A new point matching algorithm for non-rigid registration

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Tese: Robust Point Matching In Biometrics Under Severe Noise and Outliers
INESC

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1. Introdução

A basic non-rigid point matching problem can defined as follows: given two sets of points (essentially their coordinates), we would like to find the *non-rigid transformation* that best maps one set of points onto the other and/or the set of *correspondence* (including possible outliers) between the points.

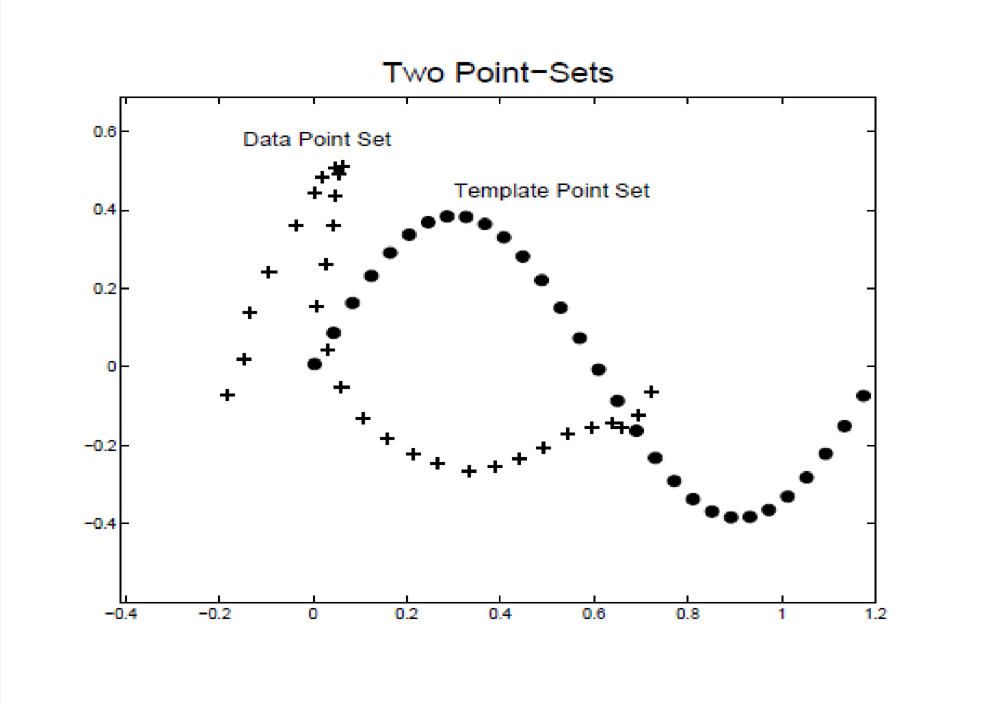

Fig. 1. A simple non-rigid point matching problem.

There are two unknown variables in the point matching problem: the **correspondence** and the **transformation**.

