Chameleon Cosmology

1) Scalar-tensor theories and chameleons
2) Thin vs Thick shell property
3) Thick shell and Cosmology
4) Thin shell and Cosmology

collaboration with A. Davis,
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to appear very soon

Conundrum :

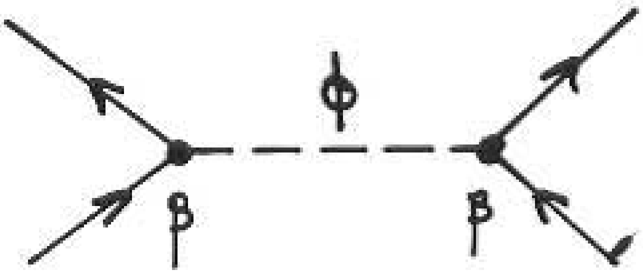
For Quintessence $V(\phi) \sim 10^{-120} m_p^4$

eg : $V(\phi) = \frac{M^4}{\phi^\alpha}$ (Ratra-Peebles)

$$\phi_{now} \sim 1$$

$m_\phi^2 \sim H_0^2$ effectively almost massless

mass of the quintessence field



$\delta F \propto \frac{\beta^2}{r^2}$

"fifth force"

no deviation from Newton's law → $\beta \ll 1$

how can this be achieved ?

I Scalar-tensor theories and chameleons

$$\begin{tikzcd}
 & & \text{gravity} \\
\text{extra dimensional models} \arrow[r] \arrow[dd] & \text{moduli} \arrow[ur] \arrow[dr] & \\
 & & \text{scalars} \arrow[d] \\
\text{low energy action 4d} \arrow[rr] & & \text{scalar-tensor theories}
\end{tikzcd}$$

- $$S = \frac{1}{2\kappa_4^2}\int d^4x \sqrt{-g}\,\left(R - (\partial\phi)^2 - V(\phi)\right)$$

  $\uparrow$ Einstein Frame

- $$S_{matter}\left(\psi, A^2(\phi)\, g_{\mu\nu}\right)$$

  $\uparrow$ matter field $\qquad$ $\nwarrow$ minimally coupled

eg: the radion in the R-S model

$$\begin{tikzcd}
 & R \gg 1 \quad \frac{1}{2} e^{\frac{R}{\sqrt{6}}} \\
A(\phi) = \cosh\frac{R}{\sqrt{6}} \arrow[ur] \arrow[dr] & \\
 & R \ll 1 \quad 1 + \frac{R^2}{12}
\end{tikzcd}$$

