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Stress testing a retail loan portfolio: an error correction model approach

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The use of stress testing for risk monitoring has increased considerably over the last decade. Stress testing – a simulation technique used to assess the strength of a portfolio or a financial institution under unusual economic conditions – emerged as a powerful tool that was originally used in market risk. Its use has subsequently been extended into credit risk. To stress test a credit risk portfolio, practitioners focus on the key parameters that allow the risk of a credit portfolio to be assessed. These parameters, also known as Basel II parameters, are probability of default, loss given default, exposure at default and asset correlation. In this paper, using a time series approach (specifically, an error correction model), we focus on the probability of default parameter that is related to macroeconomic factors. Such an approach involves dealing with the nonstationarity of economic time series and cointegration issues. Hence, when the model is estimated, the probability of default can be simulated by measuring the effects of macroeconomic shocks applied to the model. In turn, these probabilities of default can be used to measure the impact on the probability of default under a given macroeconomic scenario, and then to improve the credit risk monitoring. The results of our study suggest that error correction models are well-suited to macroeconomic stress testing. Indeed, the fitting of the historical probability of default and the results under the stress scenarios considered here are satisfactory.

1 INTRODUCTION

The recent economic and financial crisis led to the collapse of many firms, and macroeconomic uncertainty and lower household creditworthiness have reinforced the importance of credit risk, the central risk faced by banks (more than 80% of a

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bank's overall risk). Supervisors have therefore promoted the setting up of a stress-testing process, and central banks took the lead in stress-test research. In addition, the need for an intensification of credit risk monitoring tools has enhanced the role played by macroeconomic stress testing.

To assess the credit exposure and potential losses that they face, banks have to estimate a set of parameters (probability of default (PD), loss given default (LGD) and exposure at default) that are required to calculate regulatory capital in the Basel framework. Practitioners commonly begin by estimating the PD parameter, which is the basic input when evaluating a portfolio's credit risk especially during recessions. Therefore, risk managers and regulators are interested in the prediction of PD (and potential losses) under a given macroeconomic scenario. They wish to improve the monitoring of their portfolio credit risk through stress testing.

Two main approaches have been provided in the literature (see Sorge (2004)) for macroeconomic stress testing: the piecewise approach and the integrated approach. Risk managers adopting the integrated approach combine the analysis of multiple risk factors into a single portfolio loss distribution, while the piecewise approach involves forecasting models of individual financial soundness indicators. We adopt the piecewise approach, which allows us to design a broader stress scenario than is possible in the integrated approach. In addition, this approach is more intuitive and has a lower computational burden.

The literature has produced a lot of models for macroeconomic stress testing in the piecewise framework. The most common method used for performing macroeconomic stress testing on a credit portfolio is a time series approach, particularly the vector autoregressive (VAR) model. These studies have been made for a corporate loans portfolio (see Avouyi-Dovis *et al* (2009)) and for a whole banking or financial system (Hoggarth *et al* (2005)), but retail portfolios have not received as much attention. Bucay and Rosen (2001) develop a methodology for measuring the credit risk of a retail portfolio (which can be used to perform macroeconomic stress testing) using an integrated approach. Since we adopt a piecewise approach, the distinctive feature of this paper is that it applies statistical methods that are usually used for a corporate portfolio to a retail portfolio.

A VAR model is inappropriate for our study since, in a VAR model, all the variables used (macroeconomic variables and a portfolio's default rates) are considered endogenous. From our point of view, there is no causal relationship between the PD of a consumer loan portfolio and France's gross domestic product (GDP) or three-month interest rate. So we choose to consider a model based on a time series approach where macroeconomic factors are exogenous. Note that Rösch and Scheule (2007) use a credit risk model derived from Merton's (1974) model to carry out a stress test on credit risk parameters with an application to retail loan portfolios, but they do not use a time series approach. In this paper we relate the PD of two retail loan portfolios to key macroeconomic factors using a time series approach. Macroeconomic-based models are motivated by an *a priori* link between the PD and the macroeconomic environment. It is well-known that PD increases during a recession.

This paper is organized as follows. In Section 2 we discuss the use of stress testing as a risk management tool. Section 3 introduces the model setup and some intuitions are given about the mechanism of an error correction model (ECM). We then explain the estimation procedure. Section 4 describes the data set. In Section 5 we perform a macroeconomic stress test under two scenarios and discuss the results. Section 6 concludes.

2 STRESS TESTING AS A RISK MANAGEMENT TOOL

According to the Bank for International Settlements, a stress test is described as "the evaluation of the financial position of a bank under a severe but plausible scenario to assist in decision making within the bank" (Basel Committee on Banking Supervision (2009)). Stress testing has become an important risk management tool used by banks as part of their internal risk management process and promoted by supervisors through the Basel II capital adequacy framework. Moreover, stress testing provides banks with another tool with which to supplement other risk management approaches and measures. Again following the Bank for International Settlements definition, stress testing plays a particularly important role in

- providing forward-looking assessments of risk,
- overcoming limitations of models and historical data,
- supporting internal and external communication,
- feeding into capital and liquidity planning procedures,
- informing the setting of banks' risk tolerance,
- easing the development of risk mitigation or contingency plans across a range of stressed conditions.

Since stress testing is an essential element of the Basel II framework, practitioners (risk and business managers) and regulators are interested in quantitative methods for assessing the potential risk of their bank or specific loans portfolio under a hypothetical but plausible stress scenario. Although the use of stress testing by banks appears to have been growing prior to the crisis, a 2005 Bank for International Settlements study showed that the focus at that time was primarily on applications to market risk: 80% of the tests were related to market risk. However, since the crisis, more attention has been paid to the integration of other types of risk, especially the credit risk portfolio. Stress testing appears to be useful in a broad variety of contexts, from regulatory reporting and risk management to newer uses such as strategic planning (assessing banks' risk appetite and determining which business segments to grow or stem), improvement of banks' risk management (by anticipating future risk level) or banks' budget planning.