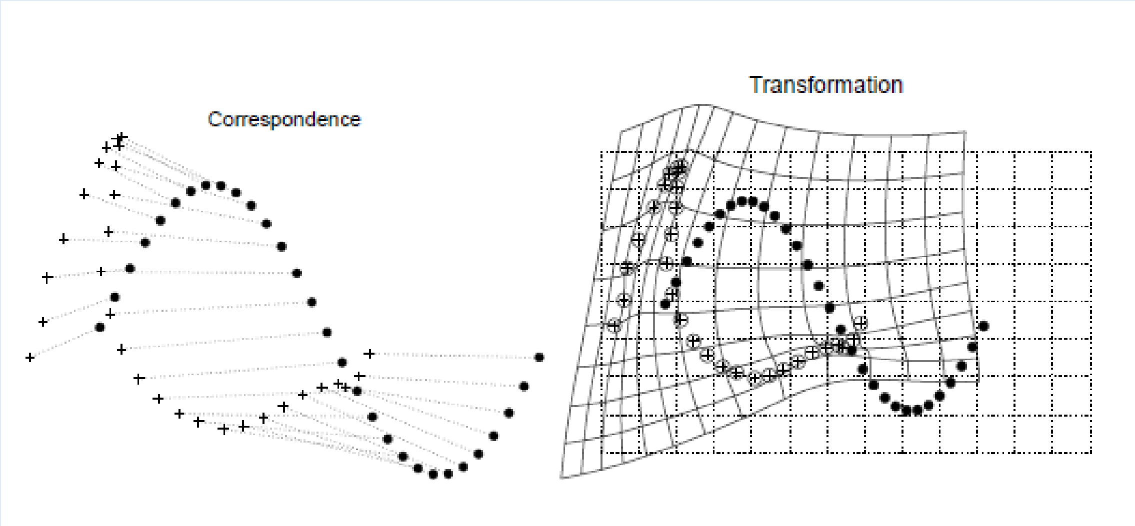

Fig. 2 The correspondence and transformation example

Methods that solve only for the spatial transformation:

- method of moments (Hibbard and Hawkins, 1988)
- Hough Transform (Ballard, 1981; Stockman, 1987)
- tree searches (Baird, 1984; Grimson and Lozano-Perez, 1987)
- the Hausdor distance (Huttenlocher et al., 1993),
- geometric hashing (Lamdan et al., 1988; Hummel and Wolfson, 1988)
- and alignment method (Ullman, 1989)

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Methods that solve only for the correspondence:

- dense feature-based methods (Tagare et al., 1995; Metaxas et al., 1997; Szeliski and Lavallee,1996; Feldmar and Ayache, 1996).
- more sparsely distributed points-sets
- weighted graph matching (Shapiro and Haralick, 1981; Cross and Hancock (1998)

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2. Previous Work

Methods that solve both the correspondence and the transformation:

Solving for just the correspondence or the transformation in isolation seems rather difficult, if not impossible. It would be much easier to estimate the non-rigid transformation once correspondences were given. However, before good correspondences can be estimated, a reasonable transformation is clearly needed.

This leads to consider joint approach for the point matching problem—alternating estimation of the correspondence and the transformation.

Methods that Treat the Correspondence as a Binary Variable

ICP – iterative closest point

Methods that Treat the Correspondence as a Continuous Variable

probabilistic approach

EM – expectation-maximization

When applied to the point matching problem, the E-step basically estimates the correspondence under the given transformation, while the M-step updates the transformation based on the current estimate of the correspondence.

3.1 A Binary Linear Assignment-Least Squares Energy Function

Suppose we have two point-sets V and X (in $\mathbb{R}^2$or in $\mathbb{R}^3$) consisting of points $\{v_a, \ \ a = 1,2, \dots, K\}$ and $\{x_i, i = 1,2, \dots, N\}$respectively. For the sake of simplicity, we will assume for the moment that the points are in 2D. We consider general function f representing the non-rigid transformation by a general function f and introduce an operator L and our chosen *smoothness* measure is $||Lf||^2$.

We would like to match the point-sets as closely as possible while rejecting a reasonable fraction of the points as outliers. The correspondence problem is cast as a linear assignment problem (Papadimitriou and Steiglitz, 1982), which is augmented to take outliers into account.

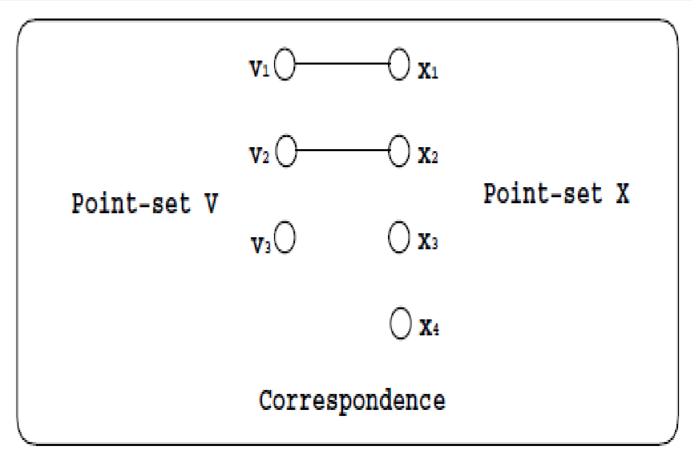

Fig. 3

z_{ai}	x_1	x_2	x_3	x_4	outlier
v_1	1	0	0	0	0
v_2	0	1	0	0	0
v_3	0	0	0	0	1
outlier	0	0	1	1	