$R \gg 1$ short distance

$R \ll 1$ large distance

The Klein-Gordon equation

$$\Box \phi = x_4^2 \frac{\partial V}{\partial \phi} - x_4^2 \alpha_\phi T$$

↑ coupling to the trace of the energy-momentum tensor

$$\alpha_\phi = \frac{\partial \ln A}{\partial \phi}$$

The non-conservation equation

$$D_\mu T^\mu_\nu = \alpha_\phi (D_\nu \phi) T$$

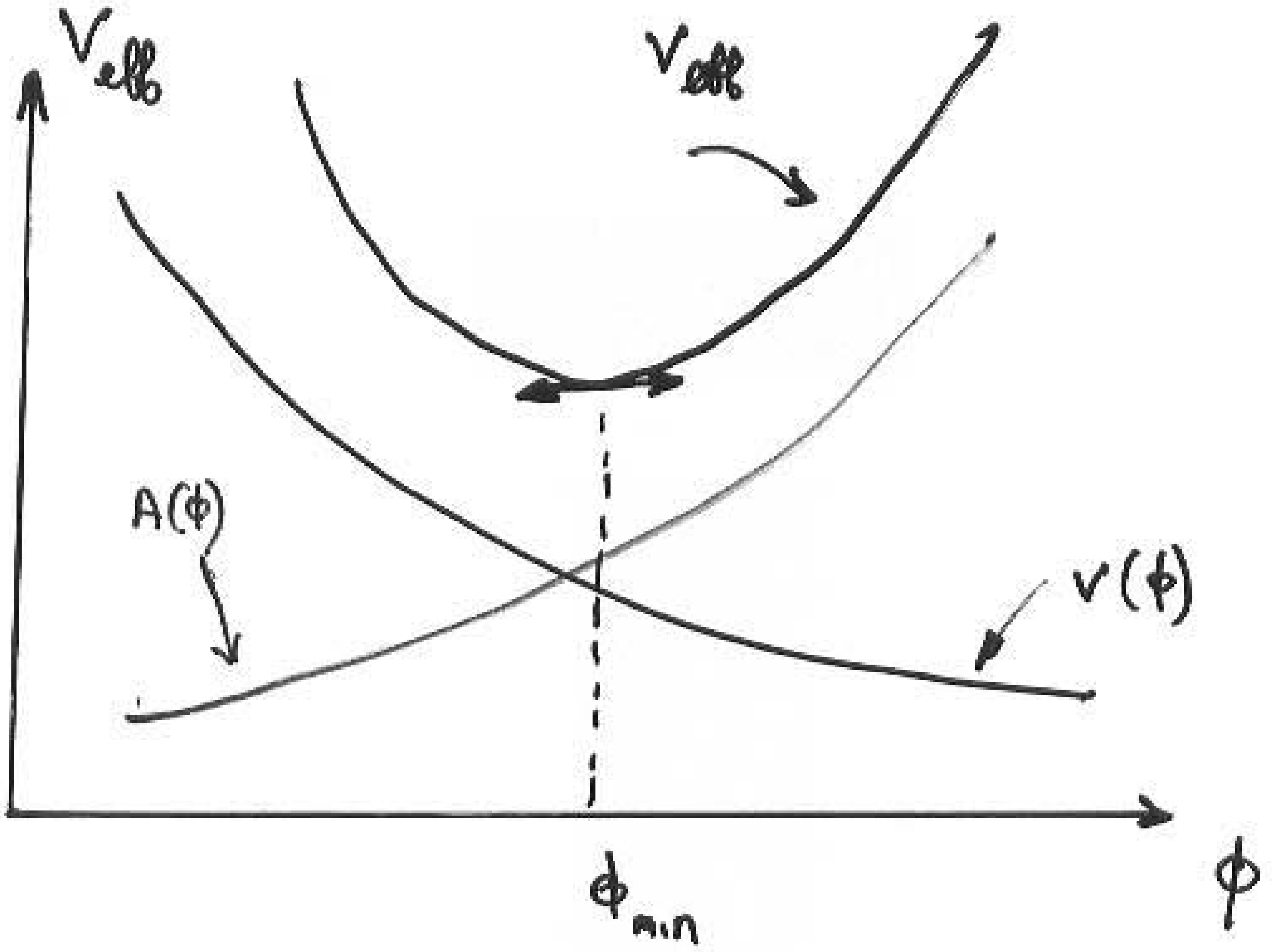
In a FRW universe ⟶

$$\rho = A(\phi)\ \rho_m \qquad \rho_m = \frac{\rho_0}{a^{3(1+w_m)}}$$

↑ conserved

$$\dot{\phi} + 3H\dot{\phi} = -x_4^2 \frac{\partial V}{\partial \phi} - x_4^2 (1-3w_m) \frac{\partial A}{\partial \phi}$$

$$V_{eff}(\phi) = V(\phi) + \rho_m(1-3w_m)\,A(\phi)$$



Cosmologically

$m^2_{min} \propto H^2 \Rightarrow$ almost massless

Solar system $m^2_{min} \longrightarrow$ thick vs thin shell

$$F_\phi = - m\,\alpha_\phi \frac{\partial \phi}{\partial x_\mu}$$

eg : radion α_ϕ ↗ $(\text{const})\,\phi$ $R \ll 1$

↘ const $R \gg 1$

typical potential

- $$V(\phi) = \frac{\Lambda^4}{\phi^\alpha}$$ (quintessence Ratra-Peebles)