Stress tests can be divided into two categories: scenario tests and sensitivity tests. In the first case, the stress scenarios are based on a portfolio-driven approach or an event-driven approach. Event-driven scenarios, which are generally requested by senior management and motivated by recent news, are based on plausible events and how these events might affect the relevant risk factors for a bank or a given portfolio. In contrast, in the portfolio-driven approach, risk managers discuss and identify risk drivers of a given portfolio and then design plausible scenarios under which these factors are stressed. For example, if risk managers identify unemployment rate as their main risk factor, stress tests will be designed around changes in the unemployment rate.

Moreover, under each approach, events can be categorized as either historical or hypothetical scenarios. Historical scenarios are based on historical data and rely on a crisis experienced in the past, while hypothetical scenarios are based on plausible scenarios that have not yet happened.

In sensitivity tests, risk factors are moved instantaneously by a unit amount and the source of the shock is not identified. Moreover, the time horizon for sensitivity tests is generally shorter in comparison with scenarios.

In this paper we perform a scenario stress test for a retail loan portfolio and use hypothetical scenarios instead of historical ones.

3 A MACROECONOMIC CREDIT RISK MODEL

Wilson (1997a,b) develops a credit risk model linking macroeconomic factors and corporate sector default rates. The idea was to model the relationship between default rates and macroeconomic factors and, when a model is fitted, to simulate the evolution of default rates over time by applying a stress scenario to the model. The simulated default rates in turn make it possible to obtain estimates of default rates for a defined credit portfolio under a given stress scenario.

Our approach, derived from Wilson's approach, considers a one-year observed PD instead of the default rates. In the following sections we use PD for the observed probability of default.

The PD is modeled by a logistic functional form:

$$\mathrm{PD}_t = \frac{1}{1 + \exp(-y_t)} \Leftrightarrow y_t = \ln\left(\frac{\mathrm{PD}_t}{1 - \mathrm{PD}_t}\right) \tag{3.1}$$

where PD_t is the observed default rates of the portfolio at time t and y_t is a macroeconomic index. For more convenience, we transform Wilson's original formulation[1] so that the lower value of the macroeconomic index implies lower values of PD, which means a better state of the economy. In (3.1), y_t is assumed to be related to a set of macroeconomic factors that are assumed to be exogenous:

$$y_t = f(X_t, X_{t-1}, X_{t-2}, \dots) \tag{3.2}$$

where $f(\cdot)$ is a functional specification[2] and X_{t-i} for all $i \in \{0, \dots, p\}$ is a vector of macroeconomic factors at time $t - i$.

The literature provides several statistical specifications, such as linear regressions and vector autoregressions, to relate such an index to macroeconomic factors. In this paper, unit root tests reveal that some macroeconomic variables are not stationary (see (A.2) for details) and this nonstationarity could lead to spurious regressions in the case of a linear specification. To address this issue we choose to take advantage of the nonstationarity property of the macroeconomic time series and thereby specify an ECM to model the relationship between the macroeconomic index and the macroeconomic factors.

Recall that ECMs are based on the behavioral assumption that two or more time series exhibit an equilibrium relationship that determines both short- and long-term behavior. An ECM takes into account the joint evolution of the macroeconomic factors and the PD in terms of level and dynamics. Then consideration of the correction's mechanism improves the results in comparison with what we would have been able to obtain with a linear regression or a VAR model. Through its two equations, an ECM considers a long-term relationship (actually a long-term trend) between the PD (Y), the macroeconomic factors (X) and the dynamics of short-term adjustments. Finally, an ECM provides a richer modeling than a linear regression (which only considers the variables in terms of level or first difference) and vector autoregression (which models the first difference of the time series).

3.1 Estimation procedure

In Appendix A we present a brief review of ECMs, unit roots tests and cointegration methods for readers that are unfamiliar with these techniques.

This section discusses the estimation procedure used to obtain the credit risk model. The procedure mainly involves estimating a parsimonious model with a limited number of factors. Moreover, due to the number of observations in the data set, we choose to constrain the number of lags for the macroeconomic variables to be lower than five. So we do not consider lag effects over five quarters but different lags for each

[1] Wilson's original formulation is $\mathrm{PD}_t = 1/(1 + \exp(y_t))$.

[2] Wilson chose the linear specification $y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \dots + \beta_n x_{n,t} + \varepsilon_t$.

macroeconomic variable have been tested. Moreover, in line with the economists of Crédit Agricole, we set up *a priori* signs for each macroeconomic factor with respect to economic theory: the interpretation of our model is therefore ensured. Finally, in order to be included in a model, each macroeconomic variable has to fulfill two conditions:

(1) its coefficients must be statistically significant and improve the overall model's performance;

(2) its coefficients must be consistent with economic theory.

Two methods exist in the literature for estimating the ECM: the one-step procedure and the two-step procedure.

Assuming Equation (A.1), the one-step method reduces the estimation procedure to the estimation of the following linear regression:

$$\Delta Y_t = \theta Y_{t-1} - (\gamma + \theta\alpha)X_{t-1} + \gamma X_t + \mu Z_t + v_t \tag{3.3}$$

This one-step procedure, which was popularized in economics by Davidson *et al* (1978), is quite easy to perform and requires weak exogeneity as an appropriate assumption. The validity of this assumption affects the estimation's result and could lead to (3.3) being both biased and inefficient and, therefore, the t-tests based on the model's parameter would be highly misleading.

The two-step procedure, introduced by Engle and Granger (1987), proceeds as follows. In the first step, we assume that Y_t and X_t (a vector of macroeconomic variables) are integrated of the same order and that Z_t is a vector of stationary component. Having defined the integration order and potential cointegrated variables, the long-term relationship is estimated as the linear regression of Y_t on X_t:

$$Y_t = \alpha X_t + \varepsilon_t \tag{3.4}$$

A more complex lag structure could be tested for X_t. If ε_t, the long-term relationship's residual, is stationary, Y_t and X_t are said to be cointegrated. Thus, the second step could be performed. Note that, if X_t and Y_t are cointegrated, an ordinary least-squares (OLS) regression yields a superconsistent estimator of α, since rate convergence equals T rather than $\sqrt{T}$, as in the standard context.

