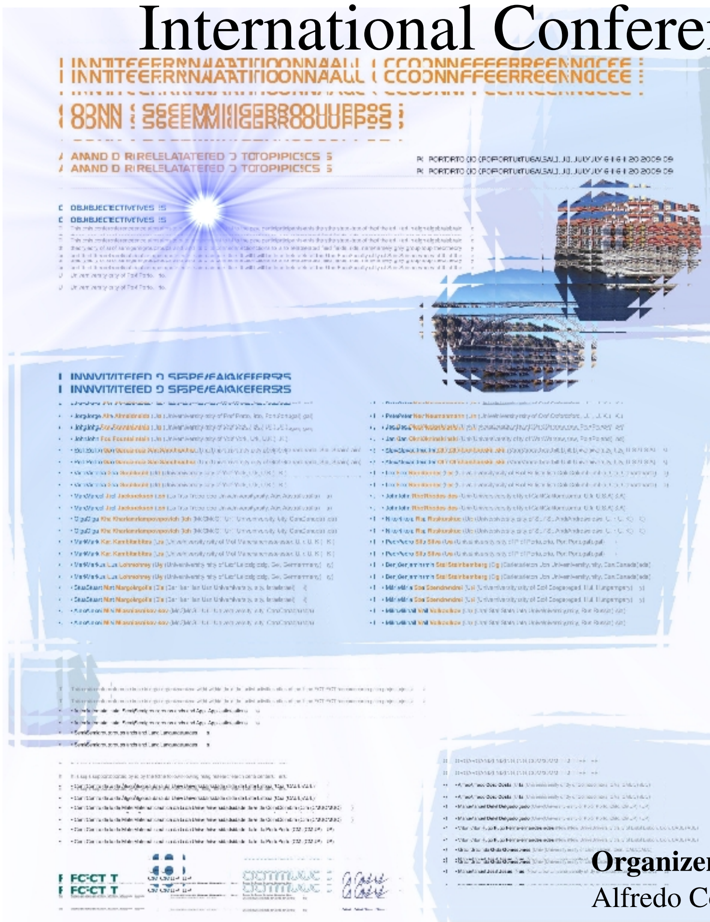
International Conference on Semigroups
and related topics

Porto 2009



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Timetable

	Monday, July 6	Tuesday, July 7	Wednesday, July 8	Thursday, July 9	Friday, July 10	Saturday, July 11
08:00 - 08:30						
08:30 - 09:00	Registration					
09:00 - 09:30			Pedro García-Sánchez (05)			
09:30 - 10:00	9:45 Welcome	Mikhail Volkov (05)		John Fountain (05)	Stuart Margolis (05)	Alexei Miasnikov (05)
10:00 - 10:30	Jorge Almeida (05)		Peter Neumann (05)			
10:30 - 11:00		Break		Break	Break	Break
11:00 - 11:30	Break	Boris Plotkin (05)	Break	Valentina Barucci (05)	László Márki (05)	Karl Auinger (05)
11:30 - 12:00	Nik Ruškuc (05)	Markus Lohrey (05)	João Araújo (05)	Marcel Jackson (05)	Victoria Gould (05)	Pedro Silva (05)
12:00 - 12:30			Alexander Olshanskii (05)			
12:30 - 13:00	Lunch	Lunch		Lunch	Lunch	Close
13:00 - 13:30						
13:30 - 14:00						
14:00 - 14:30						
14:30 - 15:00	Mark Kambites (05)	Mária Szendrei (05)		P. Catarino (04) / E. Emtander (06)	Libor Polák (05)	
15:00 - 15:30				T. Quinteiro (04) / A. Robles-Pérez (06)	Maria Bras-Amorós (05)	
15:30 - 16:00	E. Khazova (04) / A. Malheiro (06)	K. Cvetko-Vah (04) / T. Plotkin (06)		Jörg Koppitz (05)	M. Novák (04) / M.Almeida (06)	
16:00 - 16:30	A. Moura (04) / Z. Juhász (06)	E. Aladova (04) / A. Zhuchok (06)		Break	Y. Zhuchok (04) / R. N. Heale (06)	
16:30 - 17:00	Break	Break		Benjamin Steinberg (05)	Break	
17:00 - 17:30	Jan Okniński (05)	Olga Kharlampovich (05)			John Rhodes (05)	
17:30 - 18:00				Free discussion session (05)		
18:00 - 18:30						
18:30 - 19:00						
19:30 - 20:00				Conference Dinner		

Program and Abstracts

Monday, July 6

			(page)	Room
08:30 - 09:45		***Registration***		0.04
09:45 - 10:00		***Welcome***		0.05
10:00 - 11:00	*J. Almeida*	**McCammond's normal form and applications**	(7)	0.05
11:00 - 11:30		***Break***		
11:30 - 12:30	*N. Ruškuc*	**Rewriting Generators for Semigroups**	(77)	0.05
12:30 - 14:30		***Lunch***		
14:30 - 15:30	*M. Kambites*	**Groups acting on semimetric spaces and quasi-isometries of monoids**	(37)	0.05
15:30 - 16:00	*E. Khazova*	**On free semigroups of regular languages**	(41)	0.04
15:30 - 16:00	*A. Malheiro*	**Complete rewriting systems and homotopy bases in semigroup theory**	(47)	0.06
16:00 - 16:30	*A. Moura*	**The word problem for ω-terms over** DA	(55)	0.04
16:00 - 16:30	*Z. Juhász*	**Filters in (quasiordered) semigroups, lattices of filters and a generalisation of Caratheodory number**	(35)	0.06
16:30 - 17:00		***Break***		
17:00 - 18:00	*J. Okniński*	**Algebras, groups and monoids determined by set theoretic solutions of the Yang-Baxter equation**	(61)	0.05

Tuesday, July 7

			(page)	Room
09:30 - 10:30	*M. Volkov*	**The finite basis problem for finite semigroups revisited**	(85)	0.05
10:30 - 11:00		***Break***		
11:00 - 11:30	*B. Plotkin*	**Logically perfect and homogeneous algebras**	(65)	0.05
11:30 - 12:30	*M. Lohrey*	**The compression technique for solving the word problem**	(45)	0.05
12:30 - 14:30		***Lunch***		
14:30 - 15:30	*M. Szendrei*	**Almost factorizable locally inverse semigroups**	(83)	0.05
15:30 - 16:00	*K. Cvetko-Vah*	**Rings whose idempotents form a band**	(21)	0.04
15:30 - 16:00	*T. Plotkin*	**Decompositions and complexity of linear automata**	(67)	0.06
16:00 - 16:30	*E. Aladova*	**Polynomial varieties of representations of groups**	(5)	0.04
16:00 - 16:30	*A. Zhuchok*	**On idempotent dimonoids**	(87)	0.06
16:30 - 17:00		***Break***		
17:00 - 18:00	*O. Kharlampovich*	**Algorithmic Problems for Fully Residually Free Groups**	(39)	0.05

Wednesday, July 8

			(page)	Room
09:00 - 10:00	*P. García-Sánchez*	**Notable elements for counting numerical semigroups**	(27)	0.05
10:00 - 11:00	*P. Neumann*	**Synchronizing Groups**	(57)	0.05
11:00 - 11:30		***Break***		
11:30 - 12:00	*J. Araújo*	**A Duet With Occasional Chorus**	(11)	0.05
12:00 - 13:00	*A. Olshanskii*	**Actions of maximal growth**	(63)	0.05

