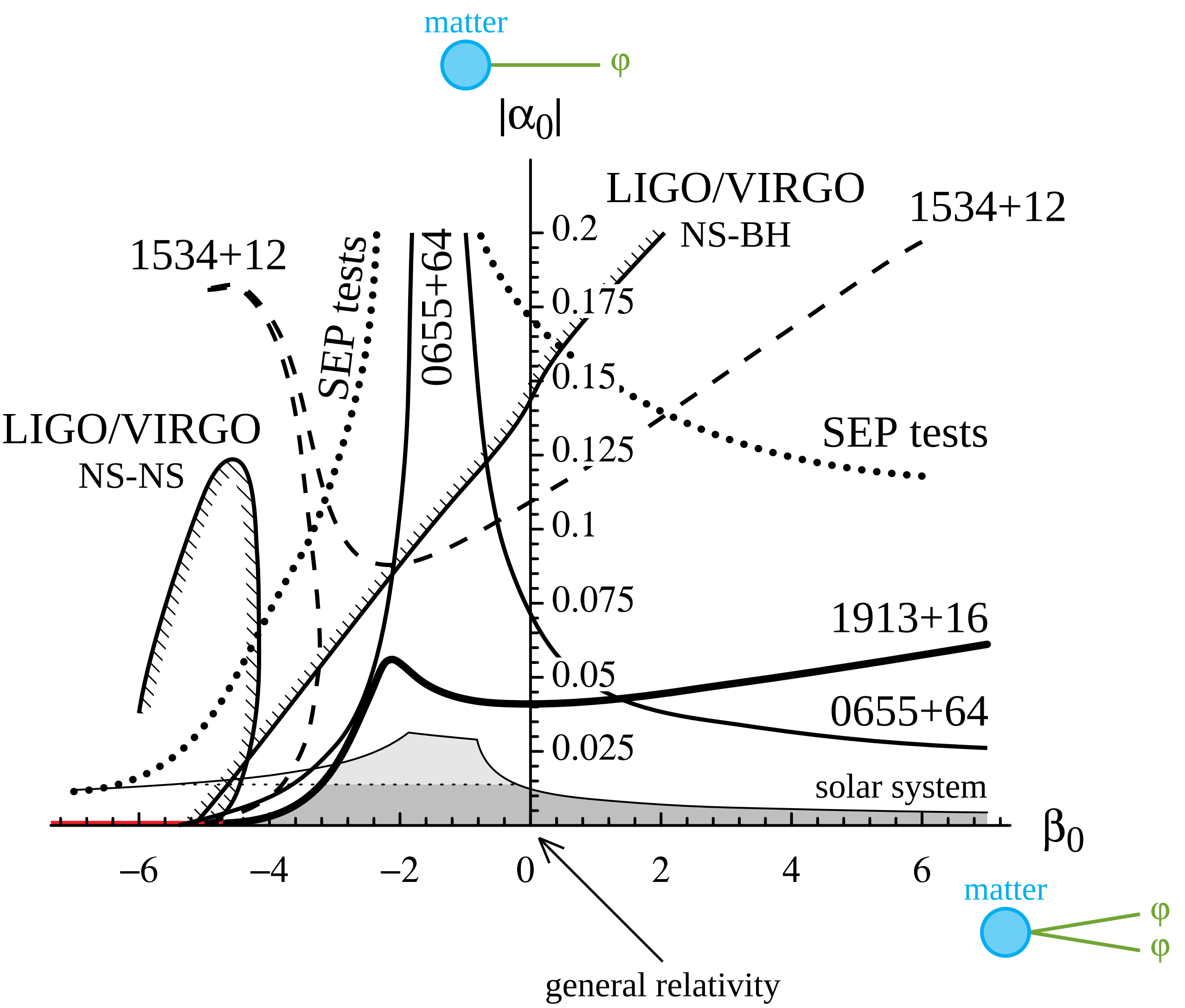



Vertical axis ($\beta_0 = 0$) : Jordan–Fierz–Brans–Dicke theory $\alpha_0^2 = \dfrac{1}{2\,\omega_{BD} + 3}$

Horizontal axis ($\alpha_0 = 0$) : perturbatively equivalent to G.R.




# Solar-system and several binary-pulsar constraints on tensor–scalar theories

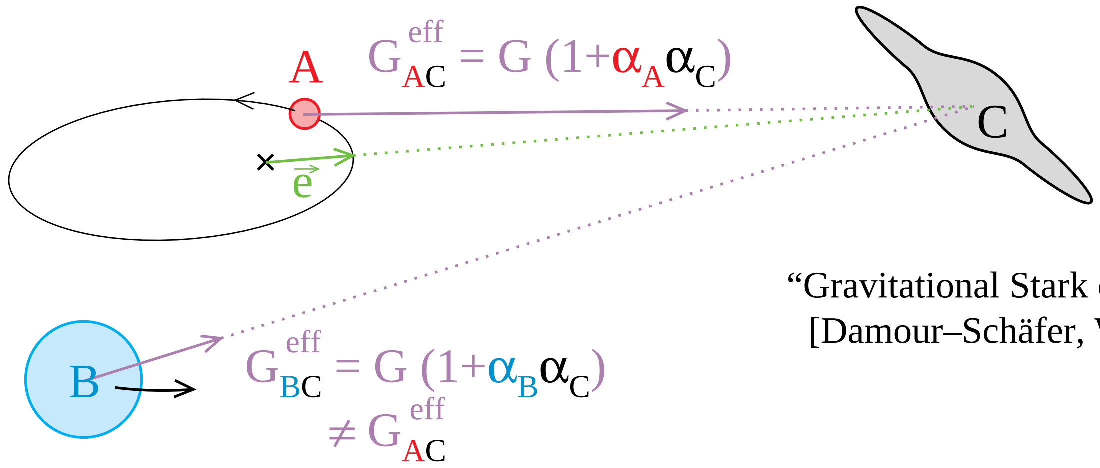



[T. Damour & G.E-F 1998]

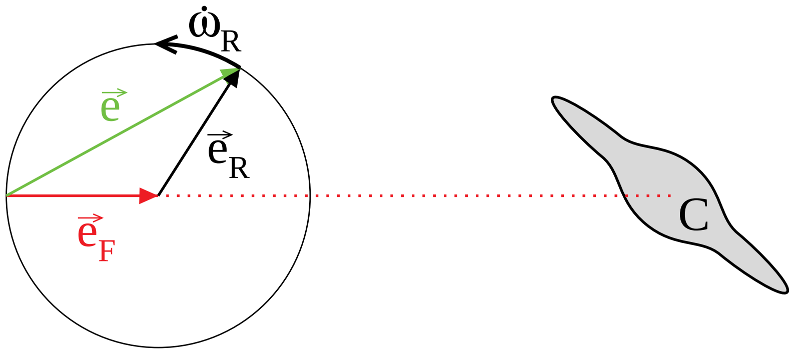
N.B.: if not enough "post-Keplerian" observables are measured, the masses $m_A$ and $m_B$ cannot be accurately determined, but Keplerian "mass function" can still be used

$$\frac{(m_B \sin i)^3}{(m_A + m_B)^2} = \left(\frac{2\pi}{P}\right)^2 \frac{(x\,c)^3}{G}$$