ingeneral

$$\rho_m \nearrow \qquad \phi_{min} \searrow$$

- $$V(\phi) = M^4 f\left(\frac{\phi m_p}{M}\right)$$

f diverges at $\phi_\#$

f verifies the tracker condition

$$\frac{f'' f}{f'^2} > 1$$

crucial sign

eg: $$V(\phi) = M^4 e^{\left(\frac{\phi m_p}{M}\right)^{-n}}$$

$\phi m_p \gtrsim M \qquad V(\phi) \simeq M^4 + \frac{M^{4+n}}{m_p^n \phi^n}$

$\phi m_p \lesssim M \qquad V(\phi) \simeq M^4$

solar system tests

only relevant on cosmological scales

II Thick-vs-Thin shells

typical setting for gravity tests:

massive compact object

$$\phi'' + \frac{2}{r}\phi' = \kappa_4^2 \frac{\partial V_{\rm eff}}{\partial \phi}$$

$r > R_0$ ϕ close to its minimum

$$\phi'' + \frac{2}{r}\phi' = m_\infty^2(\phi - \phi_\infty)$$

$r < R_0$ depends crucially on $A(\phi)$

$$\boxed{A(\phi) = 1 + \beta \frac{\phi^{m+1}}{m+1}}$$

$m = 0, 1, 2 \ldots$

determines thin and thick shells

Thick Shells $(m=1)$

$r < R_\circ$ $$\phi \sim \frac{\sinh(r/R_r)}{r} \qquad R_r^2 = \frac{m_p^2}{\beta \rho_\circ}$$

$r \geqslant R_\circ$ $$\phi = C + \frac{D}{r}$$

$$F_\phi = -\frac{\beta^2 \phi_\infty^2}{4\pi} \frac{m M_\circ}{m_p^2 r^2} = \Theta F_{newton}$$

$$\boxed{\Theta = 2\beta^2 \phi_\infty^2}$$

cassini type experiments : $\Theta \lesssim 10^{-5}$

↓

$\beta = O(1)$ ϕ_∞ severely constrained

↑ determined cosmologically

↓

restrictions on the types of potentials

thick-shell catastrophe

$m > 1$ $\quad F_\phi \underset{r\to\infty}{=} O\left(\frac{m}{r}\right)$!

Thin shells $(m=0)$

obtained for a constant α_ϕ

minimum inside

$r \leqslant R_s \qquad \phi = \phi_b$

mass inside the $r \leq R_s$ region

$$R_s \leqslant r \leqslant R_0 \qquad \phi = \phi_b - \frac{3\beta}{8\pi}\frac{x_4^2 M_s}{R_s} + \frac{\beta x_4^2 \rho_0}{6} r^2 + \frac{\beta x_4^2}{4\pi}\frac{M_s}{r}$$

minimum outside

$$r \geqslant R_0 \qquad \phi = \phi_\infty + \frac{\beta x_4^2}{4\pi}\frac{M_s - M_0}{r}$$

$$\frac{R_0 - R_s}{R_0} = \frac{1}{6\beta}\frac{\phi_\infty - \phi_b}{\Phi_0}$$

$$\Phi_0 = \frac{x_4^2 M_0}{8\pi R_0}$$ Newton's potential

thin shell $\Leftrightarrow \dfrac{R_0 - R_s}{R_0} \ll 1$

Crucial consequence: $$\Theta = \frac{9 G_N (\phi_\infty - \phi_b)}{R_0^2}$$

Thin shell $\longrightarrow$ $\Theta \ll \frac{\rho_\circ}{m_p^4} \ll 1$

Hence Thin shell $\Rightarrow$ no violation of gravity

condition for thin shell : $\Phi_\circ \gg 1$

$\uparrow$ large bodies (earth - sun)

In satellite experiments (MICROSCOPE etc...) $\rightarrow$ no thin shell

$$\Theta = 2\beta^2 = O(1)$$

$\rightarrow$ large deviations from Newton's law in satellite experiments

what about lab experiments? test bodies $m \sim 40g$ $R_\circ \sim 1cm$

$\downarrow$ $\Phi_\circ \sim 3.10^{-27}$

$$\phi_\infty \lesssim 10^{-28} \Rightarrow M \lesssim 10^{-3} eV$$

III Thick shell Cosmology

how to achieve $\phi_\infty \ll 1$?