Thursday, July 9

			(page)	Room
09:30 - 10:30	*J. Fountain*	**Zappa-Szép products of groups and right cancellative monoids**	(25)	0.05
10:30 - 11:00		***Break***		
11:00 - 11:30	*V. Barucci*	**Decompositions of ideals in irreducible ideals in numerical semigroups**	(15)	0.05
11:30 - 12:30	*M. Jackson*	**Semigroups of relations**	(33)	0.05
12:30 - 14:30		***Lunch***		
14:30 - 15:00	*P. Catarino*	**Complete semigroups of transformations on a finite chain**	(19)	0.04
14:30 - 15:00	*E. Emtander*	**Betti numbers of some semigroup rings**	(23)	0.06
15:00 - 15:30	*T. Quinteiro*	**Bilateral semidirect product decompositions of transformation monoids**	(71)	0.04
15:00 - 15:30	*A. Robles-Pérez*	**Modular numerical semigroups with embedding dimension equal to three**	(75)	0.06
15:30 - 16:00	*J. Koppitz*	**On the structure of IM_n**	(43)	0.05
16:00 - 16:30		***Break***		
16:30 - 17:30	*B. Steinberg*	**Semigroup representation theory and the Černý conjecture**	(81)	0.05
17:30 - 18:00		***Free discussion session***		0.05
19:30 -		***Conference Dinner***		

Friday, July 10

			(page)	Room
09:30 - 10:30	*S. Margolis*	**On Maximal Subgroups of Free Idempotent Generated Semigroups**	(49)	0.05
10:30 - 11:00		***Break***		
11:00 - 11:30	*L. Márki*	**A matrix construction for certain inverse semigroups with application in universal algebra**	(51)	0.05
11:30 - 12:30	*V. Gould*	**Adequate semigroups *or, losing our identity***	(29)	0.05
12:30 - 14:30		***Lunch***		
14:30 - 15:00	*L. Polák*	**Operators on varieties of monoids related to polynomial operators on classes of regular languages**	(69)	0.05
15:00 - 15:30	*M. Bras-Amorós*	**Configuration Semigroups**	(17)	0.05
15:30 - 16:00	*M. Novák*	***Ends lemma* based semihypergroups of certain transformation operators**	(59)	0.04
15:30 - 16:00	*M. Almeida*	**Testing the Equivalence of Finite Automata**	(9)	0.06
16:00 - 16:30	*Y. Zhuchok*	**L- and R-cross-sections of the symmetric inverse 0-category**	(89)	0.04
16:00 - 16:30	*R. Heale*	**The Idempotent Problem**	(31)	0.06
16:30 - 17:00		***Break***		
17:00 - 18:00	*J. Rhodes*	**Geometric Semigroup Theory with Applications to Upper Bounds for Complexity**	(73)	0.05

Saturday, July 11

			(page)	Room
09:30 - 10:30	*A. Miasnikov*	**Limit algebras**	(53)	0.05
10:30 - 11:00		***Break***		
11:00 - 11:30	*K. Auinger*	**On the decidability of gV**	(13)	0.05
11:30 - 12:30	*P. Silva*	**Fixed points of endomorphisms over special confluent rewriting systems**	(79)	0.05
12:30 -		***Close***		0.05

See page 91 for other accepted abstracts.

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Polynomial varieties of representations of groups

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(This is a joint work with B. Plotkin)

Keywords: Varieties of representations of groups, semigroup of varieties of representations of groups, varieties of associative algebras, polynomial varieties of representations of groups, dimension subgroup for the variety of representations of groups.

Extended Abstract

It is well known that there are close connections between varieties of groups and varieties of representations of groups, and between varieties of algebras and varieties of representations of algebras. We consider some relations between varieties of representations of groups and varieties of associative algebras.

Let K be a commutative and associative ring with unit. Let (V,G) be a representation of the group G in the K-module V and let G act on the module V by the rule $(v,g) \to v \circ g$. Note that the group algebra KG also acts on V.

Let F be a free group of countable rank with the free generators $x_1, x_2, \dots$, and let KF be a group algebra of F. Let $u(x_1,\dots,x_n)$ be an element of KF.

Definition. *We say that a representation (V,G) satisfies an identity $y \circ u(x_1,\dots,x_n) \equiv 0$ if for all $v \in V$ and all $g_i \in G$ we have $v \circ u(g_1,\dots,g_n) = 0$.*

Definition. *A class* $\mathbf{X}$ *of all representations of groups over a fixed ring K satisfying some set of identities is called a variety of representations of groups.*

For example, the representation (V,G) satisfies the identity

$$x \circ (y_1 - 1)(y_2 - 1)\dots(y_n - 1) \equiv 0$$

if the module V has a G-invariant series of submodules of length n:

$$0 = V_0 \subset V_1 \subset \cdots \subset V_n = V,$$

such that the group G acts trivially on the factors. Denote by $\mathbf{S}^n$ the variety satisfying the identity $x \circ (y_1 - 1)(y_2 - 1)\dots(y_n - 1) \equiv 0$. The variety $\mathbf{S}^n$ is generated by the representation $(K^n, UT_n(K))$ of the group of $n \times n$ unitriangular matrices.

All varieties of representations of groups form a semigroup $\mathbf{M} = \mathbf{M}(K)$ under operation of the multiplication of varieties. It is well-known [1] that if K is a field then the semigroup $\mathbf{M} = \mathbf{M}(K)$ is a free semigroup.

Now let K be a field. Let Σ be a variety of associative K-algebras with unit. Define the class $\eta\Sigma$ of representations of groups as follows:

Definition. *A representation (V,G) belongs to the class $\eta\Sigma$ if the algebra $\overline{KG} = KG/Ker(V,KG)$ is an algebra from the variety Σ.*

In other words, if $(V,\overline{G})$ is the faithful representation of the group $\overline{G} = G/\mathrm{Ker}(V,G)$ corresponding to the representation (V,G) then the linear span of $\overline{G}$ in the algebra $\mathrm{End}_K V$ is an algebra from the variety Σ. The class $\eta\Sigma$ forms a variety of representations of groups [2].

Definition. *A variety of representations of groups* $\mathbf{X}$ *is called a polynomial variety if there exists a variety of associative algebras Σ such that* $\mathbf{X} = \eta\Sigma$.

It is known [2] that all polynomial varieties of representations of groups constitute a subsemigroup in the semigroup $\mathbf{M}$. We consider the following

Problem. *Is the semigroup of all polynomial varieties free?*

To solve this problem we study some properties of polynomial varieties of representations of groups. In particular, we are looking for interesting examples of polynomial and non-polynomial varieties. For example, we have the following

Proposition. *The variety of representations of groups* $\mathbf{S}^n$ *is not a polynomial variety.*

Let $I_{\mathbf{X}}(KG)$ be the verbal ideal of KG for the variety $\mathbf{X}$. The kernel of the representation $(KG/I_{\mathbf{X}}(KG), G)$ is a normal subgroup $D_{\mathbf{X}}(G) = (1 + I_{\mathbf{X}}(KG)) \cap G$.

Definition. *The subgroup $D_{\mathbf{X}}(G)$ is called a dimension subgroup of the group G for the variety of representations of groups* $\mathbf{X}$.

We consider some problems about dimension subgroup for varieties of representations of groups and for the polynomial varieties of representations of groups. For example, let a group G admit a representation from the variety of representations of groups $\eta\Sigma$, and let $G/D_{\eta\Sigma}(G)$ be a group from the variety of groups Θ. Let B be a verbal subgroup of the group G for the variety Θ. Then $B \subset D_{\eta\Sigma}(G)$. There is the following

Problem. *What can we say about the structure of the group $D_{\eta\Sigma}(G)/B$?*

Let Σ be a variety of algebras with n-nilpotent derived ideal. Note that the variety of representations of groups $\eta\Sigma$ is generated by the representation $(K^n, T_n(K))$ of the group of $n \times n$ triangular matrices. Let B be a verbal subgroup of the group G for the variety of groups $\mathbf{N}_{n-1} \cdot \mathbf{A}$, where $\mathbf{N}_{n-1}$ is the variety of nilpotent groups of nilpotency class $(n-1)$, and $\mathbf{A}$ is the variety of abelian groups. We have

Proposition. *Let a group G admit a representation in the variety of representations of groups $\eta\Sigma$. If the group G/B is torsion-free then $D_{\eta\Sigma}(G) = B$.*

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McCammond's normal form and applications

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(This is a joint work with José Carlos Costa and Marc Zeitoun)

Extended Abstract

By a *pseudovariety* we mean a class of finite semigroups which is closed under taking homomorphic images, subsemigroups, and finite direct products.

