Stabilization of Homogeneous Extra Dimensions

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Phi in the Sky

The Quest for Cosmological Scalar Fields

Porto, 8-10 July 2004

based on:

[1] T. B. and M. Eriksson, JCAP **10**, 6 (2003) [astro-ph/0308498].

[2] T. B., M. Eriksson and M. Gustafsson,
Phys. Rev. D **68**, 063516 (2003) [astro-ph/0303497].

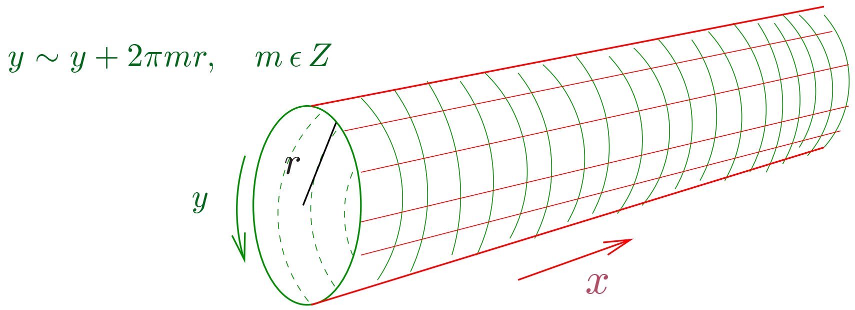
Stable ED during matter domination?

outline:

- extra dimensions (ED): basic ideas
- cosmological field equations
- case study: Universal Extra Dimensions
- a general framework for stabilization
- dimensional reduction and conformal transformation
- ways out?
- connection to brane worlds (outlook)
- summary

Basic idea

Assume extra spatial dimensions, compactified on some small scale r.



'From a distance, this looks like ordinary 4D.'

$\longrightarrow$ Low-energy physics should (in a certain sense) give standard 4D physics.

Fig.: Greene '00

The spectrum

- A massless 5D scalar field Φ can be described by the Klein-Gordon equation:

  $$\left(\partial_t^2 - \nabla^2 - \partial_y^2\right) \Phi(x^\mu, y) = 0\,.$$

  The Fourier-decomposition of $\Phi(x^\mu, y) = \sum_{k=0}^{\infty} \Phi_k(x^\mu) e^{i\frac{k}{r}y}$ then gives

  $$\left(\partial_t^2 - \nabla^2 + m^2\right) \Phi_k(x^\mu) = 0\,,$$

  i.e. a 'KK'-tower of massive states, with

  $$m = \tfrac{k}{r}, \quad k = 0, 1, 2, \ldots$$

- A massless 5D vector field A^M decomposes into a 4D vector A^μ and a 4D scalar A^4. For A^μ one will have a KK-tower as before, while the KK-components of A^4 are 'eaten up' through the vector field acquiring a mass.

$$A^M \longrightarrow \begin{array}{ll} A^\mu_{(0)},\ A^4_{(0)} & \qquad (m = 0) \\ A^\mu_{(k)} & \qquad (m = \tfrac{k}{r},\ k \geq 1) \end{array}$$

Variation of coupling constants

Consider (for example) gravity in $4+n$ dimensions:

$$S = \int \mathrm{d}^{4+n}X \sqrt{-g}\ \frac{R}{2\kappa^2}\ .$$

Assume that the metric can be written as
$g_{AB}\ \mathrm{d}X^A \mathrm{d}X^B = \bar{g}_{\mu\nu}(x^\mu)\ \mathrm{d}x^\mu \mathrm{d}x^\nu + \tilde{g}_{pq}(x^\mu, y^p)\ \mathrm{d}y^p \mathrm{d}y^q$,
so integrating out the ED results in

$$S = \int \mathrm{d}^4x \sqrt{-\bar{g}}\ \frac{\bar{R}}{2\kappa^2} \int \mathrm{d}^n y \sqrt{-\tilde{g}} + \ldots$$

The 4D gravitational coupling 'constant' is thus related to the (fundamental) higher dimensional one by:

$$\bar{\kappa}^2 = \tilde{V}^{-1} \kappa^2,$$

where

$$\tilde{V} = \tilde{V}(x^\mu) = \int \mathrm{d}^n y \sqrt{\tilde{g}} \sim r^n$$

is the volume of the internal space.