- $V(\phi) = \frac{\Lambda^4}{\phi^\alpha}$ → modified quintessence due to minimum

$$\Lambda \sim 10^{-3}\,\text{eV} \qquad \underset{\substack{\uparrow \\ \text{extremely flat potential}}}{\alpha \lesssim 10^{-5}}$$

- $V(\phi) = M^4 e^{\left(\frac{M}{m_p \phi}\right)^n}$

$M = 10^{-3}$ eV → acceleration now

$$\phi_{now} = O\left(\left(\frac{M}{m_{pl}}\right)^{\frac{2n}{n+2}}\right) \overset{n=1}{=} O(10^{-10})$$

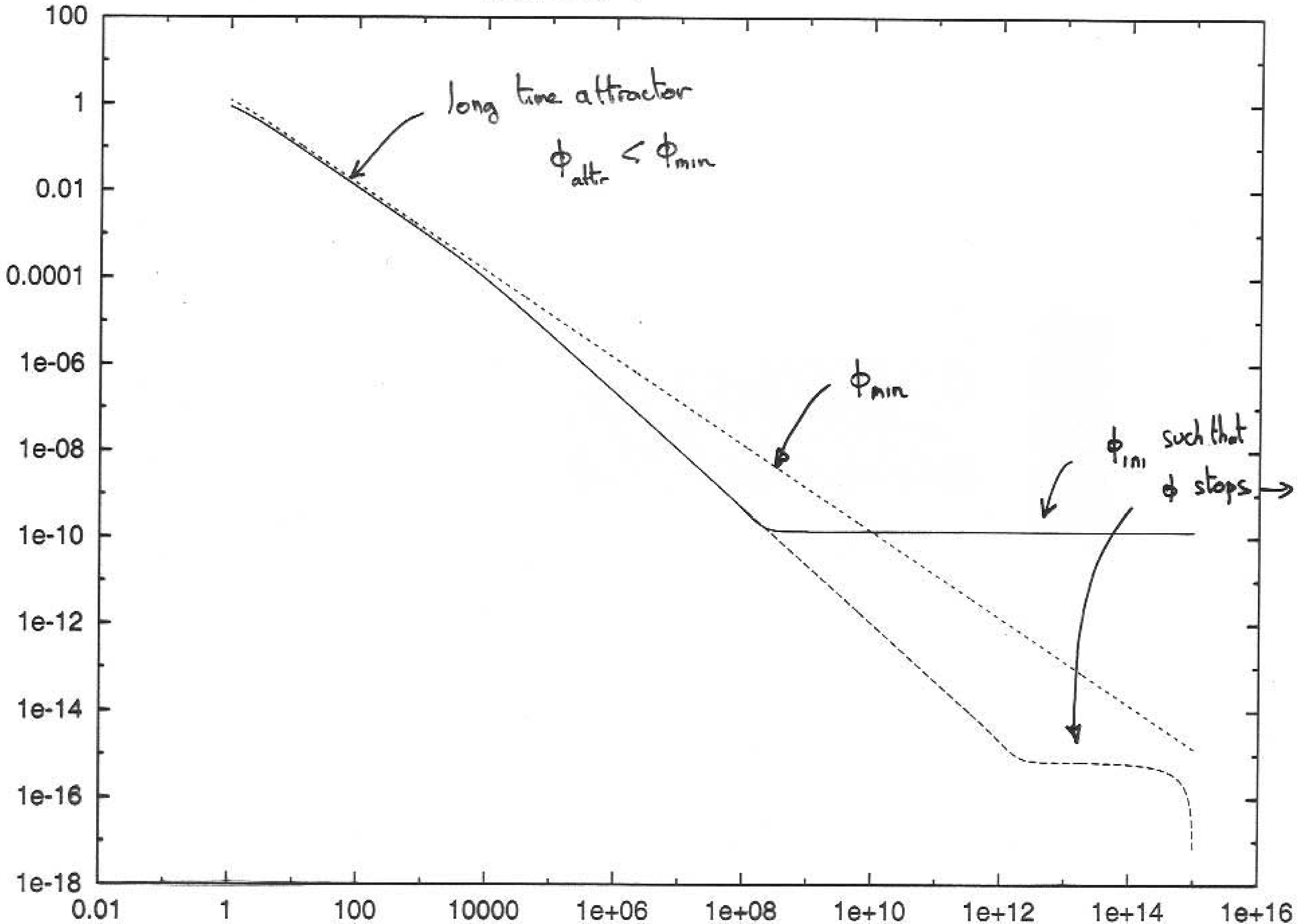
$$\downarrow$$

$$\phi_{now} \ll 1$$

Dynamically

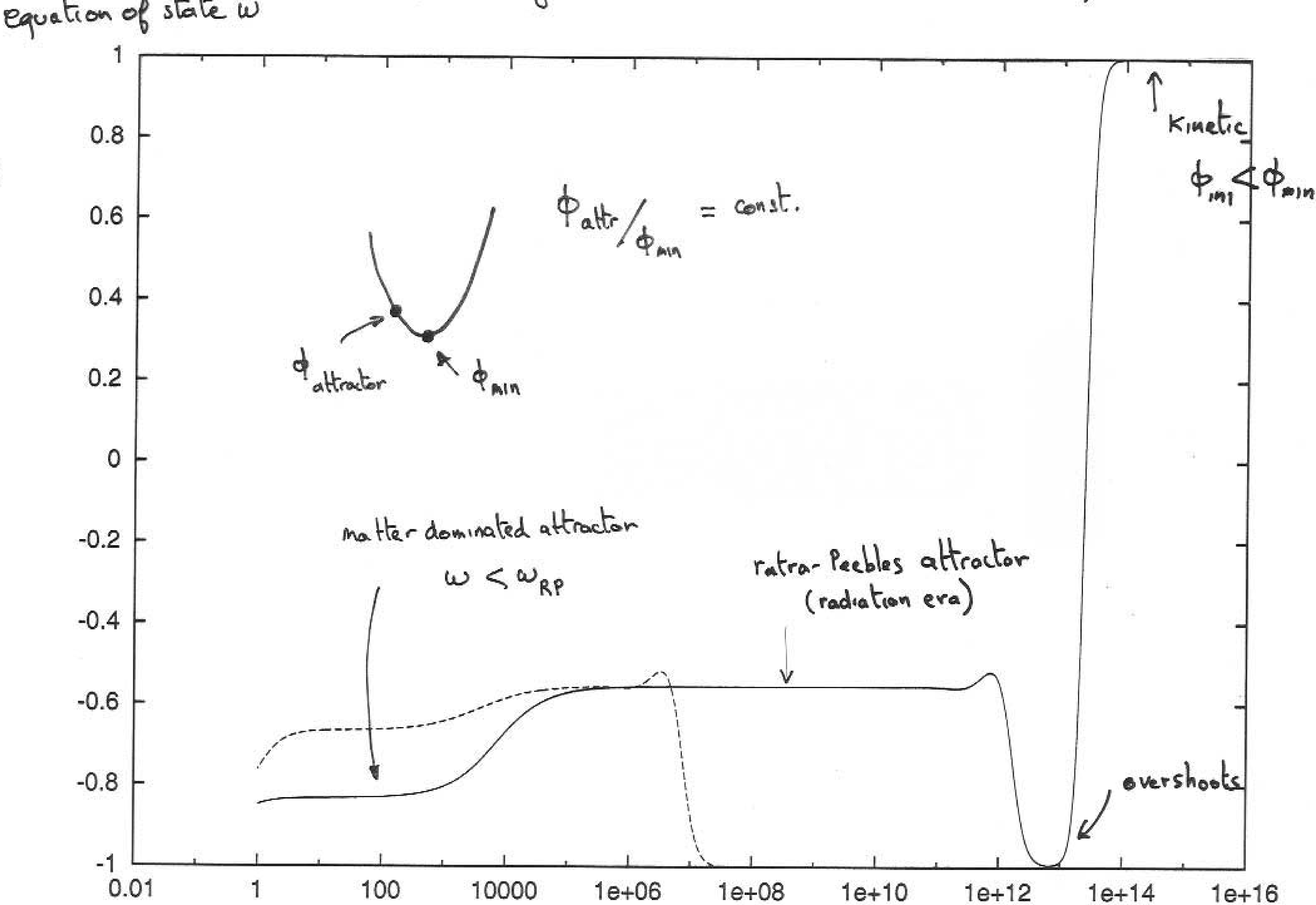
- $m_p \phi \ll M$ → mass ≫ H → minimum is an attracto
- $m_p \phi \gg M$ → previous case → almost on the minimum