Free profinite semigroups relative to a given pseudovariety have long been recognized as important mathematical objects whose properties and structure encode and reveal essential information about the pseudovariety, which can often be used to solve decidability questions [1, 2, 8]. Such questions, which most often originate from the connections with theoretical computer science in the framework described by Eilenberg [5] and later significantly extended [7], have since the 1960's been intimately connected with most developments in the theory of finite semigroups and served as strong external motivation.

In general, little is known about the structure of relatively free profinite semigroups, specially for pseudovarieties containing semigroups in which some regular elements do not lie in groups (in the usual notation, outside DS) or that are not constructed using some natural operators from pseudovarieties for which the structural problem has already been solved. Recently, there has been a surge of interest in the structure of free profinite semigroups over much larger pseudovarieties, such as pseudovarieties characterized by the groups which appear in them, which form a pseudovariety of groups; if this is denoted H, then the pseudovariety in question is usually denoted $\overline{\mathsf{H}}$. The results obtained so far concentrate mostly on the local structure of such semigroups, particularly the structure of their (maximal) subgroups using connections with symbolic dynamics, wreath product techniques, or both (cf. recent work of the first author, A. Costa, Rhodes, and Steinberg).

Because of its connection with the two main open problems in finite semigroup theory, namely the decidability of the Krohn-Rhodes complexity of a finite semigroup and of the dot-depth of a star-free rational language, the pseudovariety $\mathsf{A} = \overline{\mathsf{I}}$ of all finite aperiodic semigroups, corresponding to the trivial pseudovariety (of groups) I under the overline operator, deserves very special attention.

Motivated by the work of Steinberg and the first author [3], which announced a reduction of the complexity problem which turned out to be based on a faulty result from [4], as explained in [8], McCammond [6] solved the word problem for the ω-subsemigroup $\Omega_X^\kappa\mathsf{A}$ of the free pro-A semigroup $\overline{\Omega}_X\mathsf{A}$ generated by its free generators X, namely the closure for multiplication and ω-powers. The solution consists in obtaining a normal form for ω-terms over A and using the solution of the word problem for (suitable) free Burnside semigroups to distinguish different normal forms over A.

Let us call A-*pseudowords* elements of $\overline{\Omega}_X\mathsf{A}$ and ω-words the elements of $\Omega_X^\kappa\mathsf{A}$. It is convenient to represent ω-words as words in the extended alphabet obtained by adding to X parentheses, where matching parentheses denote the ω-power.

The purpose of this talk is to survey some recent results of the authors related to McCammond's normal form. At the basis of this work is a new direct proof of uniqueness of the normal form consisting in associating to each

ω-term in normal form a sequence of rational languages, to show that they are eventually star-free, and that they suffice to separate different normal forms. This technique affords many new applications including the following results:

1. all A-pseudoword factors of ω-words are again ω-words;
2. an A-pseudoword w is an ω-word if and only if the following conditions hold: (a) there are no infinite sets of factor-incomparable factors of w; (b) the set of normal forms of ω-factors of w is a rational language;
3. the pseudovariety A is κ-full in the sense of [3] and the analogue of the Pin-Reutenauer procedure for computing closures of rational languages in $\Omega_X^\kappa \mathsf{A}$ works.

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Testing the Equivalence of Finite Automata

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(This is a joint work with Nelma Moreira and Rogério Reis)

Keywords: finite automata, regular expressions, regular languages equivalence, minimal automata, regular expressions derivatives.

Extended Abstract

The uniqueness of the minimal deterministic finite automaton for each regular language is in general used for determining regular languages equality. Whether the languages are represented by deterministic finite automata (DFA), non-deterministic finite automata (NFA) or regular expressions (r.e.), the usual procedure uses the equivalent minimal DFA to decide equivalence. The best known algorithm, in terms of worst-case analysis, for DFA minimisation is loglinear [Hop71], and the problem is PSPACE-complete for both NFA and regular expressions. Based on the algebraic properties of regular expressions, Antimirov and Mosses proposed a terminating and complete rewrite system for deciding their equivalence [AM94]. On a paper about testing the equivalence of regular expressions, Almeida *et al.* [AMR08a] presented an improved variant of the rewrite system. Experimental results suggested a better average-case performance than the classical methods which, nevertheless, resort to minimisation algorithms applied to the DFAs obtained from the regular expressions.

Hopcroft and Karp [HK71] presented, in 1971, a linear algorithm for testing the equivalence of two DFAs that avoids minimisation. Considering the merge of the two DFAs as a single one, the algorithm computes the finest right-invariant relation which identifies the initial states. The state equivalence relation that determines the minimal DFA is the coarsest relation in that condition. Although apparently forgotten by the scientific community in the course of the last 30 years, applications of this algorithm can only be found in some applications to unification theory and type systems [Eme91, MPS84, AD08].

These algorithms are also closely related with the recent coalgebraic approach to automata developed by Rutten [Rut03]. Of special interest is their extension to Kleene algebras to model program properties, which have been successfully applied in formal program verification [Koz08].

We developed some variants of Hopcroft and Karp's algorithm, and established a relationship between this algorithm and the one proposed in Almeida *et al.* [AMR08b]. We also extended Hopcroft and Karp's algorithm to nondeterministic finite automata and produced some experimental comparative results.

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A Duet With Occasional Chorus

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Keywords: Transformation; monoid; semigroup; endomorphism; vector space; subspace; lattice; C-independent; S-independent; M-independent; matroid; SC-ranked.

Extended Abstract

For a semigroup S we say that $f : S \to S$ is an *anti-automorphism [anti-monomorphism]* if f is bijective [one-one] and satisfies $f(ab) = f(b)f(a)$.

It is well known that any group admits at least one anti-automophism, namely, $\iota : a \mapsto a^{-1}$, and the same (obviously with a different anti-automorphism) holds for the endomorphism monoid of a finite dimensional vector space. Similarly, we could ask if there is an anti-automorphism of $T(X)$, the monoid of all selfmaps on a set X (for $|X| > 1$). Clearly the answer is no, since anti-automorphisms preserve the poset of $\mathscr{J}$-classes and hence, if such an anti-automorphism did exist, the kernel of $T(X)$ would be isomorphic to its dual, an impossibility, as one is a left zero semigroup while the other is a right zero semigroup. What, then, can be said about anti-monomorphisms? Is it true that $T(X)$ admits an anti-monomorphism? Although interesting in itself, this question arose in circumstances that make it even more interesting to consider.

While George Bergman was writing [1], on embedabilities of algebras, he proved that if a monoid M is embeddable in $T(X)$, then a certain condition on a lattice associated with M must be satisfied. And then Bergman moved on to ask the natural question about the sufficiency of that condition. As the lattices associated with $T(X)$ and with its dual are isomorphic and the property referred to above is preserved by isomorphisms, the non-embeddability of $T(X)$ into its dual would yield a negative answer to Bergman's question, leading him to propose the following problem: does $T(X)$ admit an anti-monomorphism? And, motivated by the same embeddability problem, a similar question can be (in fact was) asked about the endomorphism monoid of vector spaces.

In this talk we are going to see how these questions were solved in a very elegant way by an extraordinary and widely-known mathematician.

We are also going to see how these results were then generalized to some algebras endowed with abstract notions of independence. In the 60s Marczewski (and colleagues) introduced several notions of independence aiming to provide a concept that could unify the different notions of independence spread throughout all branches of mathematics: linear independence, set independence, (several notions of) independence of continuous functions, etc. And as a side-effect of this study Marczewski and his colleagues started to study classes of algebras defined by the coincidence of two notions of independence. For example, for an algebra A, we say that $X \subseteq A$ is C-independent if for all $x \in X$ we have $x \notin \langle X \setminus \{x\} \rangle$; we say that X is M-independent if any map $f : X \to A$ can be extended to a morphism $F : \langle X \rangle \to A$; finally we say that X is S-independent if every map $f : X \to X$ can be extended to a morphism $F : \langle X \rangle \to \langle X \rangle$. We say that an algebra A is an MC-algebra if M-independence and C-independence coincide in A. For example, absolutely free algebras are MC-algebras. Quite surprisingly, if an MC-algebra contains a C-independent generating set, then all C-independent generating sets have the same number of elements. By the time this was discovered, these were the first examples of non-matroid-like algebras satisfying such property. (*En passant* it is worth observing that *independence algebras* are precisely the matroid MC-algebras.)