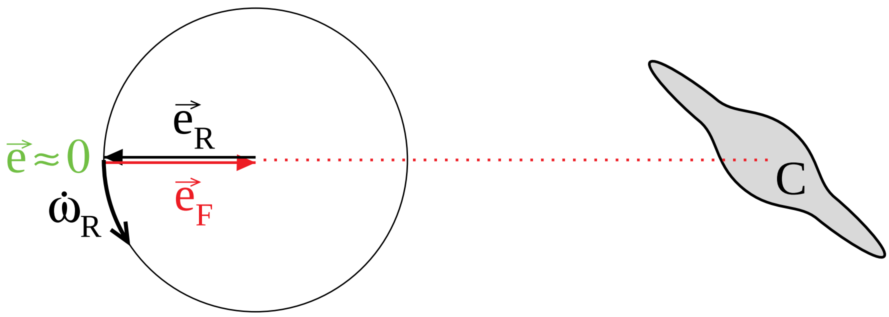
Strong equivalence principle test




“Gravitational Stark effect”
[Damour–Schäfer, Wex]




$$\vec{e}_F \propto (G_{AC}-G_{BC})\,P^2$$




$e \approx 0$ for PSRs 1713+0747, 2229+2643, 1455–3330, …

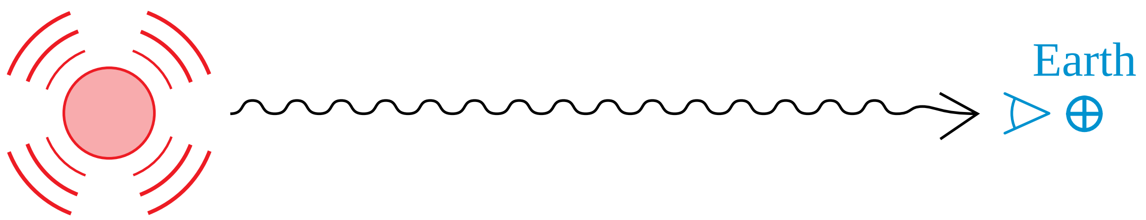
$$\boxed{\Rightarrow\ |(\alpha_A-\alpha_B)\alpha_C| \approx |1 - m_g/m_i| < 10^{-2}}$$

Detection of gravitational waves
(LIGO, VIRGO, …)

$$\text{Energy flux} = \frac{\text{Quadrupole}}{c^5} + O\left(\frac{1}{c^7}\right) \qquad \text{spin 2}$$

$$+ \frac{\text{Monopole}}{c}\left(\dot{\sigma} + \frac{1}{c^2}\right)^2 + \frac{\text{Dipole}}{c^3} + \frac{\text{Quadrupole}}{c^5} + O\left(\frac{1}{c^7}\right) \qquad \text{spin 0}$$

## Collapsing star




Factor $\alpha_0 \approx 0$

Energy flux
= (strong field)$^2$
= Monopole/c
>> usual Quadrupole/$c^5$

Detection
= (strong field) $\times$ (weak field)
= small
[J. Novak's thesis $\Rightarrow$ undetectable]

## Coalescing binary

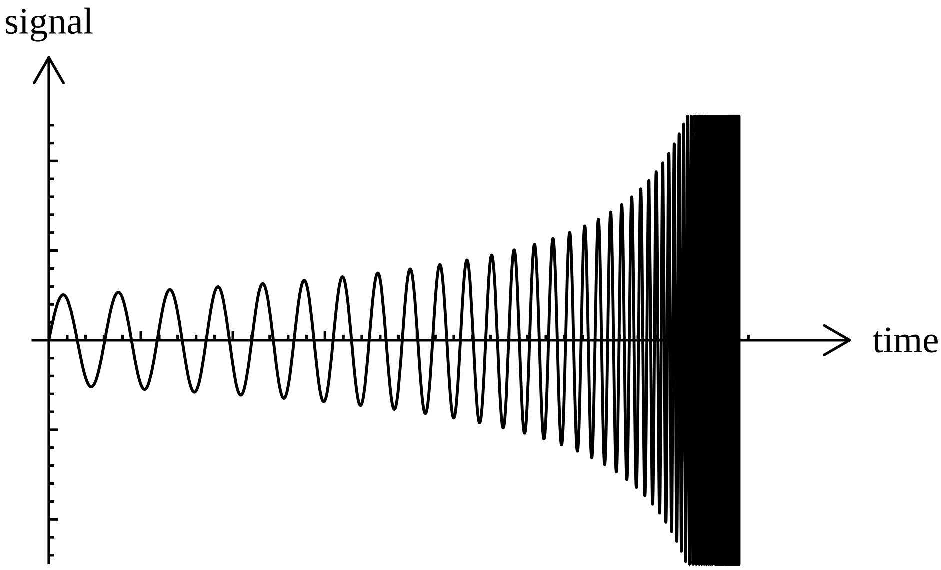
Even if no helicity-0 wave is detected, the time-evolution of the (helicity-2) chirp depends on the Energy flux = (strong field)$^2$

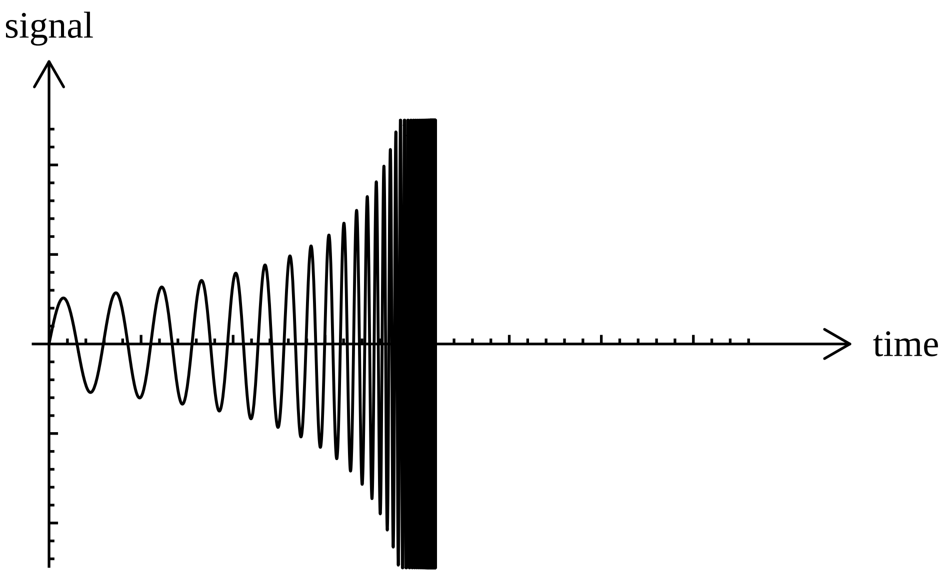
$\Rightarrow$ A priori possible to detect indirectly the presence of $\varphi$