At this step, ΔY_t is regressed on ε_{t-1} and ΔX_t. Additional stationary variables or alternative lags (and deterministic terms) may be included as well:

$$\Delta Y_t = \theta \underbrace{\hat{\varepsilon}_{t-1}}_{Y_{t-1}-\hat{\alpha}X_{t-1}} + \gamma \Delta X_t + \mu Z_t + \nu_t \tag{3.5}$$

Moreover, it is known that, in the presence of unit roots in time series and cointegrating regression without serially correlated errors, the two-step procedure performs well. However, we cannot perform any test on the long-term parameters since the limiting distribution of α parameters are nonnormal and nonstandard. Then the two-step procedure implies that any mistake introduced in the first step is carried forward in the second step.

Note that the Engle and Granger two-step procedure will produce different parameter estimates from the one-step procedure. This arises largely because the latter is a single-step estimator, whereas the former is a two-step estimator. The first step of the Engle and Granger two-step procedure estimates only the long-term parameters in a static regression, whereas the second step estimates the short-term, dynamic-adjustment parameters, conditional on the long-term estimates from the first step. Conversely, the single dynamic equation approach based on Equation (3.3) jointly estimates long-term and short-term parameters.

In this paper we perform the two-step procedure due to operational constraint.

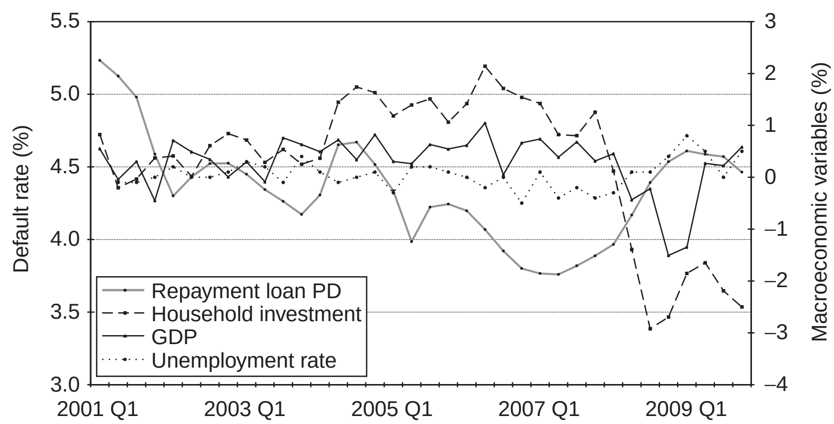
Assume p, the number of variables, and n, the number of observations of the data set. Knowing that OLS regression can be performed for parameter estimation if and only if $n \geqslant p$, the one-step procedure appears less desirable. Indeed, the one-step procedure offers limited opportunities to model, given that $n \gg p$ in our data set. Moreover, this procedure does not allow a backward selection method for the same reason. Finally, it is desirable to retain our cointegration vector to perform other cointegration tests *a posteriori* that are only allowed with the two-step procedure.

4 DATA DESCRIPTION

Two main types of empirical data have been used in this study: the macroeconomic variables and the one-year PD. The macroeconomic data was kindly provided by the Institut National de la Statistique et des Études Économiques and contains quarterly measures of thirty key macroeconomic variables (GDP, three-month interest rate, etc) and specific macroeconomic variables related to the studied portfolio (new car registration numbers, for example). The macroeconomic variables are provided over the time period from 1993 Q1 to 2010 Q4 and are seasonally adjusted.

As a variable of interest, we use loan data from one of Crédit Agricole's subsidiaries. The observed default rates are supplied for two credit retail loan portfolios, particularly revolving credit and repayment loans, which are both tailor-made for individuals. For a retail loan portfolio, the one-year PD is defined as the likelihood that a loan will not be repaid in the next twelve months. The default rate of a quarter T is obtained by dividing the number of defaulted counterparts between T and $T + 4$ (twelve months later) by the number of healthy counterparts at quarter T. Then the calculus of the default rates implies that the default rate of 2001 Q1 will only be known at 2002 Q1. Note that our default rate definition is consistent with the Basel II framework. Finally, the historical observed default rates cover a period from 2001 Q1 to 2009 Q4. This

FIGURE 1 Observed default rate and macroeconomic variables.



limited number of observations (thirty-six) could reduce the robustness of the model estimations.

Figure 1 displays the joint evolution of observed default rates and key macroeconomic factors over the period of estimation. For the sake of convenience, we use a two-scale figure. We note an increasing trend of default rates[3] of the repayment loan portfolio from 2007 to the end of 2008 and a downward trend of macroeconomic variables, particularly for household investment and GDP. This period coincides with the last economic crisis that was highlighted by the collapse of Lehman Brothers. Recall that the defaulted loans are observed between 2008 Q4 and 2009 Q4 for calculating the 2008 Q4 default rate.

5 MACROECONOMIC STRESS TESTS

5.1 Estimation results

5.1.1 Stationarity and cointegration results

In this section we introduce the results of the stationarity and cointegration tests. Several stationarity tests have been used and priority is given to the results from the Schmidt–Phillips (SP) and Elliot–Rothenberg–Stock (ERS) tests, since the SP test is more powerful and the ERS method tests the more powerful tests for stationarity.

[3] The same assessment can be made for the default rates of the revolving credit portfolio.

TABLE 1 Integration order of macroeconomic variables.

Variable	Integration order DF	PP	SP	ERS	Final integration order
PD revolving credit	$I(1)$	$I(1)$	$I(1)$	$I(1)$	$I(1)$
PD repayment loan	$I(1)$	$I(1)$	$I(1)$	$I(1)$	$I(1)$
GDP	$I(0)$	$I(0)$	$I(0)$	$I(0)$	$I(0)$
Three-month EURIBOR	$I(1)$	$I(0)$	$I(0)$	$I(0)$	$I(0)$
Consumption	$I(0)$	$I(0)$	$I(0)$	$I(0)$	$I(0)$
Unemployment rate	$I(1)$	$I(1)$	$I(1)$	$I(1)$	$I(1)$
Real disposable personal income	$I(0)$	$I(0)$	$I(0)$	$I(0)$	$I(0)$

"DF" stands for Dickey–Fuller test; "PP" stands for Phillips–Perron test.