The most general result of this talk is that the endomorphism monoid of some SC-algebras does not admit an anti-monomorphism. As a consequence it follows that the endomorphism monoid of an independence algebra, the endomorphism monoid of some (quite general) M-acts and the endomorphism monoid of some (quite general) modules do not admit anti-monomorphisms. We will also provide an example of an M-act whose endomorphism monoid admits an anti-monomorphism, but not an anti-automorphism. We will conclude this talk proposing a number of problems for experts in topology, semigroup acts, modules and universal algebra.

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On the decidability of $g\mathbf{V}$

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Keywords: pseudovariety of monoids, pseudovariety of categories, global of a monoid pseudovariety.

Extended Abstract

We shall present an example of a monoid pseudovariety $\mathbf{V}$ with decidable membership for which the global $g\mathbf{V}$ has undecidable membership. The result is based on the seminal work of Kaďourek [1] on the locality of the monoid pseudovariety $\mathbf{DG}$. More concretely, we shall construct a decidable pseudovariety $\mathbf{H}$ of groups for which membership in $g\mathbf{DH}$ is undecidable (though membership in $\mathbf{DH}$ is clearly decidable). This answers a question in the book of Rhodes and Steinberg [2]. As a by-product we also get that the operator $\mathbf{V} \mapsto \mathbf{V} * \mathbf{D}$ does not preserve decidability of membership. Indeed, for the aformentioned example $\mathbf{H}$, the semigroup pseudovariety $\mathbf{DH} * \mathbf{D}$ has undecidable membership. The latter refutes a conjecture of Straubing [3].

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Decompositions of ideals in irreducible ideals in numerical semigroups

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Keywords: Numerical semigroup, irreducible ideal.

Abstract

Given a numerical semigroup S, it is not hard to see that an integral ideal of S (i.e. a subset I of S such that $i+s \in I$, for all $i \in I$ and $s \in S$) is irreducible as integral ideal (i.e. I is not the intersection of two proper integral overideals) if and only if $I = S \setminus B(x)$, for some $x \in S$, where we set $B(x) = \{s \in S \mid s + s' = x, \text{for some } s' \in S\}$. See [4] for irreducibility of ideals in a more general setting.

On the other hand it is known that a relative ideal F of S (i.e. a subset F of $\mathbb{Z}$ such that $s+F$ is an integral ideal of S, for some $s \in S$) is irreducible as relative ideal (i.e. F is not the intersection of two proper relative overideals) if and only if $F = z + \Omega$, for some $z \in \mathbb{Z}$, where Ω is the canonical ideal of S, $\Omega = \{f - x;\ x \in \mathbb{Z} \setminus S\}$, where f is the Frobenius number of the semigroup (cf. [1]). See [3] and [2] for basic notions of numerical semigroups.

It will be proved in the talk that each integral ideal of S is in a unique way an irredundant intersection of irreducible integral ideals. The same result holds replacing “integral ideal" with “relative ideal". Given an integral ideal I of S, the two decompositions, in integral and relative ideals respectively, are essentially different. The number of components, $n(I)$ and $N(I)$ respectively, can be exactly valuated and in general $n(I) \le N(I)$. It turns out however that, in case of a principal ideal I, $n(I) = N(I)$ equals the type of the semigroup.

These results correspond to similar but weaker results which can be proved for the decompositions of integral and fractional ideals in one-dimensional local Cohen-Macaulay rings.

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Configuration Semigroups

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(This is a joint work with Jordi Castellà-Roca)

Keywords: Numerical semigroup, combinatorial block design, combinatorial configuration, user-private information retrieval.

Extended Abstract

Combinatorial configurations are a particular case of block designs which have been recently used for defining peer-to-peer communities for keeping privacy of users in front of search engines [1, 2].

A set $X = \{x_1, \ldots, x_v\}$ and a set $\mathscr{A}$ of b blocks, each block being a subset of X of length k, form a (v, b, r, k)-configuration if each element in X is exactly in r blocks of $\mathscr{A}$ and no two elements in X meet in more than one block of $\mathscr{A}$. There is a natural bijection between combinatorial configurations and bipartite graphs with girth larger than 5.

One problem when using configurations is the limited number of known configurations, specially for large v and b. We refer the reader to [3] for tables of parameters for which it is known that configurations do exist and for which it is known that they do not exist.

Particular constructions have been given with a very easy construction for any $b = v = q^2 + q + 1$, where q is a power of a prime [4]. However, although this value can be as large as desired, this puts a significant restriction on b and v.

In [2] an alternative construction is described where the only requirement is that b and v be of the form $b' + b''$ and $v' + v''$, where a (v', b', r, k) and a (v'', b'', r, k) configuration exist. This allows to obtain configurations with large number of blocks from configurations with small number of blocks. The construction is as follows: Suppose we have a (v, b, r, k)-configuration $(X = \{x_1, \ldots, x_v\}, \mathscr{A})$ and a (v', b', r, k)-configuration $(Y = \{y_1, \ldots, y_v\}, \mathscr{B})$. Swap one element in one block of $\mathscr{A}$ for one element in one block of $\mathscr{B}$ and obtain a $(v + v', b + b', r, k)$ configuration. It is trivial to check that this is indeed a configuration. By repeating this procedure combining more configurations we can obtain configurations with v and b larger than any desired values, once r and k are fixed.

Using the previous construction, one can show that the set of all tuples (v, b, r, k), for fixed r, k, such that there exists a (v, b, r, k) configuration has the structure of a commutative monoid and of a numerical semigroup if $\gcd(r, k) = 1$. Let us call $D_{r,k}$ this monoid. Some examples in which $D_{r,k}$ is a numerical semigroup are: $D_{2,3} =< 2, 3 >$, $D_{3,4} =< 3, 4, 5 >$, $D_{2,5} =< 3, 4, 5 >$, $D_{2,4} =< 5, 6, 7, 8, 9 >$, $D_{2,6} =< 7, 8, 9, 10, 11, 12, 13 >$, $D_{3,3} =< 7, 8, 9, 10, 11, 12, 13 >$. The last three examples show that the condition $\gcd(r, k) = 1$ is sufficient but not necessary.

We present the following open questions:

Question 1 Given r, k, can we assume $D_{r,k} \neq \{0\}$? That is, is there at least one pair $(v, b) \neq (0, 0)$ such that a (v, b, r, k)-configuration exists?

Question 2 All known sets $D_{r,k}$ are ordinary semigroups (i.e. semigroups of the form $\{0\} \cup [c, \infty)$). Could this be generalized?

Question 3 Can we find (r, k) with $\gcd(r, k) \neq 1$ such that $D_{r,k}$ is not a numerical semigroup?

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Complete semigroups of transformations on a finite chain

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Keywords: transformation semigroups, products of idempotents, idempotency degree, semiband.

Extended Abstract

The study of various finite semigroups of transformations makes an important contribution to semigroup theory. For elementary concepts as well as standard notation in semigroup theory see, for example, [8].

It is well-known that every semigroup can be regarded as a semigroup of transformations, and semigroups of transformations provide interesting examples of certain concepts in semigroup theory. As observed in [10], transformation semigroups are for semigroup theory as permutation groups are for group theory. Many authors dedicated to the study of certain types of finite full (partial, partial one-to-one) transformation semigroups.There are various articles with the study of important properties of some of those semigroups of transformations.(See, for example, [6], [7], [11], [5], [4], five articles, some of them about different types of finite full transformations semigroups, among many others).