Evolution of the scalar field



Cosmological evolution (thick shell $m=1$)

②



IV Thin shell cosmology

→ choose to treat $V(\phi) = M^4 e^{\left(\frac{M}{m_p \phi}\right)^n}$

$M \lesssim 10^{-3}\,\text{eV}$ (gravity tests)

- $\underline{\phi_{min}}$ is an attractor

$m_\phi^2 \gg H^2$ → field oscillates around the minimum

$$\phi_{min} < \phi_{\text{Ratra Peebles}}$$

Fast converge to the minimum

$$\langle \phi - \phi_{min} \rangle \sim a^{-\frac{3n}{4(n+1)}}$$

and the energy

$$\rho_\phi \sim a^{-\frac{3(3n+4)}{2(n+1)}}$$ (faster than radiation

→ exit from moduli probl

• Constraints on initial conditions

mass variation since BBN

$$\left|\frac{\Delta m}{m}\right| \approx \frac{\beta}{m_p}\left|\phi_{BBN}\right| \qquad \beta = O(1)$$

$$\left|\phi_{BBN}\right| \lesssim 0.1\, m_p$$

→ $$\phi_c \lesssim 0.5\, m_p$$

$$\Omega_\phi^{ini} \lesssim 4.10^{-2}$$

• effects on density perturbations

$$\delta_c'' + aH\,\delta_c' = \frac{3}{2}a^2H^2\left[1 + \frac{2\beta^2}{1 + \frac{a^2 V_{,\phi\phi}}{k^2}}\right]$$

modification of Newton's constan

Scales $\lesssim \lambda_{cham} = V_{,\phi\phi}^{-1/2}$ are affected

$$G_N\left(1 + 2\beta^2\right)$$

nothing but Θ in vacuum

$$\boxed{\delta \sim \tau^{x}}$$

$x = -\frac{1}{2} +/- \sqrt{\frac{1}{4} + 6(1+2\beta^2)}$ eg: $\beta = 1$ $x = 3.8 > \underset{\underset{G.R}{\uparrow}}{2}$

- If $\underline{\phi = \phi_{min}}$ at re combination → $V_{,\phi\phi} \sim m^2 \gg H^2$

⇒ Scales affected are ≪ Hubble horizon → nothing on CMB

- If $\underline{\phi \neq \phi_{min}}$ at recombination, $\lambda_{chan} \gtrsim 1$ kpc ⇒ $\underline{\phi_{recomb} > 10^{15} M}$

→ would be interesting to see the effect on galaxy formation

Conclusions

$$\text{Coupling} \quad A(\phi) = 1 + \beta \frac{\phi^{m+1}}{m+1} \begin{array}{l} \nearrow m=0 \quad \text{thin shell} \\ \searrow m=1 \quad \text{thick shell} \end{array}$$

$m = 0$ $\phi = \phi_{min}$ attractor

$m = 1$ $\phi_{att} < \phi_{min}$