Simulation and Scientific Computation
Report: *WaveMachine* by Sebastian Klipp, 30.5.14

1 Introduction

The NetLogo [1] model *WaveMachine* [2] simulates the propagation of a two-dimensional wave on a membrane, driven by an external force (driver). The simulation follows a multi particle approach, where the membrane consists of a bunch of particles, rather than an overall analytical description through wave modes. The physical behaviour of the membrane is described through the stiffness ϵ and the friction μ.

2 Model description

The model's state space Σ consists of several possible states σ, which assign to every particle $i \in N$ a deflection $s_i(t) \in \mathbb{R}$.

$$\Sigma = \{\sigma\}$$
$$\sigma : N \to \mathbb{R}$$

The particles are divided in three different subsets, which are firstly normal behaving particles (subset P), particles on the edge of the simulation space (subset E) and the driving particles (subset D). The subsets V_i contain the particles of the von-Neumann neighbourhood of each particle i.

The local rule $s_i(t+1)$ for the next time step $t+1$ is

$$s_i(t+1) = \begin{cases} s_i(t) + ds_i(t+1) & \text{if } i \in P \text{ (normal particle)} \\ g(t) & \text{if } i \in D \text{ (driver particle)} \\ 0 & \text{if } i \in E \text{ (edge particle)} \end{cases}$$

The equation of movement $g(t)$ of the driver particles follows a sine wave-form and is independent from its vicinity V_i

$$g(t) = A\ \sin(0.1\, f\, t) \tag{1}$$

where A is the sine's amplitude (determined in the interface by the slider DRIVER-AMPLITUDE), f its frequency (slider DRIVER-FREQUENCY) and t the momentary time-step (TICK). The driver is the external force on the system, supplying it with energy.

The edge-particles alway remain fixed $s_{i \in E}(t) = 0\ \forall t$, so we have fixed boundary conditions and waves are reflected on the boundaries.

The deflection change $ds_i(t+1)$ is the change of z-position at the new time step for the regarded particle i and is calculated through

$$ds_i(t+1) = \frac{1000-\mu}{1000}\left(ds_i(t) + 0.01\,\epsilon\,\left(\sum_{j\in V_i} s_j(t) - 4\,s_i(t)\right)\right) \tag{2}$$

Here μ is the friction (obtained from the interface's slider FRICTION) and ϵ denotes the stiffness (slider STIFFNESS). The former stands for the strength of interaction between the single particles and the latter denotes the amount of dissipated energy in the system. The sum shows that the deflection change of a normal particle $i \in P$ is proportional to the differences of the z-direction to its four neighbours.

The setup-button starts the function SETUP which defines at first the position of the membrane-edges. Afterwards on every patch of the system a turtle i is created and its state is set to $s_i(0) = 0$ (initial conditions) and if so marked as edge-particle (variable EDGE?) or driver-particle (DRIVER?). Edge-particles are colored blue, driver-particles green and normal particles red. Afterwards every turtle gets assigned an agentset of its four neighbouring turtles (variable NEIGHBOR-TURTLES).

The model gets into motion using the go-button, starting the function GO. Here for every turtle i, which is not an edge- or driver-turtle, the change in z-direction $ds_i(t+1)$ is calculated through the function PROPAGATE, using equation (2) and is then added to the momentary value $s_i(t)$. The driver-particles change their position due to equation (1), while the edge-particles do not change position. Finally the variable TICK is increased by one.

Furthermore, the driver's position on the grid, its size and the view-angle of the display can be adjusted by sliders. A switch changes the display between two- and three-dimensional view.

3 Impact of friction and stiffness on the system

First the impact of stiffness ϵ is analysed, therefor the driver is deflected only once to its maximum amplitude and its impact on the surrounding particles is regarded. Figure (1a) shows the result for small stiffnesses $\epsilon = 5$, only few particles follow the driver's movement, while in Figure (1b) the impact of the external force is spreaded further, due to high stiffness $\epsilon = 50$.