Cosmological field equations

Einstein's field equations in $D = 4 + n$ read

$$R_{AB} - \tfrac{1}{2} R g_{AB} + \Lambda g_{AB} = \kappa^2 T_{AB} \,.$$

As an ansatz, let us now take an extended FRW metric with two scalefactors:

$$g_{AB} \mathrm{d}X^A \mathrm{d}X^B = -\mathrm{d}t^2 + a^2(t)\gamma_{ij}\mathrm{d}x^i \mathrm{d}x^j + b^2(t)\tilde{\gamma}_{pq}\mathrm{d}y^p \mathrm{d}y^q .$$

This determines the energy-momentum tensor to be of the form

$$T_{00} = \rho \,, \quad T_{ij} = -p_a \gamma_{ij} \,, \quad T_{3+p\,3+q} = -p_b \tilde{\gamma}_{pq}$$

and the field equations become

$$\begin{aligned}
3\left[\left(\tfrac{\dot{a}}{a}\right)^2 + \tfrac{k_a}{a^2}\right] + 3n\tfrac{\dot{a}}{a}\tfrac{\dot{b}}{b} + \tfrac{n(n-1)}{2}\left[\left(\tfrac{\dot{b}}{b}\right)^2 + \tfrac{k_b}{b^2}\right] &= \Lambda + \kappa^2 \rho \\
2\tfrac{\ddot{a}}{a} + \left(\tfrac{\dot{a}}{a}\right)^2 + \tfrac{k_a}{a^2} + 2n\tfrac{\dot{a}}{a}\tfrac{\dot{b}}{b} + n\tfrac{\ddot{b}}{b} + \tfrac{n(n-1)}{2}\left[\left(\tfrac{\dot{b}}{b}\right)^2 + \tfrac{k_b}{b^2}\right] &= \Lambda - \kappa^2 p_a \\
3\tfrac{\ddot{a}}{a} + 3\left[\left(\tfrac{\dot{a}}{a}\right)^2 + \tfrac{k_a}{a^2}\right] + 3(n-1)\tfrac{\dot{a}}{a}\tfrac{\dot{b}}{b} + (n-1)\tfrac{\ddot{b}}{b} + & \\
\tfrac{(n-1)(n-2)}{2}\left[\left(\tfrac{\dot{b}}{b}\right)^2 + \tfrac{k_b}{b^2}\right] &= \Lambda - \kappa^2 p_b
\end{aligned}$$

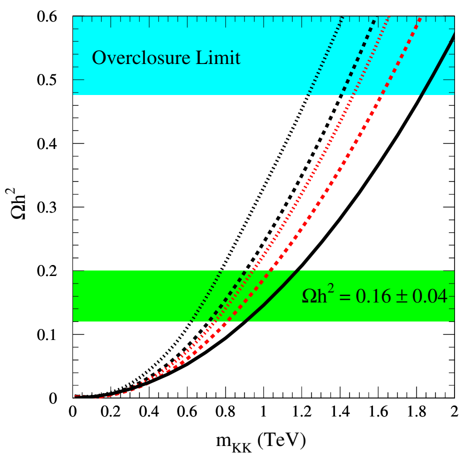
Universal Extra Dimensions

(Appelquist, Cheng & Dobrescu '01)

- *all* Standard model fields may propagate in the extra dimensions
- compactification on an orbifold
- KK *parity* is still conserved

$\longrightarrow$ The lightest KK-particle (LKP) is stable and thus a dark matter candidate (if thermally produced in the early universe)

LKP $= B^{(1)}$ (photon)



LKP $= \nu^{(1)}$ (neutrino)

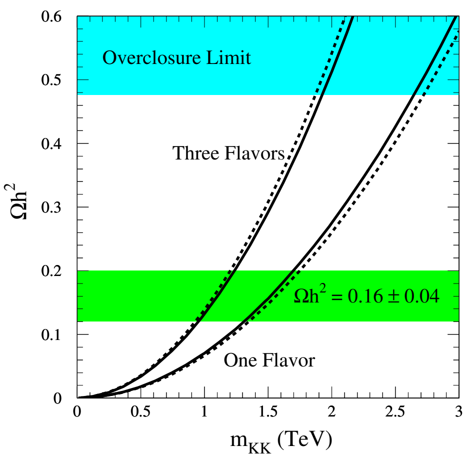


Fig.: Servant & Tait '02

Stability of UEDs

With KK dark matter, the universe has always been dominated by $(4+n)$-dim. radiation:

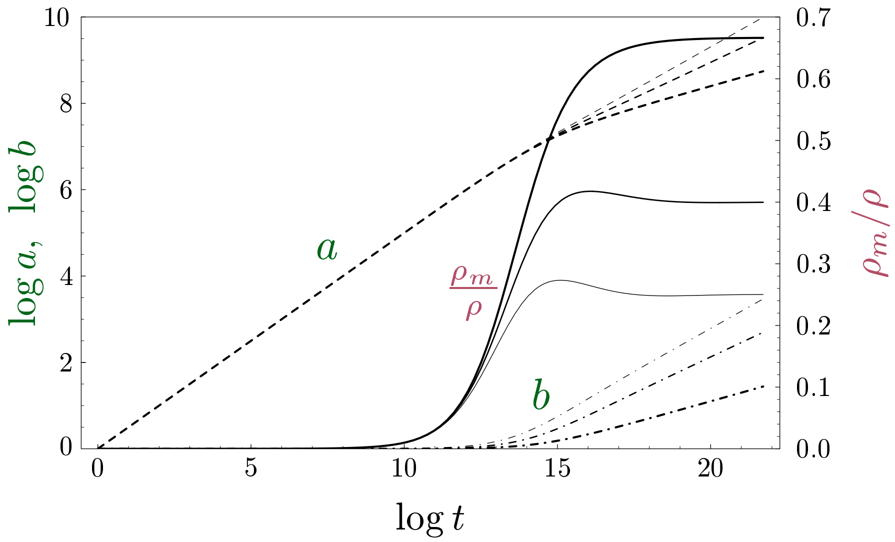
$$3p_a + np_b = \rho\,.$$

On the other hand, static solutions of the field equations are only possible for

$$\rho - 3p_a + 2p_b = 0\,.$$

$\longrightarrow$ Only for (4D) radiation domination, i.e. $p_a = \frac{1}{3}\rho,\ p_b = 0$, both equations are satisfied.

(During (4D) matter domination ($p_a = 0,\ p_b = \frac{1}{n}\rho$), in particular, there are no static solutions.)

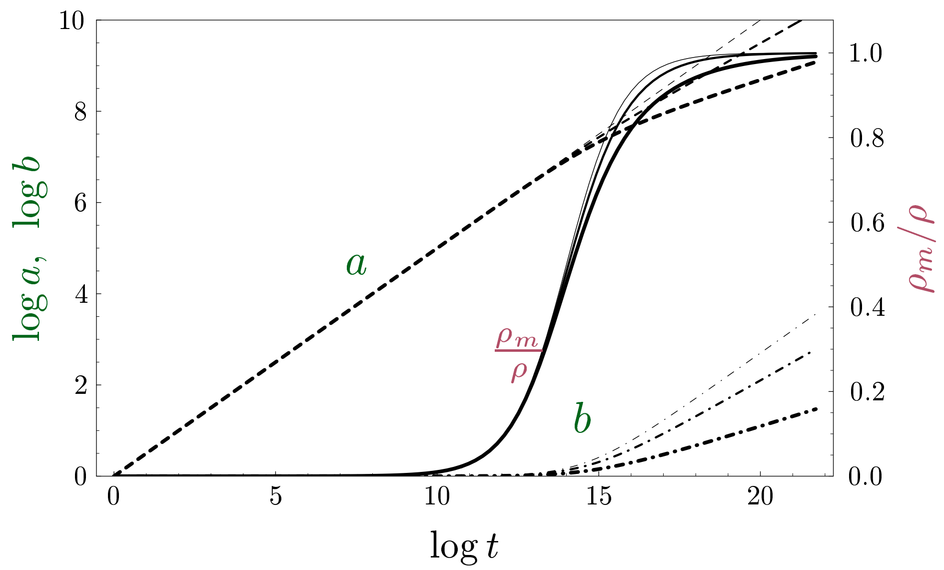


Stability of UEDs (2)

If the KK contribution to the dark matter is negligible, one has instead

$$p_a = p_b = 0$$

during matter domination. Again, there are no static solutions.



For this and the last figure it was assumed that $\left(\frac{\rho_m}{\rho}\right)_i = 10^{-7}$. The different curves correspond (with increasing thickness) to $n = 1, 2, 7$.

A framework for stabilization

Introduce background fields described by an effective action of the form

$$S^{\mathrm{bg}} = -\int \mathrm{d}^{4+n}X\sqrt{-g}\,W(b)\,,$$

which induces the following contribution to the energy-momentum tensor:

$$\rho^{\mathrm{bg}} = -p_a^{\mathrm{bg}} = W(b)$$

and

$$p_b^{\mathrm{bg}} = -W(b) - \frac{b}{n}W'(b)\,.$$

This framework includes for example:

- the Freund-Rubin mechanism

  (Cremmer & Scherk '76, Freund & Rubin '80, ...)