The full transformation semigroup T_n on a finite chain $X_n = \{1, 2, \cdots, n\}$ have been studied over the years and also many of its subsemigroups. One important subsemigroup of T_n which has been studied extensively is the semigroup O_n of order-preserving transformations on X_n. Also, another subsemigroup of T_n which is very well known is the semigroup Sing_n of all singular (non-bijective) selfmaps of X_n.

In [9], McAlister studied the structure of a semigroup S generated by its group G of units and an idempotent e. In the fourth section of [9], McAlister studied the case in which the group of units G is cyclic. Catarino and Higgins, in [1] and [2], studied the particular case of that kind of semigroups, more precisely, the subsemigroup OP_n of T_n consisting of orientation-preserving transformations on X_n. The definition of orientation-preserving mapping relies on that of a cyclic sequence of elements from a chain. We say that a sequence $A = (a_1, a_2, \cdots, a_t)$ of t elements of $X_n = \{1, 2, \cdots, n\}$ is *cyclic* if there exists no more than one subscript i such that $a_i > a_{i+1}$ (addition mod t). Also we say that A is an *anti-cyclic* sequence if there exists no more than one subscript i such that $a_i < a_{i+1}$ (addition mod t). For more details concerning these sequences see, for example, [1] and [2]. A mapping α is an *orientation-preserving* mapping if the sequence of images $(1\alpha, 2\alpha, \ldots, n\alpha)$ is a cyclic sequence and a mapping α is an *orientation-reversing* if the sequence $(1\alpha, 2\alpha, \ldots, n\alpha)$ is anti-cyclic. Note that O_n is a subsemigroup of OP_n. Also, in [2], the authors studied another subsemigroup of T_n denoted by P_n which is the union of OP_n and the collection OR_n of all orientation-reversing mappings on X_n. Note that, P_n is again a particular case of the semigroup introduced (and studied) by McAlister in the last section of [9]. Also, in [2] and [1], they introduced another subsemigroup of OP_n that is isomorphic to semigroup O_n. Such semigroup is the conjugate subsemigroups of OP_n, denoted by $O_n^k, (0 \leq k \leq n-1)$.

In 1999, in [3], Giraldes and Howie studied the completeness of certain finite full (partial) transformation semigroups on X_n. They defined, for each a in a semigroup S, a rational number $\delta(a)$ and they used this number

for the definition of *complete semigroup*, in which every element is easily shown to be a product of idempotents. This concept is used by the authors to give proofs of completeness of some finite semigroups of transformations. We can find in this article the proofs that the semigroups Sing_n and O_n are both complete. Also, in [3], we find the completeness of one finite partial transformation semigroup on X_n.

The main topic of this oral comunication is related with completeness or not of some other full transformations semigroups on a finite chain $X_n = \{1, 2, \cdots, n\}$. More precisely we study here the completeness or not of OP_n and also of some subsemigroups of the semigroup OP_n. Following [3], we use the concept of semiband for conclusion about the completeness or not completeness of those finite transformations semigroups on X_n. We show that OP_n and P_n are not complete semigroups; however we prove that O_n^k is a complete semigroup.

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Rings whose idempotents form a band

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(This is a joint work with Jonathan Leech)

Keywords: idempotent element, normal band, $\mathscr{D}$-class, skew Boolean algebra.

Extended Abstract

The talk will be based on the joint work with Jonathan Leech, [6]. A standard exercise in elementary ring theory states that given a ring with identity 1 whose idempotents are closed under multiplication, all the idempotents lie in the center of the ring and thus form a Boolean algebra with operations: $e \wedge f = ef$, $e \vee f = e + f - ef$ (i.e., the *circle operation* $e \circ f$) and $e' = 1 - e$.

What happens when $E(R)$ is multiplicative in a ring R *without* identity? In this case $E(R)$ is immediately seen to be a *normal band*. That is, $E(R)$ is a band (a semigroup of idempotents) on which $xyzw = xzyw$ holds. For some time it has been known that any band S of idempotents in a ring that is maximal with respect to being normal is likewise closed under a counter-product, $e\nabla f = (e \circ f)^2$, that is also associative and idempotent. In this case S forms a noncommutative variant of a Boolean algebra called a *skew Boolean algebra*. Its meet is again multiplication, its join is ∇ and a relative complement is given by $e \backslash f = e - efe$. All of this applies in particular to $E(R)$ when the later is multiplicative. We explore how this condition can affect the full ring R.

This requires some assumptions of a reasonably general sort about how $E(R)$ lies in R. The ring R is said to be *idempotent dominated* if every element x is a sum $x_1 + ... + x_n$ of elements x_i that are *idempotent covered*, that is, $x_i = e_i x_i = x_i f_i$ for some $e_i, f_i \in E(R)$. The class of idempotent dominated rings contains the class of rings with identity as well as the class of Von Neumann regular rings.

Like (generalized) Boolean algebras, skew Boolean algebras factor at will. Thus under appropriate conditions, skew Boolean algebras factor into direct products or direct sums (as the case may be) of atomic skew Boolean algebras. We shall describe the effect of such decompositions of $E(R)$ on all of R, and the extent to which R is likewise decomposed.

Over the past thirty years, skew Boolean algebras have been studied by various authors. In so doing, rings with a plentitude of idempotents have been a fertile source for classes of examples as well as for concepts initially observed in those examples. In particular, the fact that $E(R)$, when multiplicative, forms a skew Boolean algebra first appeared in (Leech [12]).

To our knowledge, skew Boolean algebras were first considered in the Flinders University dissertation of R. J. Bignall [1], although some relevant ideas were anticipated in the 1974 paper of Keimel and Werner [7]. These ideas were developed further in [4] by his advisor, William Cornish. Subsequent papers by Leech [9] and by Bignall and Leech [2] benefited from developments in skew lattices. Since then other papers have appeared on either skew Boolean algebras or their role in closely related topics. (See, e.g., [3], [5] and [10].) For further background on skew lattices and skew Boolean algebras see [2], [8] and [9].

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Betti numbers of some semigroup rings

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Keywords: Betti numbers, maximal embedding dimension, numerical semigroups, semigroup rings.

Extended Abstract

By a **numerical semigroup** we mean a submonoid S of $\mathbb{N}$ such that $\mathbb{N} \setminus S$ is finite. $\mathbb{N}$ is understood to be the set of non-negative integers $\{0, 1, 2, \ldots\}$. It is well know that such a semigroup is finitely generated, that is, it consists of all non-negative integer combinations of some minimal generating set $\{s_0, s_1, \ldots, s_r\}$. Given a semigroup $S = \langle s_0, \ldots, s_r \rangle$ and a field k, consider the **semigroup ring** $k[S]$. This is the k-algebra $k[t^s; s \in S]$, t an indeterminate, defined by

$$t^s \cdot t^{s'} = t^{s+s'}, \quad s, s' \in S.$$

In the talk we describe the Betti numbers of all semigroup rings $k[S]$ corresponding to numerical semigroups of maximal embedding dimension. The description, that is in terms of the generators of S, give precisely the degrees in which the non zero graded Betti numbers occur. We show that for arithmetic numerical semigroups of maximal embedding dimension, the graded Betti numbers occur symmetrically in two respects.

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Zappa-Szép products of groups and right cancellative monoids

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Keywords: Zappa-Szép products.

Extended Abstract

Consider two monoids M and N with a right action $\cdot$ of M on N and a left action $*$ of N on M. Under appropriate axioms for the actions, the set $M \times N$ is a monoid under the operation

$$(m, n)(m', n') = (m(n * m'), (n \cdot m')n').$$

This monoid is denoted by $M \bowtie N$ and called the *Zappa-Szép product* of M and N.

This construction was introduced for groups by Zappa in 1940 and investigated by Redei and Sz'ep in a series of papers. It was rediscovered by Takeuchi and independently by Majid under the name bicrossed product. The name Zappa-Szép product was coined by Brin [1] in a detailed study the product for various algebraic structures.