[C.M. Will 1994 : matched-filter analysis]

# For a given binary system

Chirp evolution in general relativity

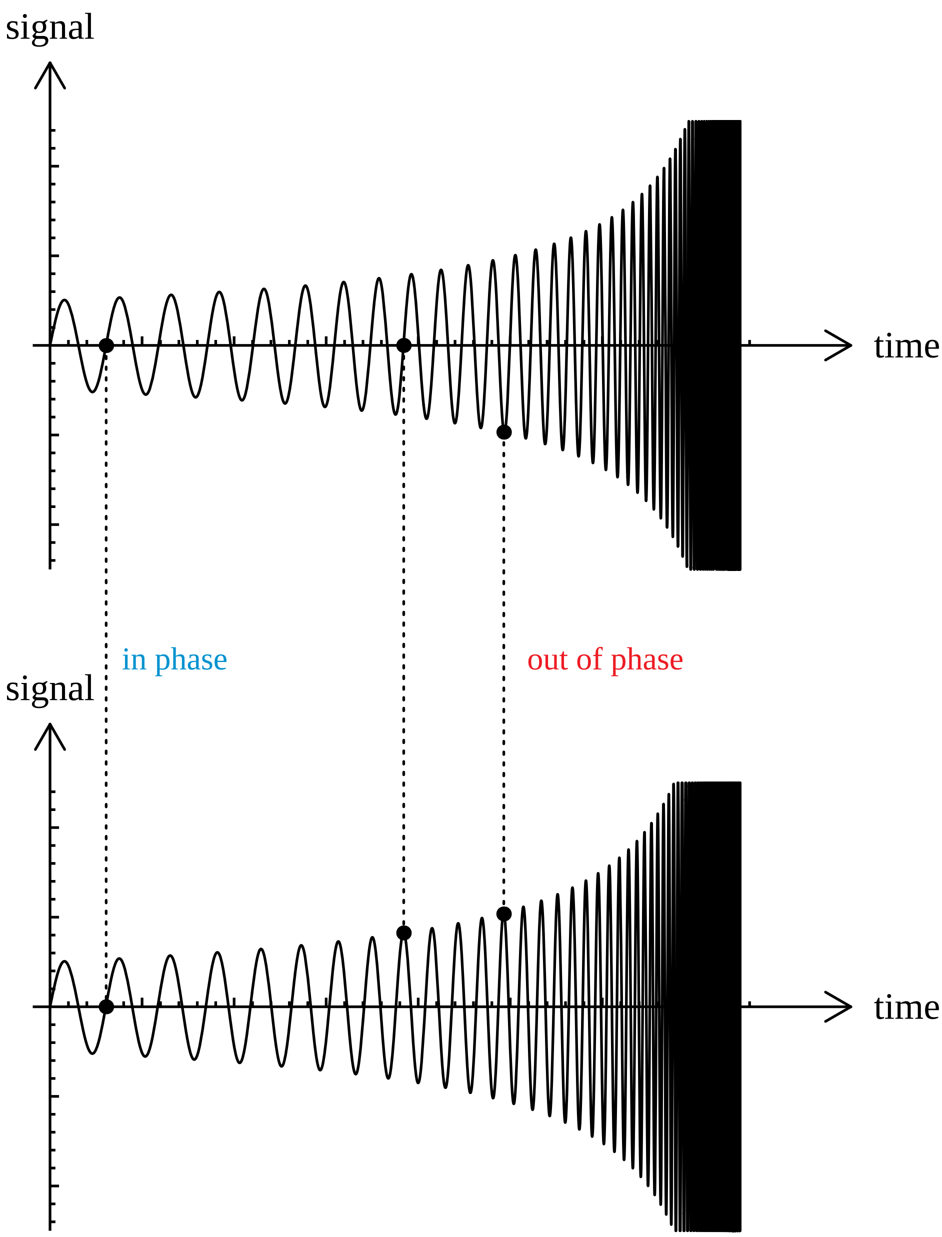


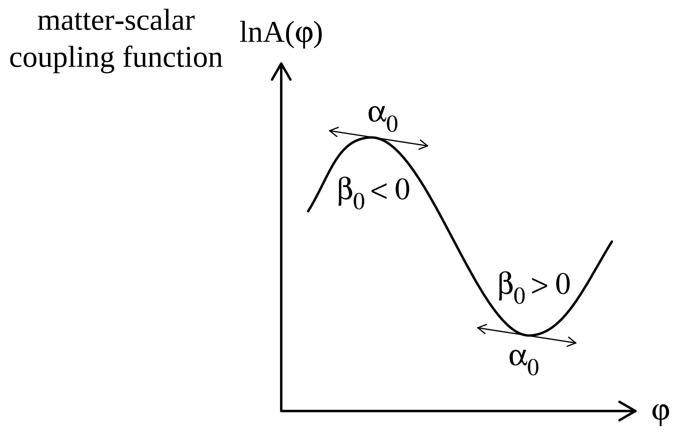

Chirp evolution in a tensor–scalar theory

# For an unknown mass of the system

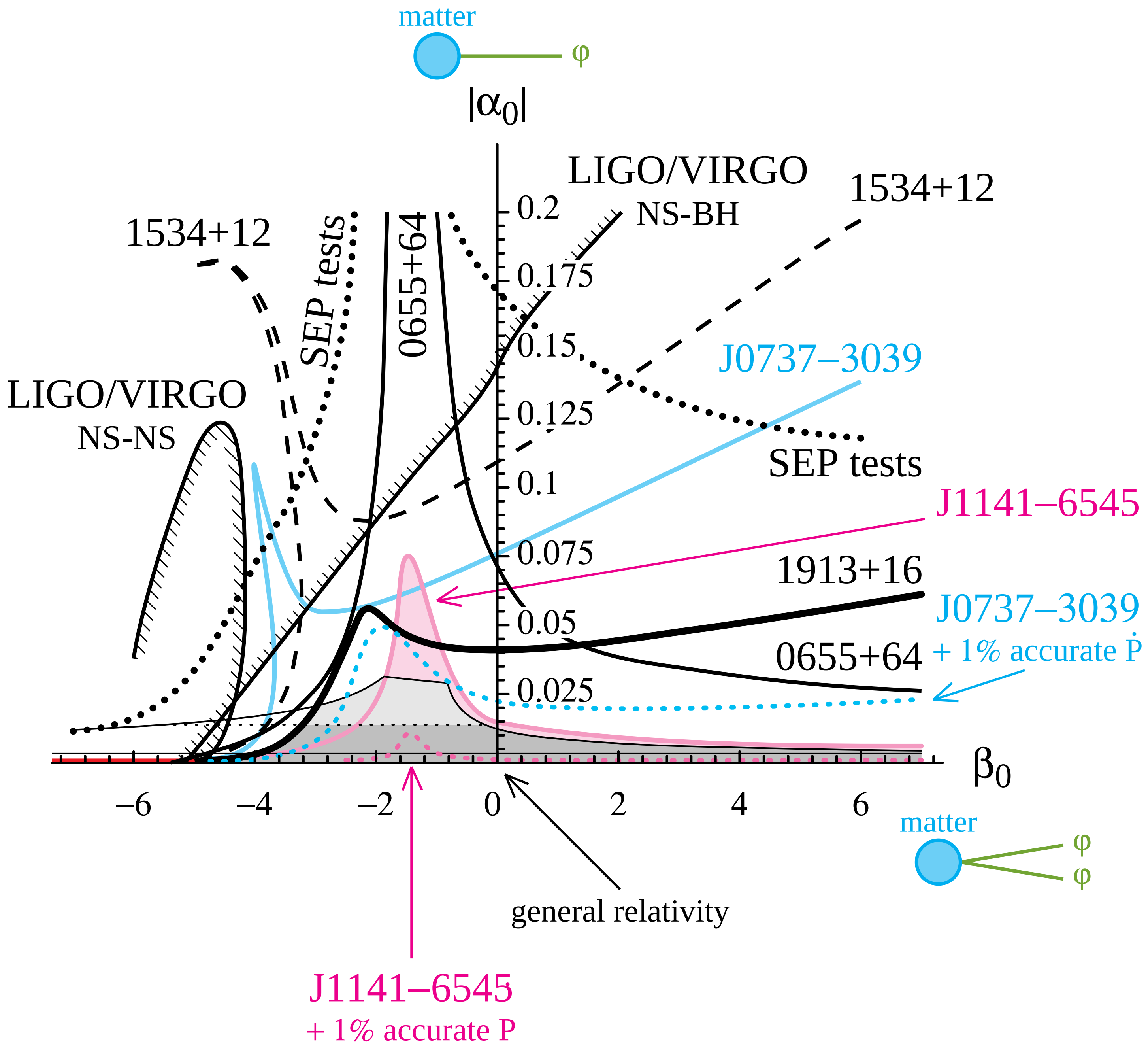
Chirp evolution in general relativity




Chirp evolution in a tensor–scalar theory



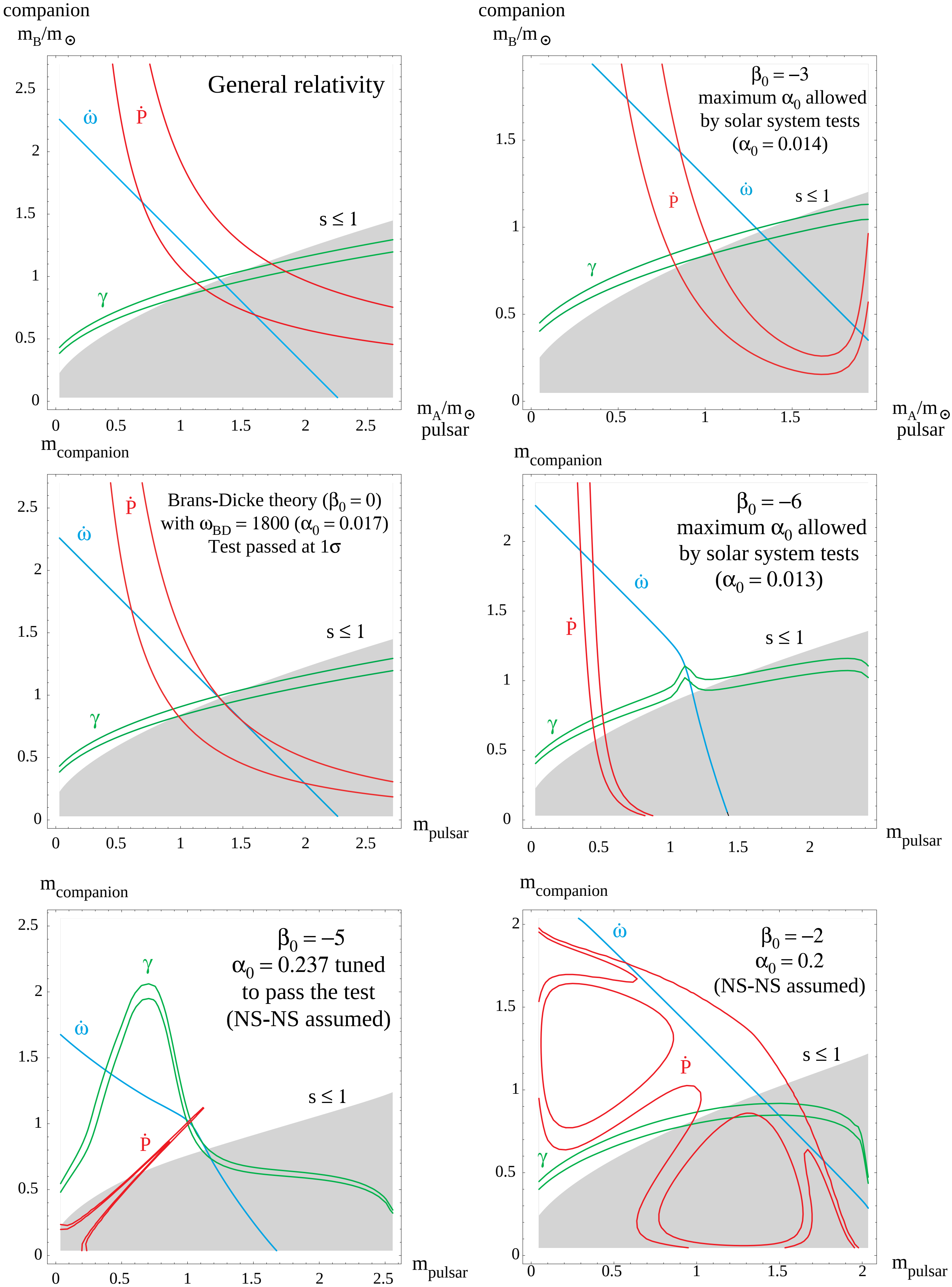

All solar-system & binary-pulsar constraints on tensor–scalar theories




[T. Damour & G.E-F 2004]

# Mass plane for PSR J1141–6545 in various scalar-tensor theories

PSR J1141–6545
rules out (by 5σ) some theories
which were consistent with
all previous experimental data



Cosmological observations give access to the full shape of $A(\varphi)$ and/or $V(\varphi)$

• **Usual cosmology:**

– Assume particular forms of $V(\varphi)$ [and $A(\varphi)$] for theoretical reasons
– Predict all observable quantities
– Compare them to experimental data

• **Phenomenological approach:**

Reconstruct $A(\varphi)$ & $V(\varphi)$ from observational data.

**Result:**
If luminosity distance $D_L(z)$ and density fluctuations $\delta_m(z) = \frac{\delta\rho}{\rho}$ are both known as functions of the redshift $z$, then $A(\varphi)$ & $V(\varphi)$ can be reconstructed.

[B. Boisseau, G.E-F, D. Polarski, A. Starobinsky 2000]

**N.B.:** A priori obvious, since one “fits” **two** observed functions [$D_L(z)$ & $\delta_m(z)$] with **two** unknown ones [$A(\varphi)$ & $V(\varphi)$] !