Fig. 4

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3.1 A Binary Linear Assignment-Least Squares Energy Function

The goal in this article is to minimize the following binary linear assignment-least squares energy function:

robusteness control

$$\min_{Z,f} E(Z,f) = \min_{Z,f} \sum_{i=1}^{N} \sum_{a=1}^{K} z_{ai} ||x_i - f(v_a)||^2 + \lambda ||Lf||^2 - \zeta \sum_{i=1}^{N} \sum_{a=1}^{K} z_{ai} \qquad (1)$$

constraint on the transformation

subject to $\sum_{n=1}^{N+1} z_{ai} = 1$ for $i \in \{1,2,\ldots,N\}$, $\sum_{a=1}^{K+1} z_{ai} = 1$ for $a \in \{1,2,\ldots,K\}$ and $z_{ai} \in \{0,1\}$.

The matrix Z or $\{z_{ai}\}$ is the binary *correspondence matrix*. The inner $N \ x \ K$ part of Z defines the correspondence. The extra $N + 1^{th}$ row and $K + 1^{th}$ column of Z are introduced to handle the outliers.

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3.2 Softassign and Deterministic Annealing

The basic idea of the softassign is to relax the binary correspondence variable Z to be a continuous valued matrix M in the interval [0,1], while enforcing the row and column constraints.

The continuous nature of the correspondence matrix M basically allows fuzzy, partial matches between the point-sets V and X.

From an optimization point of view, this fuzziness makes the resulting energy function better behaved.

With this notion of fuzzy correspondence established, another very useful technique, deterministic annealing can be used to directly control this fuzziness by adding an entropy term in the form of $T \sum_{i=1}^{N+1} \sum_{a=1}^{K+1} m_{ai} \log m_{ai}$ to the original assignment energy function.

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3.3 A Fuzzy Linear Assignment-Least Squares Energy Function

After introducing these two techniques, the original binary assignment-least squares problem is converted to the problem of minimizing the following fuzzy assignment-least squares energy function.

$$E(M,f) = \sum_{i=1}^{N}\sum_{a=1}^{K} m_{ai}||x_i - f(v_a)||^2 + \lambda||Lf||^2 + T\sum_{i=1}^{N}\sum_{a=1}^{K} m_{ai}\log m_{ai} - \zeta\sum_{i=1}^{N}\sum_{a=1}^{K} m_{ai}, \qquad (2)$$

When the temperature *T* reaches zero, the fuzzy correspondence *M* becomes binary.

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3.4 The Robust Point Matching (RPM) Algorithm

The resulting robust point matching algorithm (RPM) is quite similar to the EM algorithm.

Step 1: Update the Correspondence: For the points for $a \in \{1, \ldots, K\}$ and $i \in \{1, \ldots, N\}$

$$m_{ai} = \frac{1}{T} e^{-\frac{(x_i - f(v_a))^T (x_i - f(v_a))}{2T}} \qquad (3)$$

and for the outlier entries $a = K + 1$ and $i = 1;\ 2, \ldots, N$, and for the other outliers entries $a = 1;\ 2, \ldots, N$ and i $= N + 1$

$$m_{K+1,i} = \frac{1}{T_0} e^{-\frac{(x_i - v_{K+1})^T (x_i - v_{K+1})}{2T_0}} \qquad (4)$$

$$m_{a,N+1} = \frac{1}{T_0} e^{-\frac{(x_{N+1} - f(v_a))^T (x_{N+1} - f(v_a))}{2T_0}} \qquad (5)$$

where v_{k+1} and $x_{k+1}$are the outlier cluster centers.