$M \bowtie N$ is a generalisation of the semidirect product construction: if the action of M on N is trivial, we obtain the semidirect product $M \rtimes N$.

A monoid M is *right hereditary* if each of its right ideals is projective (as right M-acts). In [2], Lawson studied Zappa-Szép products of free monoids and groups, and characterised them as the left cancellative, right hereditary monoids such that every element is contained in only finitely many principal right ideals. He also showed that such Zappa-Szép products are intimately connected with the self-similar group actions of [3] in the sense that given a Zappa-Szép product $X^* \bowtie G$, the action of G on X^* is self-similar, and vice-versa. He also characterises these actions in terms of Mealy machines and double categories.

In the talk we consider some extensions and generalisations of Lawson's work. For example, we look at Zappa-Szép products of graph monoids and groups, and their connection with unique factorisation.

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Notable elements for counting numerical semigroups

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Keywords: Numerical semigroup, gender, Frobenius number, Apéry sets, Frobenius varieties.

Extended Abstract

A numerical semigroup is a submonoid of the set of nonnegative integers with finite complement in it. The largest integer not belonging to a numerical semigroup is known as its Frobenius number. A nonnegative integer not belonging to a numerical semigroup is a gap, and the gender of a numerical semigroup is the cardinality of its set of gaps.

In this talk we review some tools for constructing the set of numerical semigroups with a given gender or Frobenius number. We will also show that for some popular families of numerical semigroups this task becomes easier. Given a positive integer f, if a numerical semigroup S has Frobenius number f, then $\{0\} \cup \{f+1, f+2, \ldots\} \subseteq S$. Thus the numerical semigroup S is an oversemigroup of the numerical semigroup $\{0\} \cup \{f+1, f+2, \ldots\}$. Hence with a tool to construct oversemigroups of a given numerical semigroup, we can compute the whole set of numerical semigroups with fixed Frobenius number. The first step is to find for a numerical semigroup S those semigroups containing it and that differ in one element with it, that is, semigroups of the form $S \cup \{x\}$ with x a gap of S. The element x must fulfill that $x + (S \setminus \{0\}) \subseteq S$ (x is called a pseudo-Frobenius number) and that $2x \in S$. These elements are called special gaps. Later we will give alternative ways to perform this task, one of them much faster.

If a positive integer g is fixed, and we want to calculate the set of all numerical semigroups with gender g, the we can use the dual idea to the one used in the preceding paragraph. Observe that if S is a numerical semigroup with gender $g-1$, by removing an element in S so that the resulting set is a numerical semigroup, we obtain a numerical semigroup with gender g. The elements we can remove from S are precisely those that cannot be expressed as sum of two nonzero elements in the semigroup. These are the minimal generators of S (every numerical semigroup admits a unique minimal generating set; this and other facts not explicitly cited here can be found proved in [5]). In order to avoid repetitions in this procedure, one chooses only those minimal generators larger than the Frobenius number.

For S a numerical semigroup, the binary relation on the set of integers, defined by $a \leq_S b$ if $b - a \in S$, is an order relation. The set of pseudo-Frobenius numbers coincides with the maximal elements with respect to $\leq_S$ of the complement of S in the set of integers, while the minimal generating set of S is trivially the set of minimal elements of $S \setminus \{0\}$. Both sets are encoded inside any Apéry set of any element in S. Given $m \in S \setminus \{0\}$, the Apéry set of m in S is the set of elements $n \in S$ such that $n - m$ no longer belongs to S. This set contains precisely m elements that we can label by $w_0, \ldots, w_{m-1}$. For every i, w_i is the least element in S congruent with i modulo m. Selmer proved in [7] that the Frobenius number of S is the largest w_i minus m, and the gender is $\frac{1}{m}(w_1 + \cdots + x_{m-1}) - \frac{m-1}{2}$. Every w_i can be written as $k_i m + i$. We can choose m to be the smallest positive integer in S (its multiplicity) to get the smallest possible Apéry set. The tuples $(k_1, \ldots, k_{m-1})$ are solutions of a (preset) system of inequalities (see

[4]). Thus every numerical semigroup with multiplicity m corresponds to a point with integer coordinates inside a cone. If we cut these cones with the hyperplanes $x_1+\cdots+x_{m-1}=g$, the integer points in this intersection represent those numerical semigroups with gender g and multiplicity m. By moving the multiplicity one can construct with integer programming machinery the set of all numerical semigroups with gender g (see [1]). If we cut these cones with appropriate boxes, we obtain the set of all numerical semigroups with fixed Frobenius number. Unfortunately integer programming is still slow due to the large amount of variables that show up in the process. We next present some alternatives.

Let S be a numerical semigroup, and let m be its multiplicity. Assume that the Apéry set of m in S is $\{0=w_0,w_1,\dots,w_{m-1}\}$. If w_i is a minimal generator, then the Apéry set of $S\setminus\{w_i\}$ is $\{w_0,\dots,w_{i-1},w_i+m,w_{i+1},\dots,w_{m-1}\}$. By using this idea one can speed up the computation of all numerical semigroups with gender g from those of gender $g-1$ (this is precisely the way we have implemented it in the `numericalsgps` GAP package, [3]). It is still not known if in general there are more numerical semigroups with gender g than with gender $g-1$, though experiments show that the sequence of cardinalities of these sets behave like the Fibonacci sequence ([2]).

The (experimentally) better tool to compute the oversemigroups of a numerical semigroup is to encode numerical semigroups by means of their fundamental gaps ([6]). Fundamental gaps are those gaps that are maximal with respect to the divisibility relation. Special gaps are easy to determine from them, and if S is a numerical semigroup and x is a special gap of S, the fundamental gaps of $S\cup\{x\}$ are straightforward to calculate. This is why we have chosen this idea to implement the functions to calculate the oversemigroups of a numerical semigroup (and the set of numerical semigroups with fixed Frobenius number) in the `numericalsgps` package.

We will show how for some well studied families of numerical semigroups (irreducible, Arf, saturated) these procedures can be speeded up.

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Adequate semigroups *or, losing our identity*

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Keywords: Ample, adequate, proper, free.

Extended Abstract

The study of inverse semigroups forms a central plank of algebraic semigroup theory. The Wagner-Preston representation theorem tells us that they are precisely subsemigroups of symmetric inverse semigroups $\mathscr{I}_X$ closed under $\alpha \mapsto \alpha^{-1}$. This result at once indicates both the importance of inverse semigroups, and that they should be thought of as algebras of type $(2,1)$, that is, equipped with the binary operation of semigroup multiplication, and the unary operation $a \mapsto a^{-1}$. With this signature, inverse semigroups form a variety. In terms of idempotents, an inverse semigroup consisting entirely of idempotents is precisely a semilattice, whilst at the other extreme, an inverse semigroup with exactly one idempotent is precisely a group. From the point of view of structure, three major approaches stand out, all succeeding in classifying inverse semigroups via groups (or groupoids) and semilattices. These are (a) the Ehresmann-Schein-Nambooripad characterisation of inverse semigroups in terms of inductive groupoids, (b) Munn's use of fundamental inverse semigroups and his construction of the semigroup T_E from a semilattice E, and (c) McAlister's results showing on the one hand that every inverse semigroup has a proper (E-unitary) cover and on the other determining the structure of proper inverse semigroups in terms of groups, semilattices and partially ordered sets.

It becomes clear only when one looks at wider classes of semigroups containing semilattices of idempotents, that a significant factor in the success of all of these approaches is the satisfaction of the ample identities. For an inverse semigroup S and any $a \in S, e \in E(S)$ we have that $ae = ae(ae)^{-1}a, ea = a(ea)^{-1}ea$; defining x^+, x^* for $x \in S$ by $x^+ = xx^{-1}, x^* = x^{-1}x$, we therefore have $ae = (ae)^+a, ea = a(ea)^*$. These are referred to as the *ample identities*; the purist will probably prefer to write them as $ab^+ = (ab^+)^+a, b^+a = a(b^+a)^*$.