• **Semi-phenomenological approach:**

[$\delta_m(z)$ not yet measured]

– Theoretical hypotheses on $V(\varphi)$ or $A(\varphi)$
– Reconstruct the other one from $D_L(z)$

**N.B.:** A priori obvious too, since one fits **one** observed function [$D_L(z)$] with **one** unknown function [$A(\varphi)$ or $V(\varphi)$].
However, this naive reasoning works only locally (small interval).

**Result:**
$\exists$ tight constraints if $D_L(z)$ measured on a wide interval $z \in [0, \sim 2]$, even with large error bars!

[G.E-F & D. Polarski 2001]

Constraints come mainly from positivity of energy :

$$E_{\text{graviton}} \geq 0 \Leftrightarrow \quad A^2 > 0 \quad \Leftrightarrow \Phi_{BD} > 0$$
$$E_\varphi \geq 0 \Leftrightarrow - (\partial_\mu \varphi)^2 \Leftrightarrow \omega_{BD} > -3/2$$

# Reconstruction of A(φ) and V(φ)

tensor–scalar theories

$$S=\frac{c^3}{4\pi G}\int\sqrt{-g}\left\{\frac{R}{4}-\frac{1}{2}\left(\partial_\mu\varphi\right)^2-V(\varphi)\right\}+S_{\text{matter}}\left[\text{matter}\,;\tilde{g}_{\mu\nu}\equiv A^2(\varphi)g_{\mu\nu}\right]$$

spin 2 spin 0 physical metric

Change of variables (Brans-Dicke-like representation):

$$\begin{aligned}\tilde{g}_{\mu\nu}&\equiv A^2(\varphi)g_{\mu\nu} & 2\omega(\Phi)+3&\equiv A^2(\varphi)/A'^2(\varphi)\\ \Phi&\equiv A^{-2}(\varphi) & U(\Phi)&\equiv 2V(\varphi)/A^4(\varphi)\end{aligned}$$

$$\Rightarrow S=\frac{c^3}{16\pi G}\int d^4x\sqrt{-\tilde{g}}\left\{\Phi\tilde{R}-\frac{\omega(\Phi)}{\Phi}\left(\partial_\mu\Phi\right)^2-2U(\Phi)\right\}+S_{\text{matter}}\left[\text{matter};\tilde{g}_{\mu\nu}\right]$$

$H(z)$ known if $D_L(z)$ observed:

$$\frac{1}{H(z)}=\left(\frac{D_L(z)}{1+z}\right)'\times\left[1+\Omega_{\kappa,0}\left(\frac{H_0D_L(z)}{1+z}\right)^2\right]^{-1/2}$$

- $$\begin{cases}\ddot{\delta}_m+2H\dot{\delta}_m-4\pi G_{\text{eff}}\rho\delta_m\simeq 0 \qquad \text{for}\quad \lambda\ll\left(\frac{1}{H},\frac{1}{m_\varphi}\right)\\ \Rightarrow\ \frac{\Phi}{\Phi_0}\ \simeq\ \frac{3}{2}\left(\frac{H_0}{H}\right)^2\frac{(1+z)\,\Omega_{m,0}\,\delta_m}{\delta''_m+\left(\frac{H'}{H}-\frac{1}{1+z}\right)\delta'_m}\times\left(1+\frac{1}{2\omega+3}\right),\end{cases}$$

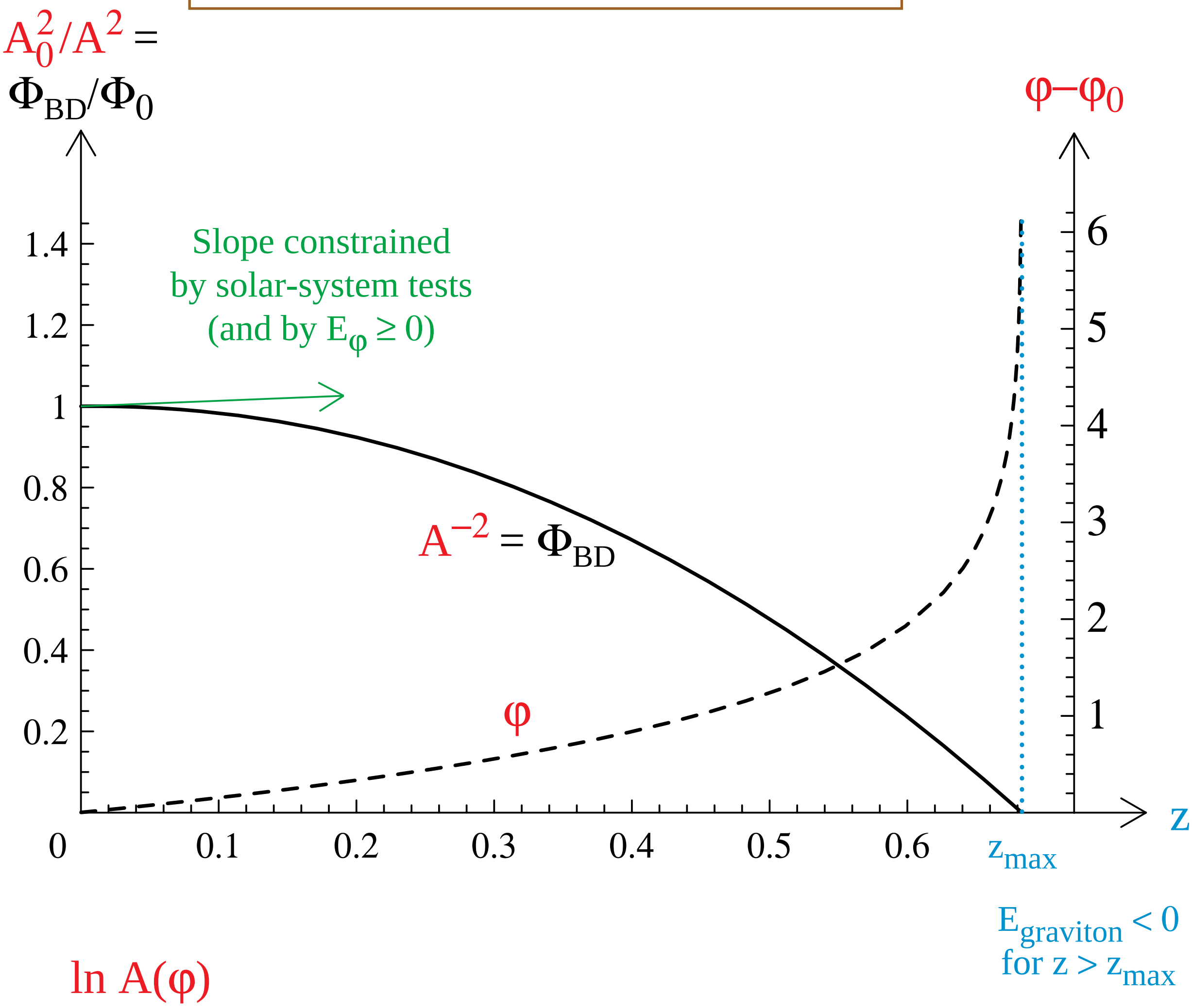
- $$\frac{2U}{(1+z)^2H^2}=\Phi''+\left(\frac{H'}{H}-\frac{4}{1+z}\right)\Phi'+\left[\frac{6}{(1+z)^2}-\frac{2}{1+z}\frac{H'}{H}-4\left(\frac{H_0}{H}\right)^2\Omega_{\kappa,0}\right]\Phi$$
$$-3\,(1+z)\left(\frac{H_0}{H}\right)^2\Phi_0\,\Omega_{m,0}\,,$$