Having performed the stationarity test, the variables are divided into two groups:

(1) the macroeconomic variables that have the same order of integration as the PD;

(2) the stationary variables.

Table 1 displays the integration order of the macroeconomic variables. For example, it shows that the PD is $I(1)$ for the three-month EURIBOR and the household investment growth rate. Recall that only $I(1)$ macroeconomic variables can be included in the long-term relationship. Table 1 highlights the results of the stationarity tests performed for other macroeconomic factors.

We then test the presence of a cointegration relationship between the PD and several combinations of these variables through the Engle and Granger two-step approach. In this way we regress the PD on macroeconomic variables. Then, for each relationship obtained, we perform a stationarity test on the regression's error term. Knowing that the latter is not observed, we use, for the stationarity test, different critical values that have been specially tabulated by MacKinnon (2010). Following the procedure above, we obtain a long-term relationship for each portfolio.

The variables that have been detected as stationary are only eligible for the short-term relationship. Moreover, we transform nonstationary variables into a stationary process by differencing or detrending, so that the transformed variables are also eligible for the short-term relationship. The short-term relationship is estimated by regressing the PD in first difference on stationary variables and the long-term relationship's error term with a one-period lag. Note that the lagged version of the first difference of the endogenous variable (Δ PD) could also be included in the short-term relationship. At this step, OLS or generalized least-squares (GLS) regressions could be used to perform the estimation.

TABLE 2 Repayment loan model.

Variables		Lag	Coefficient	P-value
Long-term relationship				
Constant term		—	−3.4463	<0.001
Household investment	(1)	$T-1$	−3.3883	<0.0001
	(1)	$T-3$	−3.8658	<0.0001
Unemployment rate	(2)	$T-5$	0.089	<0.0001
Short run relationship				
Constant term		—	0.01993	0.0183
Long-term relationship's residual		$T-1$	−0.4872	0.0013
Δ PD		$T-1$	0.4633	<0.0001
Real disposable income	(1)	$T-4$	−1.5432	<0.0001
	(1)	$T-5$	−1.267	0.0012
Quarterly inflation	(2)	$T-3$	−0.0152	0.0013

"(1)" denotes that the variable is considered in growth rate. "(2)" denotes that the variable is considered in level. "(3)" denotes that the variable is considered in first difference of growth rate.

5.1.2 The model

For each loan portfolio, several models have been tested. The criteria used to select the model are economic consistency and several statistical measures (R^2, root mean squared error, etc). Also, backtesting out of sample was used where necessary. Table 2 and Table 3 on the facing page highlight the model obtained for each portfolio. Note that a 5% significance level has been used for the estimation.

The macroeconomic variables retained in the repayment loan model are statistically significant and directly related to household. This result is consistent when considering the nature of the portfolio studied. In addition, as expected, the lag of the unemployment rate and real disposable income are higher than that of the other variables, since both variables' impact on the default rates are spread over time. So, having been made redundant, a borrower will be given unemployment benefits for several quarters.

The lagged effects of the macroeconomic variables included in the revolving credit model are often smaller than the lagged effects in the repayment loan model. In fact, a revolving credit is a short-term contract that is generally renewed by tacit agreement. Besides, the macroeconomic variables are statistically significant and closely related to the households.

5.2 Backtesting

Figure 2 on page 14 shows the observed PD on a quarterly sample from 2001 Q1 to 2009 Q4 and its corresponding forecast obtained from the macroeconomic credit

TABLE 3 Revolving credit model.

Variables		Lag	Coefficient	*P*-value
Long-term relationship				
Constant term		—	−5.97534	<0.001
Outstanding mortgage loan	(1)	T	−4.39016	<0.0001
Household debt ratio	(2)	$T-1$	0.14640	<0.0001
Unemployment rate	(2)	$T-1$	0.089	<0.0001
Three-month EURIBOR	(2)	T	0.03895	<0.0001
Short run relationship				
Constant term		—	0.00215	<0.0001
Long-term relationship's residual		$T-1$	−0.66528	0.0013
Δ PD		$T-1$	0.38996	<0.0001
Outstanding mortgage loan	(3)	$T-4$	−2.48486	<0.0001
Hard goods consumption expenditure	(1)	$T-2$	−0.415650	0.0012
New car registration	(1)	$T-1$	−0.0152	<0.0013

risk model for the repayment loan portfolio. On average, the in-sample prediction of the PD fits the observed series quite well for each loans portfolio. The results for the revolving credit portfolio are also satisfactory since we observe that the PD estimation fits the historical PD. Figure 5 on page 16 highlights this.

Furthermore, as an indicator of the goodness of fit we computed an adjusted R^2 for the whole model (both long-term and short-term relationship) as follows.

First we define:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(\mathrm{PD}_i - \widehat{\mathrm{PD}}_i)^2}{\sum_{i=1}^{n}(\mathrm{PD}_i - \overline{\mathrm{PD}})^2}$$

then:

$$R^2_{\text{adjusted}} = 1 - \frac{(1-R^2)(n-1)}{n-k-1}$$

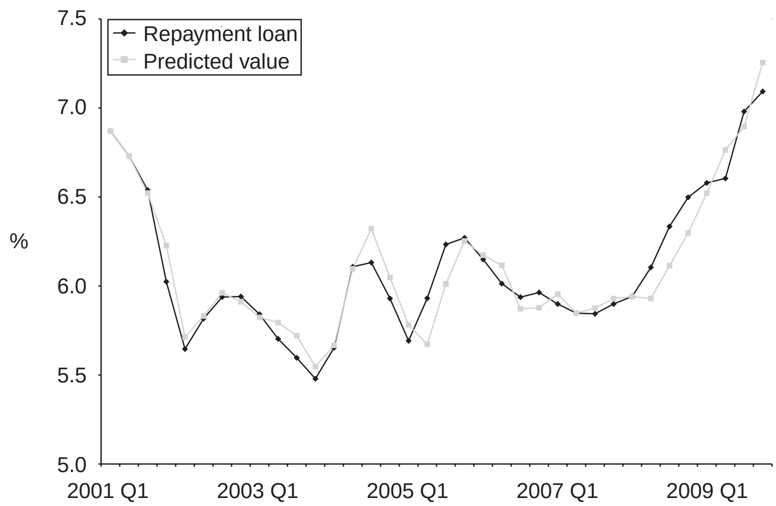
where k is the number of variables included in the model, where $\widehat{\mathrm{PD}}_i$ is the predicted PD at time i, where $\overline{\mathrm{PD}}$ is the average PD and where PD is the historical observed probability of default.