As a step away from inverse semigroups, authors of the York school have studied classes such as that of ample semigroups. These may be arrived at via consideration of the Green's *-relations $\mathscr{L}^*$ and $\mathscr{R}^*$. Whereas Green's relations are relations of mutual divisibility, these are relations of mutual cancellability. Let S be a semigroup with $E(S)$ a semilattice. Then S is *adequate* if every $\mathscr{R}^*$-class and every $\mathscr{L}^*$-class contains an idempotent; note that an adequate semigroup with one idempotent is precisely a cancellative monoid. In a regular semigroup, $\mathscr{R}^* = \mathscr{R}$ and $\mathscr{L}^* = \mathscr{L}$, so that a regular adequate semigroup is precisely an inverse semigroup. In an adequate semigroup, there is a necessarily unique idempotent in the $\mathscr{R}^*$-class ($\mathscr{L}^*$-class) of a, denoted a^+ (a^*).

As indicated above, we need to add the ample identities to obtain classes of semigroups, known as *ample* semigroups, displaying behaviour analagous to that of inverse semigroups. Ample semigroups have been successfully described in terms of semilattices and cancellative semigroups (or cancellative categories) via results analogous to those of (a), (b) and (c), by Armstrong, Fountain and Lawson, respectively.

Adequate semigroups, not satisfying the ample identities, do not behave like inverse or ample semigroups. What can we say about their structure? The pioneer in this direction was Lawson; in [6] he showed that adequate semigroups correspond to strongly cancellative Ehresmann categories. Indeed Lawson considered the wider class of *Ehresmann* semigroups and showed these correspond to Ehresmann categories. Ehresmann semigroups form a variety and with this in mind, the author and Gomes study fundamental Ehresmann semigroups in [3], developing an analogue of the Munn semigroup T_E.

Thus far, there have been no attempts to follow the route of McAlister for adequate semigroups or indeed for Ehresmann semigroups. Suppose we say that an adequate semigroup is *left proper* if $\sigma \cap \mathscr{R}^*$ is trivial, where σ is the least cancellative congruence on S. Let S be a left proper adequate semigroup and imagine as an extreme case that we can write each $a \in S$ uniquely as $a^+ a'$ where a' lies in a subsemigroup T of S and $T \cong S/\sigma$. How do we multiply elements? If S is ample, we have

$$ab = (a^+ a')(b^+ b') = a^+(a' b^+) b' = a^+(a' b^+)^+(a' b') = (ab)^+ a' b'.$$

This kind of observation is exactly that which leads to a semidirect product structure. Without the ample identity, we must think again.

In this talk we present results of [1], a joint work with Branco and Gomes. We introduce a new notion of proper for adequate semigroups and related classes, including one-sided variations. To simplify matters, we consider monoids. Our notion of proper is arrived at via a careful consideration of how free adequate monoids behave and an analysis of how an adequate monoid may be generated by $E(S)$ together with a distinguished submonoid.

We show that every adequate monoid has a proper cover and that free adequate monoids are proper. Although some way short of a structure theorem for all proper adequate monoids, we show how to construct proper left Ehresmann monoids from monoids acting via order preserving maps on semilattices. This construction was inspired by that of Fountain in [2]. Finally, we remark that we may obtain the free left adequate monoid using this technique. The structure of both the free left adequate and the free adequate monoid have also been recently discovered by Kambites [4, 5].

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The Idempotent Problem

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Keywords: word problem, idempotent problem, regular languages, context-free languages, polycyclic monoids, graph inverse semigroups.

Extended Abstract

Given a group G generated by a set X, the word problem of G is the subset of words in $(X \cup X^{-1})^*$ that represent the identity. Considering the word problem as a language has been an important bridge between group theory and formal language theory. Famous results include Anisimov and Seifert's theorem: the word problem of a finitely generated group is a regular language if and only if the group is finite; and the Muller-Schupp-Dunwoody theorem: the word problem of a finitely generated group is a context-free language if and only if the group has a free subgroup of finite index.

We introduce an analogous concept for inverse semigroups called the "Idempotent Problem". If S is an X-generated inverse semigroup, the idempotent problem consists of all words from $(X \cup X^{-1})^*$ that correspond to idempotents. There are also two very natural variations on the idempotent problem: if we specify an idempotent from S, we can look at the language of all words that correspond to this idempotent; we can also consider the language of all words that map to elements greater than or equal to the given idempotent (using the natural partial order for inverse semigroups).

Our goal is to relate the properties of an inverse semigroup (or an inverse semigroup with zero) to the language properties of its idempotent problem. We also wish to consider the relationship between the idempotent problem and its variations. This is research in progress and we share initial results. First, we describe various conditions on the inverse semigroup that guarantee that the idempotent problem is regular. Second, we show that polycyclic monoids and graph inverse semigroups both have context-free idempotent problem. To illustrate this, we construct pushdown automata that recognize their idempotent problems.

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Semigroups of relations

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Keywords: Representations of semigroups, algebras of binary relations, modal operators, dynamic algebras.

Extended Abstract

This talk reports on joint work, predominantly with Tim Stokes (University of Waikato, New Zealand) and Robin Hirsch (University College, London).

Every semigroup is isomorphic to a semigroup of transformations on a set, or to a semigroup of partial maps on a set, or to a semigroup of binary relations on a set. Moreover, finite semigroups can be represented in these ways over a finite set. These elementary facts underpin many of the applications of semigroups, and are surely amongst the most fundamental properties of semigroups.

One of the most familiar examples of this interplay is the relationship between finite state automata and finite semigroups: the "algebra of transitions" of a finite state automata is a finite semigroup, and every finite semigroup arises in this way.

More generally, the study of transitions and their composition pervades many mathematical contributions to computer science, but often there are extra properties of interest (beyond simply transitions between states). In turn, this gives rise to an interest in other kinds of operations (beyond simply a multiplicative semigroup). Nevertheless, the underlying semigroup structure remains, and the semigroup theorist may still have much to offer. In this talk we examine some "semigroup based" methods in algebras of relations and their application.

1. We examine algebras of relations arising from the formal analysis of computer programs. Here, existing logical methods in formal program verification very quickly motivate the study of (enriched) semigroups of modal operators. These turn out to be closely related to an increasingly popular brand of unary semigroup: *restriction semigroups* [1] (also known as guarded semigroups, twisted (left) closure semigroups, weakly left E-ample semigroups, type SL2 γ-semigroups, amongst others). Techniques developed by Boris Schein for the study of semigroups of partial maps (see [3] for example) enable a new representation for algebras relating to deterministic program logics, such as the *Strict Deterministic Propositional Dynamic Logic* of [2].

2. On the other hand we explain how a semigroup-theoretic approach has provided some surprisingly negative results concerning the characterisation of various kinds of algebras of relations. In particular we show how some algorithmically undecidable embedding problems arising from the study of finite semigroups can be used to show the undecidability of representability and of finite representability (as binary relations) for a wide range of enriched semigroups, including finite *lattice-ordered monoids*, finite *Boolean monoids* and *dynamic algebras with intersection*.

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Filters in (quasiordered) semigroups, lattices of filters and a generalisation of Caratheodory number

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(This is a joint work with Alexei Vernitski)

Keywords: quasiordered semigroups, filters, semilattice congruences, Caratheodory number.

Extended Abstract

Let S be a semigroup and let $\leq$ be a quasiorder on S.

Let us say that a subset F of S is *downward closed* if for all $s, t \in S$ if $s \leq t$ and $t \in F$ then $s \in F$. A *filter* F is a downward closed subsemigroup of S.

Let us say that $\leq$ is *operation-positive* if $s, t \leq st$ for all $s, t \in S$. Let us say that $\leq$ is *operation-compatible* if for all $s, t, r \in S$ from $s \leq t$ it follows $rs \leq rt$ and $sr \leq tr$. There is a 'natural' quasiorder on every semigroup, which is the smallest operation-positive operation-compatible quasiorder. In older papers, filters (also known as *faces* and *consistent subsemigroups*) were introduced as subsemigroups whose complement is an ideal, and they coincide with our filters relative to the natural quasiorder. Some results regarding filters were obtained in the 1960s and 1970s, see, for instance, [1]. Then filters relative to arbitrary operation-positive operation-compatible quasiorders were introduced, see the survey article [2]. Recently, these filters were considered in some papers because of applications to logic, see, for instance, [3].