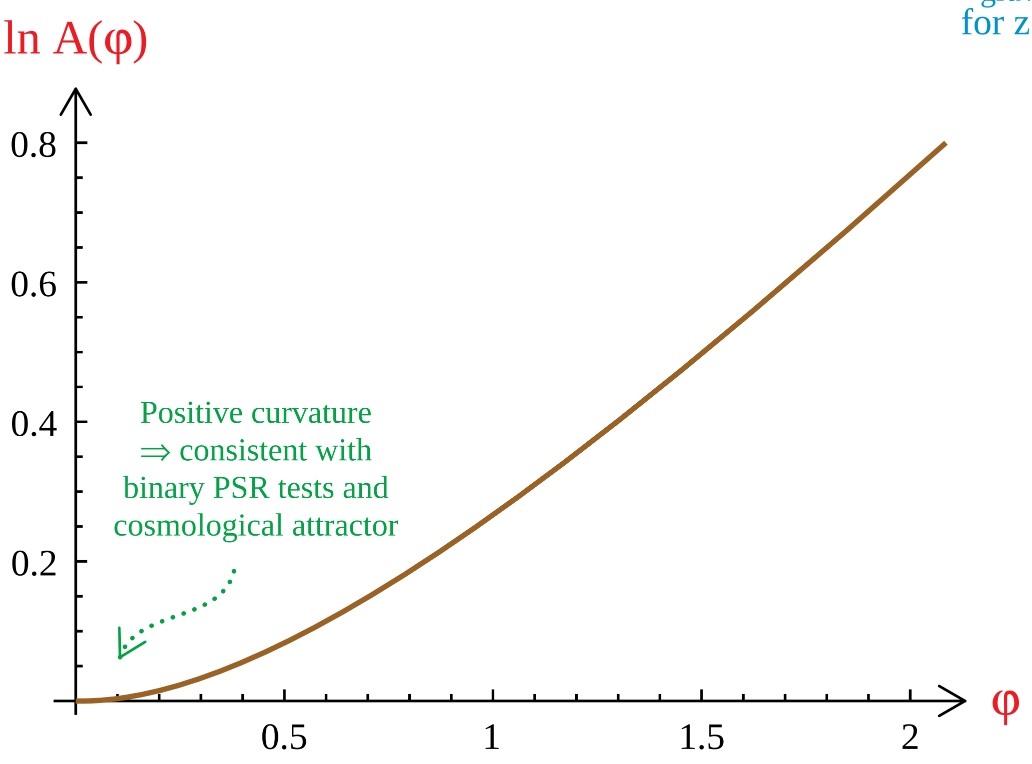
- $$\omega=-\frac{\Phi}{\Phi'^2}\left\{\Phi''+\left(\frac{H'}{H}+\frac{2}{1+z}\right)\Phi'-2\left[\frac{1}{1+z}\frac{H'}{H}-\left(\frac{H_0}{H}\right)^2\Omega_{\kappa,0}\right]\Phi\right.$$
$$\left.+3\,(1+z)\left(\frac{H_0}{H}\right)^2\Phi_0\,\Omega_{m,0}\right\}.$$

$\Rightarrow \Phi(z)$, $U(z)$ and $\omega(z)$ reconstructed $\Rightarrow U(\Phi)$ and $\omega(\Phi)$ reconstructed

## Reconstruction of $A(\varphi)$ from $D_L(z)$, assuming $V(\varphi) = 0$

same as GR + Λ



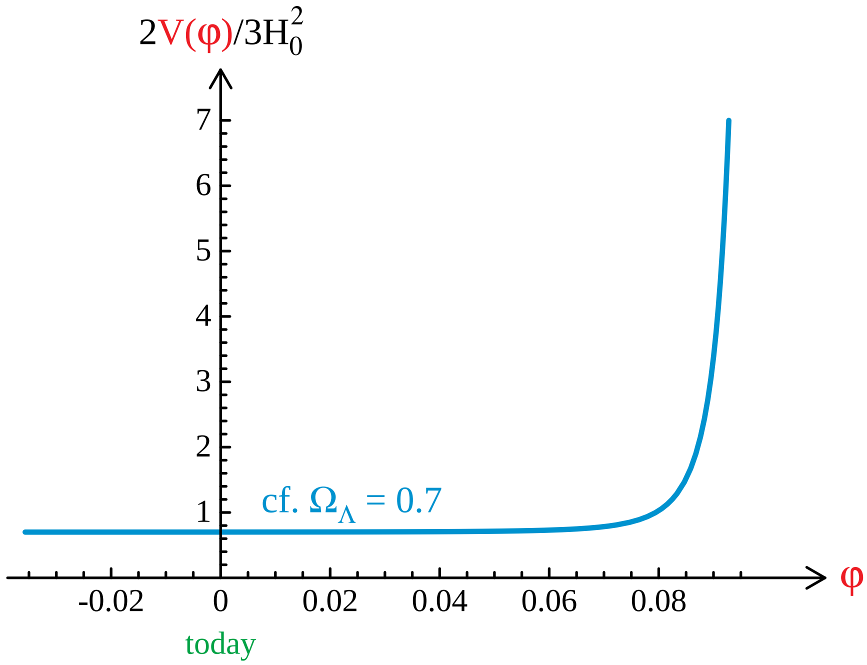




(∃ analytical solutions)

Reconstruction of $V(\varphi)$ from $D_L(z)$, assuming $A(\varphi) = 1$ (i.e. minimally coupled model) in spatially closed FRW universe