Run the iterated row and column normalization algorithm to satisfy the constraints until convergence is reached.

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Step 2: Update the Transformation: After dropping the terms independent of f, we need to solve the following least-squares problem,

$$\min_f E(f) = \min_f \sum_{i=1}^{N} \sum_{a=1}^{K} m_{ai} ||x_i - f(v_a)||^2 + \lambda T ||Lf||^2. \quad (6)$$

The solution for this least-squares problem depends on the specific form of the non-rigid transformation.

Annealing: An annealing scheme controls the dual update process. Starting at $T_{init} = T_0$, the temperature parameter T is gradually reduced according to a linear annealing schedule, $T^{new} = T^{old} r$ (r is called the annealing rate). The dual updates are repeated till convergence at each temperature. Then T is lowered and the process is repeated until some final temperature T_{final} is reached.

3.4 The Robust Point Matching (RPM) Algorithm

We normally starts the algorithm's alternating update process by setting the transformation parameters be zeros (so that the transformation is an identity transformation and points stay at their original place). Then we run the correspondence update and the transformation update while gradually lower the temperature.

The General RPM Algorithm Pseudo-code:

Initialize parameters $T = T_0$ and λ.

Initialize parameters α (or M).

Begin A: Deterministic Annealing.

Begin B: Alternating Update.

Step I: Update correspondence parameter M based on current α.

Step II: Update transformation parameter α based on current M.

End B

Decrease $T \leftarrow T \cdot r$ until T_{final} is reached.

End A

Fig.5

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4. The Thin-Plate Spline and the TPS-RPM Algorithm

Different models of transformation have their different properties and, hence are suitable for different applications. An algorithm’s ability to accommodate different transformation models can make it a general tool for many problems.

But to complete the specification of the non-rigid point matching algorithm, its used specific form of non-rigid transformation the thin-plate spline.

Because any non-rigid transformation can be put in to replace the general notion of f, and the used in this article doesn´t look interesting for the context of the thesis, I will not go in further details about this specific transformation.

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Final position. U (transformed V) and X.

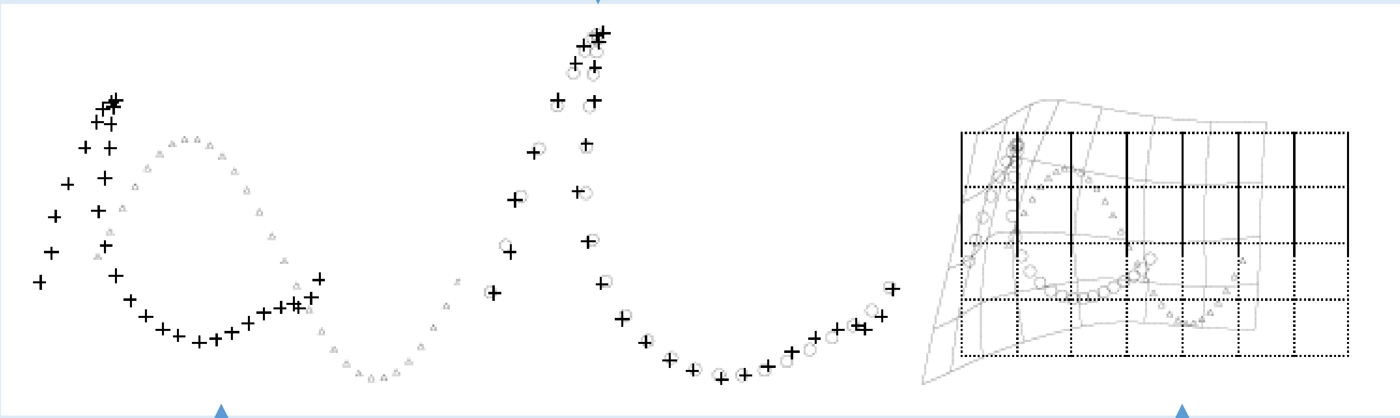

Fig. 6 A Simple 2D Example

Initial position. Two point sets V (triangles) and X (crosses).