Next the effect of friction is investigated. The driver goes one whole periode and the resulting wave is regarded. For small friction $\mu = 1$ (Figure (2a)) the particles have a much higher maximal deflection than for high friction values $\mu = 99$ in Figure (2b), for in the latter case the dissipated energy is higher.

The analysis of waves with variated physical parameters ϵ and μ shows the expected physical behaviour.

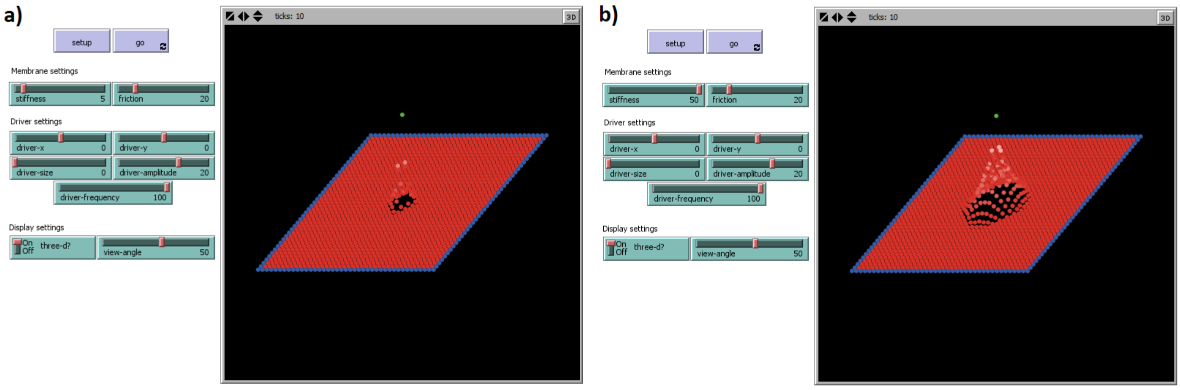

Figure 1: Comparison of small and high stiffness ϵ impacts after one quarter driver period at TICK=10 and $\mu = 20$. a) State for $\epsilon = 5$. b) State for $\epsilon = 50$.

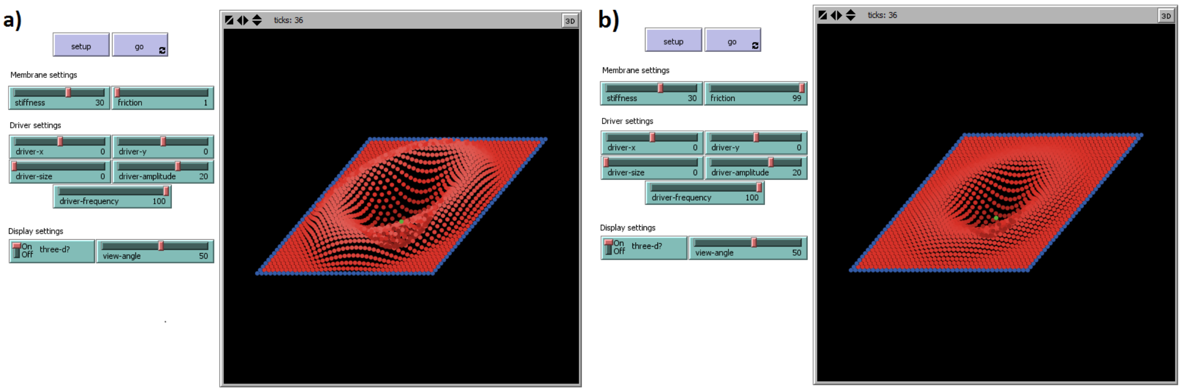

Figure 2: Comparison of small and high friction μ impacts after one driver period at TICK=36 and $\epsilon = 30$. a) State for $\mu = 1$. b) State for $\mu = 99$.

4 Sources

1) Wilensky, U. (1999). NetLogo. Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL. http://ccl.northwestern.edu/netlogo/
2) Wilensky, U. (1997). NetLogo Wave Machine model. Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL.
http://ccl.northwestern.edu/netlogo/models/WaveMachine.