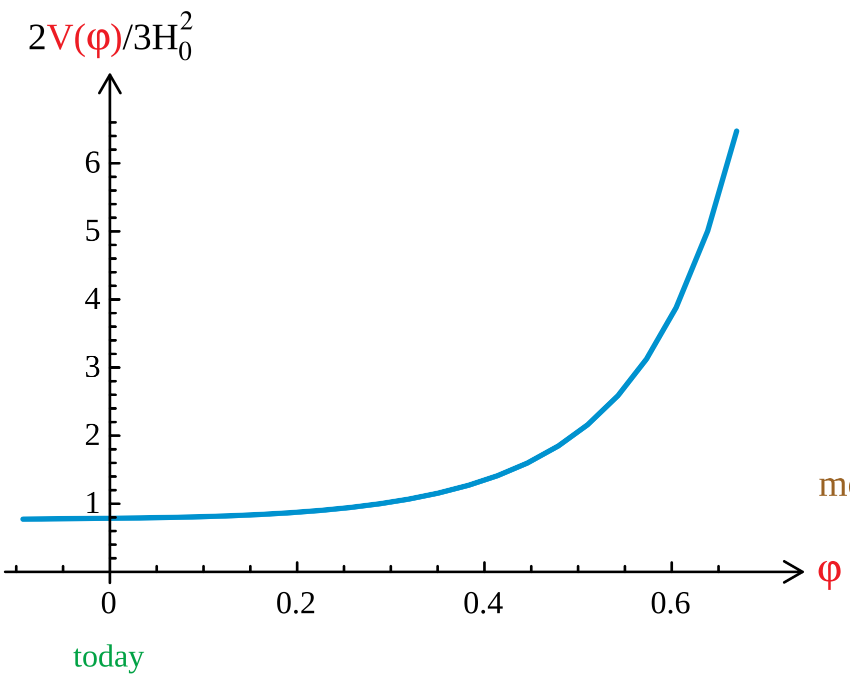
same as GR + $\Lambda$

$\Omega_\kappa = -0.001$




not natural

$\Omega_\kappa = -0.1$




more natural

($\exists$ analytical solution)

## More general tensor – scalar theories

$$S = \frac{c^3}{4\pi G}\int\sqrt{-g}\left\{\frac{R}{4}-\frac{1}{2}\left(\partial_\mu\varphi\right)^2 - V(\varphi)\right\} + S_{\text{matter}}\left[\text{matter}\,;\tilde{g}_{\mu\nu}\equiv A^2(\varphi)g_{\mu\nu}\right]$$

spin 2 spin 0 physical metric

k-essence σ-model quintessence

$$S = \frac{c^3}{4\pi G}\int\sqrt{-g}\left\{\frac{R}{4}-\frac{1}{2}F\left(g^{\mu\nu}\gamma_{ab}(\varphi^c)\,\partial_\mu\varphi^a\partial_\nu\varphi^b\right) - V(\varphi^a)\right\}$$
$$-\hbar\int\sqrt{-g}\,W(\varphi^a)\left(R^2_{\mu\nu\rho\sigma}-4R^2_{\mu\nu}+R^2\right)$$
$$+\ S_{\text{matter}}\left[\text{matter}\,;\tilde{g}_{\mu\nu}\equiv A^2(\varphi^a)g_{\mu\nu}\right]$$

Gauss-Bonnet

extended quintessence

N.B.:

- $f(R) \Longleftrightarrow$ an extra scalar field [Teyssandier & Tourrenc 1983]
- $f(R, \Box R, \ldots, \Box^n R) \Longleftrightarrow n+1$ extra scalar fields [Gottlöber *et al.* 1990; Wands 1994]
- $f(R_{\mu\nu})$ and/or $f(R_{\mu\nu\rho\sigma}) \Longleftrightarrow$ an extra massive spin-2 **ghost** [Stelle 1977; Hindawi *et al.* 1996; Tomboulis 1996]

Example of a pure scalar–Gauss-Bonnet coupling

$$S = \frac{c^3}{4\pi G}\int\sqrt{-g}\left\{\frac{R}{4}-\frac{1}{2}\left(\partial_\mu\varphi\right)^2 - 0\right\}$$
$$-\hbar\int\sqrt{-g}\,W(\varphi)\left(R^2_{\mu\nu\rho\sigma}-4R^2_{\mu\nu}+R^2\right)$$
$$+S_{\text{matter}}\left[\text{matter}\,;\tilde{g}_{\mu\nu}\equiv 1\times g_{\mu\nu}\right]$$

## Experimental constraints on W(φ) assuming V(φ) = 0 & A(φ) = 1 ?

- Solar system (& binary pulsars)

$$\Box\varphi = \frac{3r_0^2}{r^6}\left(\frac{2GM_\odot}{c^2}\right)^2\left[W_0' + W_0''\varphi + O(\varphi^2)\right]$$

$$\left(r_0^2 = \frac{16\pi G\hbar}{c^3}\right)$$

$$\begin{cases} \text{light deflection} & \Delta\theta_{\text{\ding{73}}} = \dfrac{4GM_\odot}{\rho_0 c^2} + \dfrac{1536}{35}\left(\dfrac{GM_\odot}{\rho_0 c^2}\right)^3\left(\dfrac{r_0}{\rho_0}\right)^4 W_0'^2 \\ \text{perihelion shift} & \Delta\theta_{☿} = \dfrac{6\pi GM_\odot}{pc^2} + 192\pi\left(\dfrac{GM_\odot}{pc^2}\right)^2\left(\dfrac{r_0}{p}\right)^4 W_0'^2 \end{cases}$$

OK if $|W_0'|$ small enough

- Reconstruction of $W(\varphi)$ from cosmological observation of $D_L(z)$

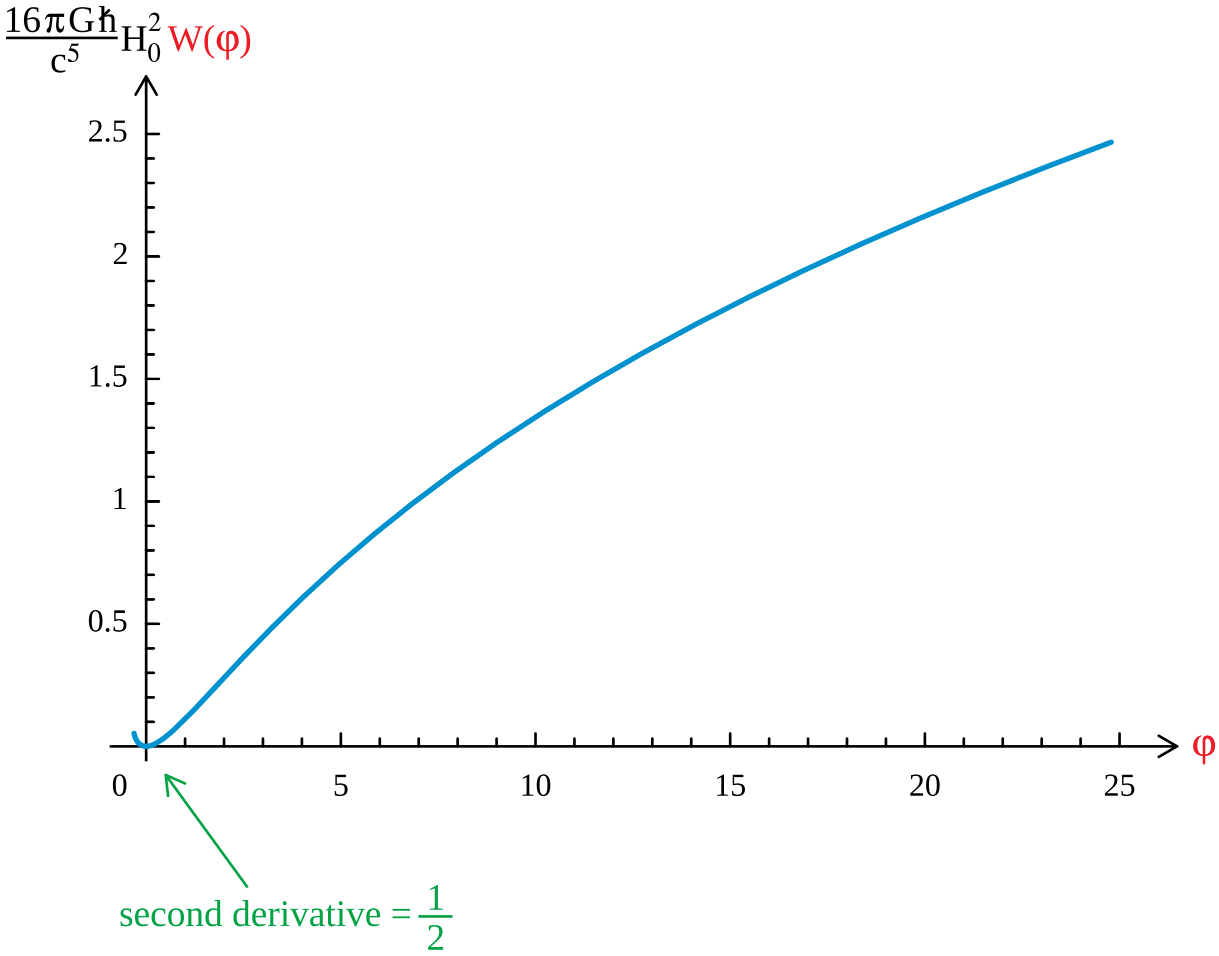
– Can always be done without any problem of negative energy.