Deformation of the space is shown by comparing the original regular grid (dotted lines) and its transformed version (solid lines).

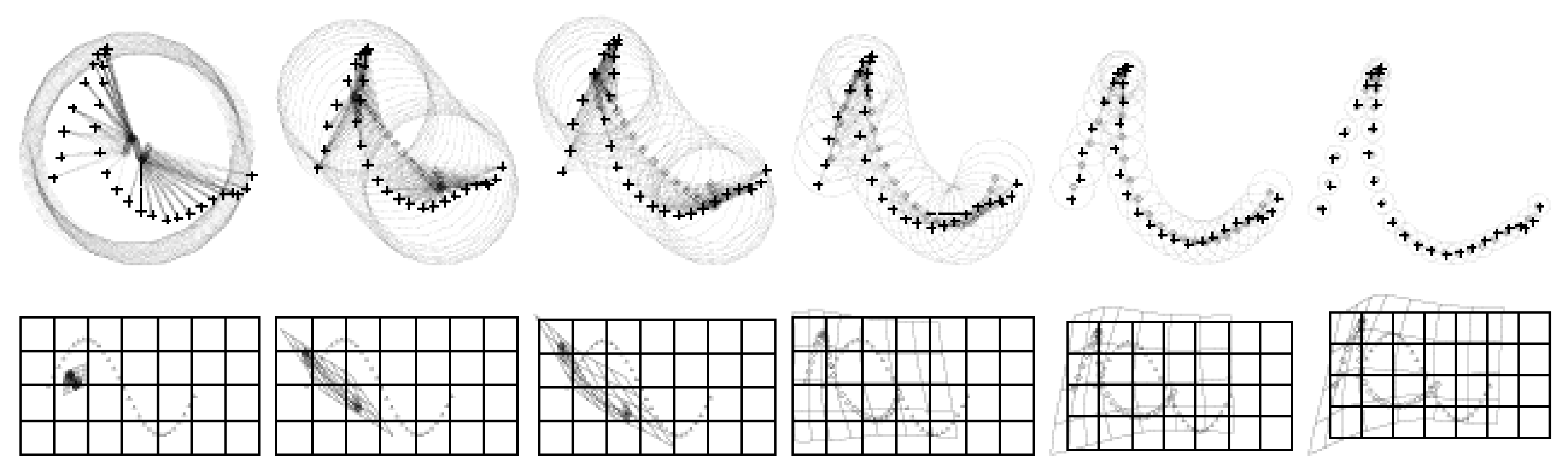

Fig. 7 Matching Process

Each column shows the state of the algorithm at a certain temperature T.
Top: Current correspondence between U (transformed V , circles) and X (crosses). The most significant correspondences $\left(m_{ai} > \frac{1}{K}\right)$ are shown as dotted links. A dotted circle is of radius is drawn around each point in U to show the annealing process.
Bottom: Deformation of the space. Again dotted regular grid with the solid deformed grid. Original V (triangles) and U (transformed V , circles).

To test RPM's performance, we ran a lot of experiments on synthetic data with different degrees of warping, different amounts of noise and different amounts of outliers and compared it with ICP.

Fig. 8

There are two important free parameters in the new non-rigid point matching algorithm - the regularization parameterλ and the outlier rejection parameter ς.

We have developed a new non-rigid point matching algorithm –TPS-RPM – which is well suited for non-rigid registration. The algorithm utilizes the softassign, deterministic annealing, the thin-plate spline for the spatial mapping and outlier rejection to solve for both the correspondence and mapping parameters.

The computational complexity of the algorithm is largely dependent on the implementation of the spline deformation [which can be $O(N^3)$ in the worst case].

We have conducted carefully designed synthetic experiments to empirically demonstrate the superiority of the TPS-RPM algorithm over TPS-ICP and have also applied the algorithm to perform non-rigid registration of cortical anatomical structures.