Table 4 on the next page provides the value of the two statistical criteria previously introduced. For this type of model, the most important feature is the ability to provide out-of-sample forecasts instead of in-sample ones. Actually, overparameterized models usually perform very well in in-sample tests, but their out-of-sample performances are often rather weak. We therefore perform an out-of-sample backtesting: the credit

TABLE 4 Statistical criteria.

	R^2_{adjusted}	Root mean squared error
Repayment loan	0.867	0.0226
Revolving credit	0.957	0.018

FIGURE 2 Observed default rates versus prediction: repayment loan.



risk model is estimated on a quarterly sample from 2001 Q1 to 2008 Q4. We then perform an out-of-sample forecast from 2009 Q1 to 2010 Q2.[4]

Figure 4 on the facing page shows the observed and predicted values of PD when the model's parameters are estimated from 2001 Q1 to 2008 Q4. The differences between observed and predicted values are generally small. However, the repayment loan's model has much more difficulty predicting the PD than the revolving credit model. Indeed, the repayment loan model underestimates the PD over the backtesting period even if it fits the PD dynamic quite well.

[4] PDs for 2010 Q1 and 2010 Q2 were not available when the model was built.

FIGURE 3 In-sample observed default rates versus prediction: revolving credit.

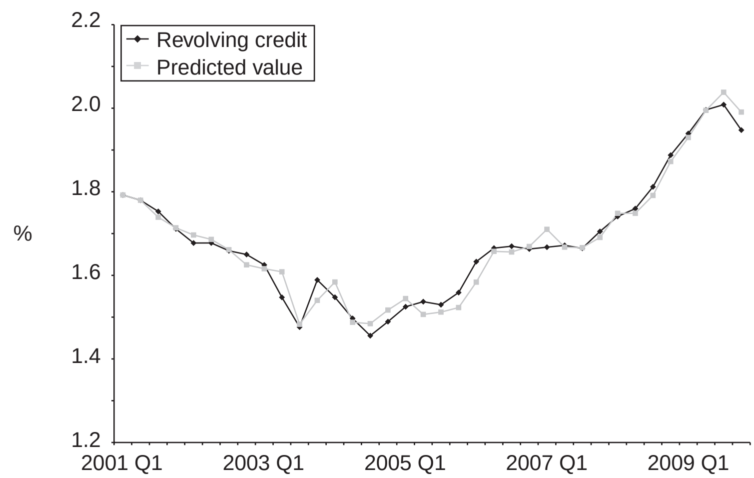
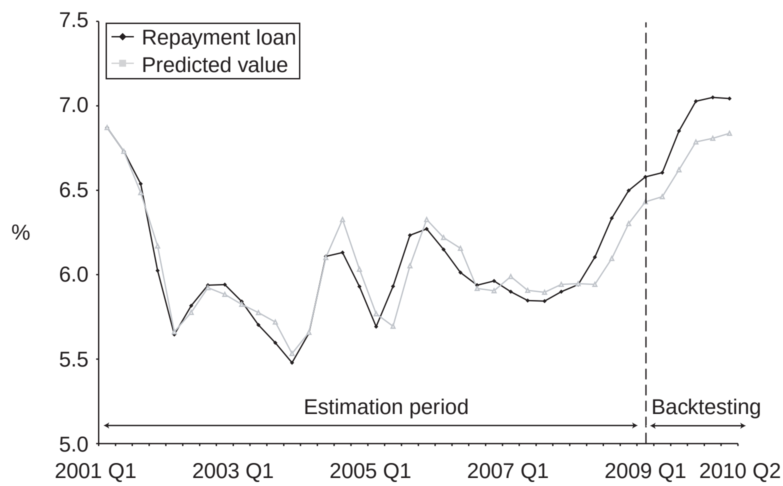


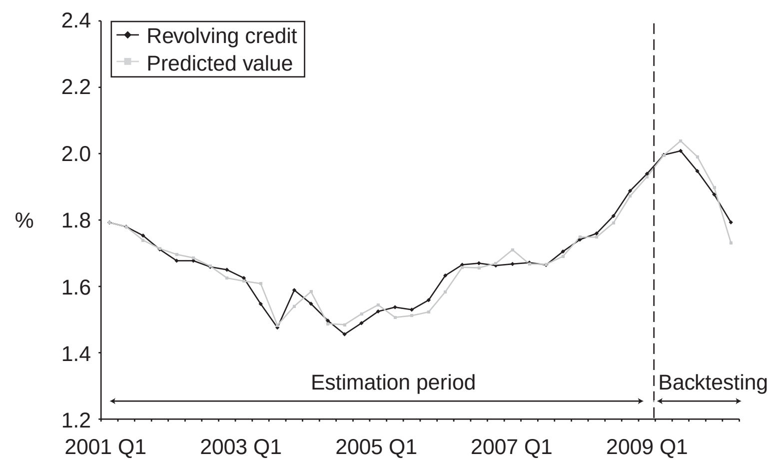
FIGURE 4 Backtesting out of sample: repayment loan.