- stabilization by the Casimir effect

  (Appelquist & Chodos '83, Candelas & Weinberg '84, Ponton & Poppitz '01 ...)

Recovering standard cosmology

Setting $b(t) = b_0$ reduces the field equations to the ordinary Friedmann equations with an effective 4D cosmological constant

$$\bar{\Lambda} = \frac{2\Lambda}{n+2} + \frac{\kappa^2}{n+2}\left[2W(b_0) + b_0 W'(b_0)\right]$$

plus a consistency relation:

$$(n+2)(n-1)\frac{k_b}{b_0^2} = 2\Lambda + 2\kappa^2\left[W(b_0) - b_0 W'(b_0)\right] + \kappa^2(\rho - 3p_a + 2p_b).$$

This can easily be satisfied for:

- constant T^{AB}
- radiation domination ($\rho = 3p_a$, $p_b = 0$)

... but *not* for matter domination ($p_a = 0$)!

(Unless one allows for $p_b = -\frac{1}{2}\rho$)

A different perspective

Start from the higher dimensional action

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^{4+n} X \sqrt{-g} \, \left(R - 2\Lambda - 2\kappa^2 \mathcal{L}_m \right)$$

and make the ansatz

$$g_{AB} \, \mathrm{d}X^A \mathrm{d}X^B = \bar{g}_{\rho\sigma}(x^\mu) \, \mathrm{d}x^\rho \mathrm{d}x^\sigma + b^2(x^\mu) \tilde{g}_{rs}(y^p) \, \mathrm{d}y^r \mathrm{d}y^s .$$

A Weyl transformation to $\hat{g}_{\mu\nu} = b^n \bar{g}_{\mu\nu}$ gives

$$\boxed{S = \int \mathrm{d}^4 x \sqrt{-\hat{g}} \left(\frac{1}{2\hat{\kappa}^2} \hat{R} - \frac{1}{2} \left(\partial_\mu \Phi \right)^2 + V_{eff}(\Phi) \right)}$$

whith $\Phi \sim \ln b$ and

$$V_{eff}(\Phi) = c_1 \tilde{R} e^{-C_1 \Phi} + (c_2 \Lambda + c_3 \mathcal{L}_m) e^{-C_2 \Phi} .$$

The equation of motion for the scalar field is then given by

$$\hat{\nabla}_\mu \hat{\nabla}^\mu \Phi = -V'_{eff}(\Phi) \, .$$

$\rightarrow$ For $\mathcal{L}_m = \rho \sim a^{-3(1+w_a)} b^{-n(1+w_b)}$ this gives the same consistency relation for $\dot{b} = 0$ as before.

Ways out?

In order to evade the presented problems with a period of matter domination one has to invent new stabilization mechanisms that do not only depend on b. One possibility would be to have

$$W(b) \to W(b, \rho),$$

as one could expect from interaction terms between the background fields and the matter Lagrangian.

In such a case

- one would get additional time-dependent terms in the consistency condition, so it may be satisfiable even during matter domination. ($\to$ fine tuning... !?)
- the minimum of V_{eff} is still most likely to be different during for example radiation and matter domination, i.e. one expects a time variation during transition periods.

... can this be related to branes?

Up to now, the discussion has been restricted to homogeneous ED, but:

- Naively, an empty (or Λ-dominated) bulk should allow for static solutions.
- In some models, gauge fields or even fermions are allowed to leave the brane. $\rightarrow$ at least for a changing equation of state in the bulk, one would expect similar conclusions as discussed before.

Some examples of brane world scenarios:
[Arnowitt, Dent & Dutta, hep-th/0405050]

- In the Randall-Sundrum (I) case, one expects the distance between the branes to change by a (small) amount proportional to ρ_{nr}.
- In Horava-Witten cosmology, a static fifth dimension does not exist for non-relativistic matter (not even to $\mathcal{O}(\rho)$).

Conclusions

In order to match observations, it is important to have (homogeneous) ED not only compactified but also stabilized.

- Naturally possible only during radiation domination or periods with constant energy density and pressure
- Inherent problems otherwise, e.g. during matter domination
- During transition periods one would generally expect a time-variation of the size of the ED (and thus of coupling constants)
- Results do not only apply to homogeneous ED. Particularly, one would expect a similar behaviour for all models with a changing equation of state in the bulk