– ∃ attraction mechanism towards a minimum of $W(\varphi)$

$\Rightarrow$ $|W_0'|$ small is expected.

Reconstruction of the scalar–Gauss-Bonnet coupling function $W(\varphi)$

[for $V(\varphi) = 0$ and $A(\varphi) = 1$]




Conclusion : $dW(\varphi)/d\varphi = 0$ possible

but $d^2W(\varphi)/d\varphi^2 \approx 7 \times 10^{119}$

(cf. $\Lambda \approx 3 \times 10^{-122}\, c^3/\hbar G$)

[G.E-F & E. Semboloni]

## Experimental constraints on W(φ) assuming V(φ) = 0 & A(φ) = 1 ? (continued)

- Solar system again

If $|W_0''\varphi| \gg |W_0'|$, we cannot neglect it in

$$\Box\varphi = \frac{3r_0^2}{r^6}\left(\frac{2GM_\odot}{c^2}\right)^2\left[W_0' + W_0''\varphi + \cancel{O(\varphi^2)}\right]$$

assume parabolic $W(\varphi)$

$$\Rightarrow \boxed{\begin{aligned} \varphi &= \frac{W_0'}{W_0''}\sum_{n\geq 1}\frac{1}{(3\times 4)(7\times 8)\cdots(4n-1)(4n)}\left(\frac{12r_0^2G^2M_\odot^2W_0''}{r^4c^4}\right)^n \\ &\simeq \frac{W_0'}{W_0''}\left[\begin{matrix}\cosh\\ \cos\end{matrix}\underbrace{\left(\frac{GM_\odot r_0}{r^2c^2}\sqrt{3|W_0''|}\right)}_{\substack{\text{if } '' \gg 1 \\ \sim 10^8}} - 1\right] \qquad \begin{matrix}\text{if } W_0'' > 0\\ \text{if } W_0'' < 0\end{matrix}\end{aligned}}$$

- $\varphi \longrightarrow 0$ for $r \longrightarrow \infty$

$\Rightarrow$ theory $\simeq$ G.R. for $r > 4 \times 10^{14}$ m

(farther than solar system + comet cloud)

- In the solar system, ∃ highly nonlinear corrections in $\dfrac{1}{r^{4n}}$

- $\varphi \longrightarrow 0$ for $W_0' \longrightarrow 0$

$\Rightarrow$ no nonperturbative effect (like spontaneous scalarization)

- Solar system tests

$$ds^2 = -\left(1 + \sum_n \frac{\beta_n}{\rho^n}\right) c^2 dt^2 + \left(1 + \sum_n \frac{\alpha_n}{\rho^n}\right) d\rho^2 + \rho^2 d\Omega^2$$

light deflection $\quad \Delta\theta_{\star} = \sum_n 2^{n-1} \frac{\Gamma\left(\frac{n+1}{2}\right)^2}{\Gamma(n+1)} \frac{\alpha_n - n\beta_n}{\rho_0^n} + O(\alpha_n, \beta_n)^2$

perturbative

perihelion shift $\quad \Delta\theta_{☿} = \frac{6\pi G M_\odot}{pc^2} - \sum_n \frac{n(n-1)\beta_n c^2}{2GM_\odot p^{n-1}} \pi + O(\alpha_n, \beta_n)^2$

$\Rightarrow |W_0'| < 10^{-2\times10^{11}}$ !!!

if we take the $W_0'' > 0$ given by the cosmological reconstruction.

**Hyperfine tuning**, which cannot last for more than a fraction of a second.

$\Rightarrow$ The model $A(\varphi) = 1,\ V(\varphi) = 0,\ W(\varphi) \neq 0$ is already ruled out

*

* *

- N.B.: If $W_0'' < 0$, then

$$\varphi \simeq \frac{W_0'}{W_0''}\left[\cos\left(\frac{GM_\odot r_0}{r^2 c^2}\sqrt{3|W_0''|}\right) - 1\right]$$

and it suffices to have $|r_0^2 W_0'| \ll r^2$

to get negligible effects in the solar system,

even if $|W_0''| \sim 10^{120}$.

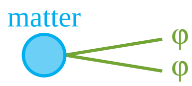
$\Rightarrow$ Not so trivial that a $R^2$ term in the Lagrangian must have larger effects on small scales than on large ones.

# Conclusions

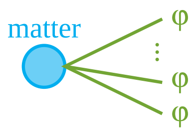
- Scalar–tensor theories are the best motivated alternatives to general relativity.

- Solar-system tests constrain the first derivative of the scalar–matter coupling function $A(\varphi)$.


- Binary-pulsar data constrain the second derivative of $A(\varphi)$.




- Knowledge of the two cosmological functions $D_L(z)$ and $\delta_m(z)$ suffices to reconstruct both $A(\varphi)$ and the potential $V(\varphi)$ on a finite interval of $\varphi$.




- Knowledge of $D_L(z)$ alone over a wide redshift interval strongly constrains the theories if one takes into account

  – solar-system (& binary-pulsar) data

  – **positivity of energy**

  – stability of the theory

  (– naturalness)

[N.B.: less constraining if universe marginally closed]

$\Rightarrow$ SN Ia data allow us to discriminate between G.R.+$\Lambda$ and scalar-tensor theories.

- Scalar–Gauss-Bonnet coupling strongly constrained by combination of solar-system & cosmological data.

  N.B.: A model with $V(\varphi)$, $A(\varphi)$ & $W(\varphi)$ is experimentally allowed.

  $W(\varphi)$ will change the behaviour at small scales (clustering, Big Bang).