5.3 Stress scenario

The stress scenarios (both baseline and adverse) used in this paper have been defined by the European Banking Authority for the 2011 stress-test exercise. The objective

FIGURE 5 Backtesting out of sample: revolving credit.



was to assess the resilience of a large sample of banks in the EU[5] against an adverse but plausible scenario. The scenario assesses banks against a deterioration from the baseline forecast in the main macroeconomic variables such as GDP, unemployment and house prices. For example, GDP would fall four percentage points from the baseline. Moreover, changes in interest rates and sovereign spreads also affect the cost of funding for banks under the stress. Crédit Agricole economists, with respect to the European Banking Authority's stress scenarios, designed these scenarios for more specific variables such as household investment, household real disposal income, outstanding mortgage credit, etc. Table 5 on the facing page presents both scenarios used for stress testing.

5.4 Application to the credit risk model

5.4.1 Repayment loan

Let us recall the general framework of our macroeconomic credit risk model built for both portfolios:

$$\left.\begin{aligned} Y_t &= \alpha X_t + \varepsilon_t \\ \Delta Y_t &= \theta \hat{\varepsilon}_{t-1} + \gamma \Delta X_t + \mu Z_t + \nu_t \end{aligned}\right\} \tag{5.1}$$

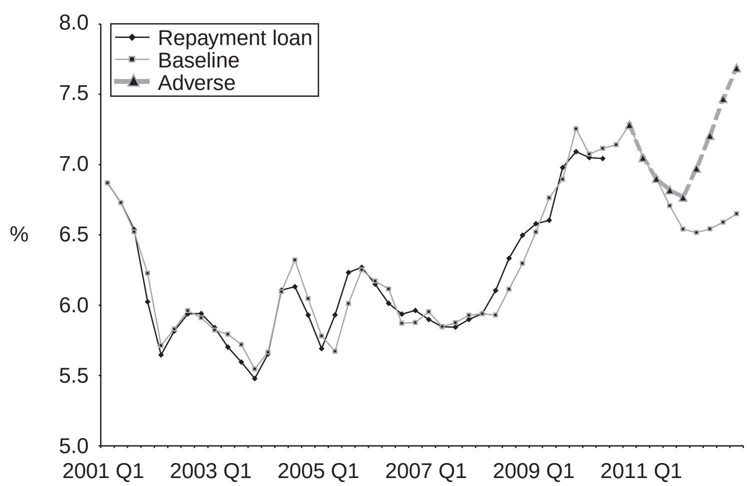
where Y_t is the historical PD, where X_t is a vector of $I(1)$ variables, where Z_t is a vector of stationary variables and where ε_t and ν_t are two random terms assumed to

[5] This includes non-EU European Economic Area banks where appropriate.

TABLE 5 Stress scenario.

Variables	2010 Q4 (%)	Baseline 2011 (%)	Baseline 2012 (%)	Adverse 2011 (%)	Adverse 2012 (%)
(1) Level					
Unemployment rate	9.3	9.5	9.2	9.6	9.8
Inflation rate	+1.5	+1.6	+1.6	1.3	0.9
Three-month EURIBOR	1	1.5	1.8	2.75	2.75
Household debt ratio	79.3	82.1	85.2	78.9	76.9
(2) Growth rate					
Outstanding mortgage loan	+8.2	+8.4	+7.9	+2.0	−2.0
Household investment	−2.2	+2.3	+2.6	−2.0	−5.5
Real disposable income	+2.6	+3.0	+2.8	+1.5	+0.5
New cars registration	−3.0	−2.0	−3.0	−6.0	−3.0

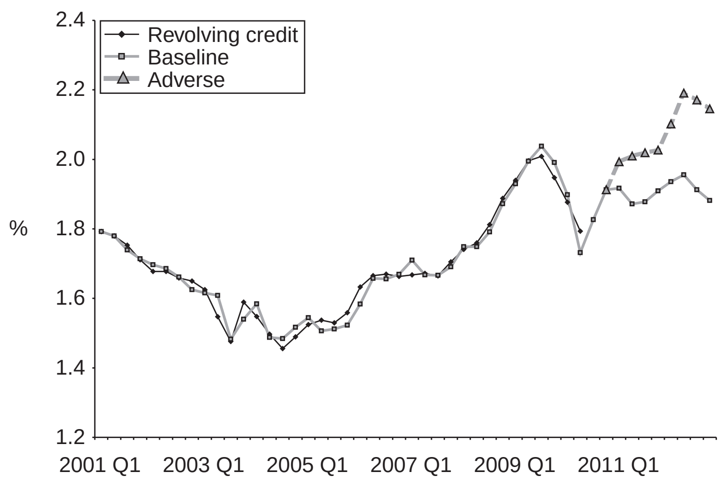
FIGURE 6 Stress testing for the repayment loan portfolio.



be independent, identically and normally distributed. By iterating the model forward over the two-year horizon we obtain Figure 6.

Figure 6 presents the evolution of the default rates of the repayment loan portfolio under the baseline and the adverse scenarios. Figure 7 on the next page presents the evolution of the default rates of the revolving credit portfolio under the baseline and the

FIGURE 7 Stress testing for the revolving credit portfolio.



adverse scenarios. For the repayment loan portfolio under the baseline scenario, the default rate has decreased from 7.28% in 2010 Q4 to 6.65% in 2012 Q4; by contrast, the PD reaches 7.69% under the adverse scenario. Finally, the model provides a −8.68% impact on the baseline scenario and a 5.54% increase on the adverse one. For the revolving credit portfolio, as presented in Figure 3 on page 15, the default rate has decreased under the baseline scenario from 1.91% to 1.88% between 2010 Q4 and 2012 Q4, which means an impact on the PD of −1.61%. In contrast, under the adverse scenario, the default rate rises to 2.15%. These results (for both portfolios) show the sensitivity of the PD to the macroeconomic variables in the scenario, defending our stress-test approach. Indeed, experts expected a slight decrease in the default rates under the baseline scenario and a sharp rise under the adverse scenario. Table 6 on the facing page gives more details.

6 CONCLUSION

In this paper we have investigated a time series model, specifically an ECM, for stress testing a retail loan portfolio. A distinguishing feature of the study is the application of this approach for stress testing to a retail loan portfolio. Indeed, this time series approach is normally developed for corporate or sovereign loans portfolios. Within the time series framework, we choose an ECM since we consider key macroeconomic variables to be exogenous to the model. Moreover, this macroeconomic credit risk model is based on the assumption that the PD of a retail loan portfolio is influenced by

TABLE 6 PD impact.

Scenario	2010 Q4 (%)	2011 Q4 (%)	Impact: 2010 versus 2011 (%)	2012 Q4 (%)	Impact: 2010 versus 2012 (%)
Repayment loan					
Baseline	7.28	6.54	−10.18	6.65	−8.68
Adverse	7.28	6.77	−7.07	7.69	+5.54
Revolving credit					
Baseline	1.91	1.91	−0.19	1.88	−1.61
Adverse	1.91	2.03	+5.98	2.15	+12.19

key macroeconomic variables through a cointegration relationship. This assumption has been validated by several stationarity and cointegration tests.

The empirical results show a significant and robust relationship between the portfolio's PD and several macroeconomic variables, including some consumption-loan-specific variables. In addition, the macroeconomic credit risk models obtained are economically consistent with respect to the out-of-sample backtesting results. The repayment loan model suggests a decrease of PD under the baseline scenario and a sharp increase under the adverse scenario. On the other hand, the revolving credit portfolio model suggests a steady evolution of the PD with some fluctuations under the baseline scenario, whereas the results under the adverse scenario show a sharp rise in PD. These results are close to what stress-test managers expected. Finally, the impacts in terms of PD under the adverse scenario are always greater than the baseline ones. We believe that these very simple models are well-suited to retail portfolios. Indeed, the outstanding loan balance of a retail loan portfolio being less than the corporate portfolio, assuming the endogeneity of all the variables (PD and macroeconomic factors) as in a VAR model, is an unrealistic assumption. In other words, it is unrealistic to consider that a change in PD of a Crédit Agricole repayment loan or revolving credit portfolio affects France's GDP.

An interesting direction for future research would be the integration of the other risk parameters, especially LGD, to this type of model. We would then be able to compute a loss distribution without considering LGD parameters as a constant term. Another idea would be to use more robust stationarity and cointegration tests due to the weakness of the number of observations. The improvement of historical data is obviously a way to improve the robustness of the models.

APPENDIX A: ERROR CORRECTION MODEL

A.1 Principles

An ECM assumes the following specification:

$$\left.\begin{aligned} Y_t &= \alpha X_t + \varepsilon_t \\ \Delta Y_t &= \theta \hat{\varepsilon}_{t-1} + \gamma \Delta X_t + \mu Z_t + \nu_t \end{aligned}\right\} \tag{A.1}$$

where Δy_t is the first difference of y_t, ie, $\Delta y_t = y_t - y_{t-1}$, and where α, θ and γ are a set of coefficients to be estimated. ε_t and ν_t are random terms assumed to be independent, identically and normally distributed. The first equation of (A.1), which is called a long-term relationship, considers a long-term equilibrium between X and Y. Then X and Y are supposed to have a joint evolution and not to differ significantly over time. When they differ, ε is meant to correct the temporary imbalance in the short-term relationship (second equation of (A.1)). Therefore, at each step t, ε_{t-1} allows the correction of the previous estimation of Y. This is the error correction mechanism.

Note that lagged values of the first difference of Y could be included in (A.1).

Equation (A.1) is valid if and only if

- Y and X are integrated (see (A.3) for details) of the same order,
- ε is stationary (see (A.2) for details), which implies that X and Y are cointegrated,
- Z is stationary,
- θ is both negative and statistically significant.

The following section introduces two concepts that are fundamental to the understanding of an ECM.

A.2 Stationarity and unit root tests

A stationary process has the property that the mean, variance and autocorrelation structure do not change over time. Consider the following processes:

$$x_t = \rho x_{t-1} + u_t, \quad |\rho| < 1 \tag{A.2}$$

$$y_t = y_{t-1} + v_t \tag{A.3}$$

The error terms u_t and v_t are assumed to be normally independently identically distributed with zero mean and unit variance, $u_t, v_t \sim \text{iin}(0, 1)$, ie, a purely random process. Both x_t and y_t are AR(1) models. By calculating the mean, variance and autocovariance x_t and y_t, we can show that the means of the two series are:

$$\mathbb{E}(x_t) = 0 \quad \text{and} \quad \mathbb{E}(y_t) = 0 \tag{A.4}$$

and the variances are:

$$\mathbb{V}(x_t) = \sum_{i=0}^{t-1} \rho^{2i} \operatorname{var}(u_{t-i}) \to [t \to \infty] \frac{1}{1-\rho^2}$$

$$\mathbb{V}(y_t) = \sum_{i=0}^{t-1} \operatorname{var}(v_{t-i}) = t$$

The autocovariances of the two series are:

$$\gamma^x(h) = \mathbb{E}(x_t, x_{t+h}) = \sum_{i=0}^{t+h-1} \rho^i \rho^{h+i}$$

$$\gamma^y(h) = \mathbb{E}(y_t, y_{t+h}) = t - h$$

If the means of x_t and y_t are equals, they differ by their variances and their autocovariances. But the most important result is that the variance and autocovariance of y_t are functions of t, contrary to x_t. Then, as t increases, the variance and autocovariance of y_t increase while those of x_t converge to a constant. By considering the definition of a stationary process, we conclude that x_t is stationary, whereas y_t is a nonstationary process. y_t is a nonstationary process because of the unit root.[6] As a consequence, the presence of a unit root indicates that a time series is not stationary. x_t is said to be difference-stationary.

There exists another type of nonstationary process. Thus, if we considered a constant (drift) term with or without a deterministic trend in (A.2) and (A.3), we would have the same type of results about the mean,[7] the variance and the autocovariance of x_t and y_t. Within a deterministic trend, y_t is said to be trend-stationary.

With respect to the presence of a constant term and/or a deterministic trend, y_t could be transformed to a stationary process using one of the following methods.

- Differencing once, ie, $\Delta y_t = (1 - L)y_t = y_t - y_{t-1} = v_t$, where L is a lag operator.
- Detrending (the trend βt is subtracted), ie:

$$y_t = \alpha + \beta t + \varepsilon_t \Rightarrow y_t - \beta t = \alpha + \varepsilon_t$$

- Detrending, then differencing the detrended process.

---

[6] y_t is a special case of an x_t process when $\rho = 1$.
[7] With a constant term, $\mathbb{E}(x_t) = \alpha/(1 - \rho)$ and $\mathbb{E}(y_t) = \alpha t$.

Since Dickey and Fuller (1979), an enormous number of studies of stationarity tests have appeared, and several statistical tests have been implemented. In this paper we perform the Dickey–Fuller (DF) test (and its augmented version when necessary), the ERS test, the Phillips–Perron (PP) test and the SP test. Maddala and Kim (1998) give more details on these stationarity tests.

Since the decision regarding the integration order (see Section A.3 for details) is a determining step of the modeling process, we chose to use several stationarity tests. Thus, the integration order is chosen by considering the power and the robustness of each test. Salanié (1999) shows that the SP test for stationarity is the most robust and that the ERS stationarity test is the most powerful among the stationarity tests. As a result, priority is given to the results of these two stationary tests because the reduced size of the data (thirty-six observations) could lead a time series to be wrongly considered as stationary.

A.3 Integration order and cointegration

If a nonstationary time series can be transformed into a stationary one by differencing once, then this series is said to be integrated of order one or $I(1)$. Some time series could require k repeated differences to obtain a stationary process, they are said to be integrated of order k. In addition, stationary variables are said to be $I(0)$.

In general, regression models for nonstationary variables give spurious results. For example, Granger and Newbold (1974) present some examples with artificially and independently generated data so that there is no relationship between two time series $z_{1,t}$ and $z_{2,t}$. However, the correlations between $z_{1,t}$ and $z_{1,t-1}$ and between $z_{2,t}$ and $z_{2,t-1}$ were high. The regression of $z_{1,t}$ on $z_{2,t}$ gave a high coefficient of determination (R^2) but a low Durbin–Watson statistic. They also run the regression in first difference, for which the R^2 is close to zero and the Durbin–Watson statistic is close to two. This demonstrates that there is no relationship between $z_{1,t-1}$ and $z_{2,t}$ and that the R^2 obtained was spurious.

Regression models for nonstationary time series only make sense if they are said to be cointegrated. The concept of cointegration was first introduced by Engle and Granger (1987) so that two $I(1)$ variables y_t and x_t are said to be cointegrated if there exists β such that $y_t - \beta x_t$ is $I(0)$. The concept could be generalized for two or more $I(d)$ variables: two $I(d)$ variables y_t and x_t are said to be cointegrated if there exists β such that $y_t - \beta x_t$ is $I(d - b)$ with $b > 0$.

Several cointegration tests have been introduced in the literature and used in practice. The principle of these tests is to test whether two or more integrated variables deviate significantly from a certain relationship. While cointegration tests can be performed in a single equation framework or in systems of multiple equations, this paper will only discuss cointegration tests in a single equation framework.

TABLE A.1 Integration order of macroeconomic variables.

Variable	Integration order DF	PP	SP	ERS	Final integration order
Inflation (twelve months)	$I(1)$	$I(1)$	$I(0)$	$I(1)$	$I(1)$
Household debt ratio (consumer credit)	$I(1)$	$I(1)$	$I(0)$	$I(0)$	$I(0)$
Household investment	$I(1)$	$I(1)$	$I(1)$	$I(1)$	$I(1)$
Ten-year interest rate	$I(1)$	$I(1)$	$I(1)$	$I(1)$	$I(1)$
Outstanding mortgage credit	$I(0)$	$I(1)$	$I(1)$	$I(1)$	$I(1)$
CAC 40 index	$I(0)$	$I(0)$	$I(0)$	$I(0)$	$I(0)$
Corporate debt ratio	$I(1)$	$I(1)$	$I(1)$	$I(1)$	$I(1)$
Consumption loan interest rate	$I(1)$	$I(1)$	$I(1)$	$I(1)$	$I(1)$
Household debt ratio	$I(1)$	$I(1)$	$I(1)$	$I(1)$	$I(1)$
Outstanding consumer credit	$I(0)$	$I(1)$	$I(0)$	$I(0)$	$I(0)$
Corporate investment	$I(0)$	$I(0)$	$I(0)$	$I(0)$	$I(0)$
Mortgage's interest rate	$I(1)$	$I(1)$	$I(0)$	$I(1)$	$I(1)$
New car registration	$I(0)$	$I(0)$	$I(0)$	$I(0)$	$I(0)$
Quarterly inflation	$I(1)$	$I(1)$	$I(0)$	$I(1)$	$I(1)$
Industrial output	$I(0)$	$I(0)$	$I(1)$	$I(0)$	$I(0)$
Hard goods consumption expenditure	$I(0)$	$I(0)$	$I(1)$	$I(0)$	$I(0)$

The most commonly used cointegration test is the Engle and Granger (1987) two-step approach, which is known as a residual-based test (the first test of its kind for cointegration). Consider a set of $(k+1)$ $I(1)$ variables: a vector of k explanatory variables X_t and a variable of interest y_t. If there exists a vector β such that $y_t - \beta' X_t$ is $I(0)$, then β is the cointegrating vector. The residual-based tests consider the equation:

$$y_t = \beta' X_t + \varepsilon_t \tag{A.5}$$

If ε_t has a unit root, ie, it is not stationary, then $y_t - \beta' X_t$ is not a cointegrating relationship. Thus, a test for a unit root in ε_t is meant to test that the variables y_t and X_t are not cointegrated. In practice, β and ε are not observed; they are estimated by OLS or GLS regressions and a unit root test is performed on $\hat{\varepsilon}_t$. Yet because ε_t is not observed, different critical values have been tabulated to test the presence of a unit root in the Engle and Granger approach. The stationarity of ε implies that y and X are cointegrated, so the cointegration's hypothesis will be rejected in the case of nonstationarity of ε_t. In this paper the PP test is used to test the stationarity of $\hat{\varepsilon}_t$.

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