We study lattices of filters. For instance, we prove that for every quasivariety V of (finite) semigroups, every semigroup in V is a homomorphic image of a semigroup in V whose lattice of filters (relative to the natural quasiorder) is Boolean.

The concept of filters enables us to generalise the Caratheodory number from lattices to semigroups.

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Groups acting on semimetric spaces and quasi-isometries of monoids

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(This is a joint work with Robert Gray)

Keywords: quasi-isometry, finitely generated monoid, group action, Švarc-Milnor lemma.

Extended Abstract

I shall describe recent joint work with Robert Gray on the development of geometric methods for finitely generated monoids and semigroups. We study a natural notion of quasi-isometry between spaces equipped with asymmetric, partially defined distance functions, and hence between finitely generated semigroups and monoids. It transpires that, just as for groups, many natural algebraic and geometric properties of monoids and semigroups are quasi-isometry invariants. A key tool is an extension of the Švarc-Milnor Lemma to the setting of groups acting by length-preserving transformations on asymmetric distance spaces.

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Algorithmic Problems for Fully Residually Free Groups

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Keywords: algorithmic problems, free group, fully residually free group, limit group.

Extended Abstract

In this talk I will discuss some methods and techniques designed to deal with algorithmic problems for fully residually free groups (and groups acting freely on Λ-trees). These methods were extensively, though sometimes implicitly, used in our (joint with A. Myasnikov) solution of the Tarski's problems. It seems it is worthwhile to introduce them explicitly. Our key players in this area are infinite non-Archimedean words and Elimination Processes.

Let G be a subgroup of H. We say that a family of G-homomorphisms (homomorphisms identical on G) $\mathscr{F} \subset Hom_G(H,K)$ *separates* [*discriminates*] H into K if for every non-trivial element $h \in H$ [every finite set of non-trivial elements $H_0 \subset H$] there exists $\phi \in \mathscr{F}$ such that $h^\phi \neq 1$ [$h^\phi \neq 1$ for every $h \in H_0$]. In this case we say that H is *G-separated* [*G-discriminated*] by K. Sometimes we do not mention G and simply say that H is separated [discriminated] by K. In the event when K is a free group we say that H is *freely separated* [*freely discriminated or fully residually free*].

Equivalent definitions ([6],[3],[2],[1])

Let either G be an equationally Noetherian group and H a finitely generated (fg) group, or H be finitely presented (fp). Then for H the following conditions are equivalent:

1. $Th_\forall(G) \subseteq Th_\forall(H)$, i.e., $H \in \mathbf{Ucl}(G)$;
2. $Th_\exists(G) \supseteq Th_\exists(H)$;
3. H embeds into an ultrapower of G;
4. H is discriminated by G;
5. H is a limit group over G;
6. H is defined by a complete atomic type in the theory $Th_\forall(G)$;
7. H is the coordinate group of an irreducible algebraic set over G defined by a system of coefficient-free equations.

Algorithmic problems

Let G be a fg fully residually free group.

1. The Word Problem is decidable in G - follows from the fact that G is residually finite and f.p.

2. The Conjugacy Problem is decidable in G [5] also follows from decidability of the problem for relatively hyperbolic groups (Bumagin, 2004) and relative hyperbolicity of f.g. fully residually free groups (Dahmani, 2002).

   Related results: CP was proved to be decidable for $F^{\mathbf{Z}[\mathbf{t}]}$ by Ribes and Zalesski (1996) and using different methods by E. Liutikova (2003).
3. There is an algorithm to construct the JSJ decomposition [4]
4. The Isomorphism Problem is decidable in G (Bumagin, Karlampovich, Miasnikov, 2007).
5. Dahmani and Groves later solved the isomorphism problem in relatively hyperbolic groups with abelian parabolics.

Let G be a f.g. fully residually free group and $H, K \leq G$ be f.g.

1. the Membership Problem is decidable for H (Miasnikov, Remeslennikov, Serbin, 2003),
2. $H \cap K$ is f.g (Howson Property) and can be found effectively [5],
3. up to conjugation by elements from K there are only finitely many subgroups of G of the type $H^g \cap K$, where $g \in G$ [5],
4. it can be decided effectively if H is malnormal [5],
5. it can be decided if $H^g = K$ (and $H^g \leq K$) for some $g \in G$ and if yes such g can be found effectively [5],
6. for any $g \in G$, its centralizer $C_G(g)$ is G can be found effectively [5],
7. homological and cohomological dimensions of G can be computed effectively [5],
8. it can be decided if $|G : H| < \infty$ (A. Nikolaev, D. Serbin, 2008),
9. $Comm_G(H)$ can be found effectively, hence it is possible to find effectively $n(H) \in \mathbf{N}$ such that for any $P \leq G$ if $|P : H| < \infty$ then $|P : H| < n(H)$ (NS, 2008).

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On free semigroups of regular languages

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(This is a joint work with Sergey Afonin)

Keywords: semigroup of languages, regular language, freeness.

Extended Abstract

Let Σ be a finite alphabet and $\text{Reg}(\Sigma)$ denotes the set of regular languages over Σ. Every regular substitution $\varphi : \Delta \to \text{Reg}(\Sigma)$ can be naturally extended to the morphism $\varphi : \Delta^+ \to (\text{Reg}(\Sigma), \cdot)$ between the free semigroup Δ^+ and finitely generated semigroup of regular languages over Σ with language concatenation as a product, denoted as S_φ. Such semigroups appear in many applications, e.g. data management or cryptography. In a previous work the authors conjectured that finitely generated semigroup of regular languages are automatic [1]. It was proved that in case of unary alphabet such semigroups are rational, i.e. form a proper subclass of automatic semigroups, and one can effectively construct rational transducer that represents the semigroup [3]. In the general case there exists no algorithm that, given a set of finite Σ-automata corresponding to semigroup generators, computes respective rational transducer. This means that either there exist non-rational semigroups of regular languages, or there is no algorithm for checking that given rational semigroup (rational transducer) is isomorphic to the semigroup generated by a given finite set of regular languages. This talk deals with an instance of the later problem — the decidability of semigroup freeness.

Algorithms for checking freeness of F-semigroups are well known. It is also well known that prime prefix codes constitute free semigroup. As every prefix code could be uniquely decomposed into prime ones it is possible to check the freeness of a semigroup generated by prefix codes. In general there exist prime languages that satisfy non-trivial relations and even semigroups of finite languages are not F-semigroups. Thus the above mentioned algorithms could not be applied. Moreover, finitely generated semigroups of finite languages are not left cancellation semigroups (e.g., $\{a + a^2\}\{a + a^3\}\{a\} = \{a + a^2\}^3$).

We show that every semigroup of regular languages over unary alphabet generated by more than one generative element satisfies non-trivial relation. That means that "almost all" semigroups of regular languages over unary alphabet are not free. In contrast, in case of binary alphabet "almost all" semigroups are free in the following sense. Define the distance between languages L and M be $d(L, M) = 2^{-|w|}$, where w is the shortest word in the symmetric difference of L and M, and the distance between the semigroups $d(S_\varphi, S_\psi) = \max_{L \in S_\varphi, M \in S_\psi} d(L, M)$. Then every finitely generated semigroup of regular languages could be approximated by free semigroups. Using decidability of rational subset membership problem for semigroups of regular languages [2], we also present decidability results for checking of specific types of relations. The general freeness problem remains open.

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On the structure of IM_n

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Keywords: partial transformations, maximal subsemigroups.

Extended Abstract

We study the structure of the semigroup IM_n of all partial injections on $X_n := \{1, 2, \ldots, n\}$ which are monotone i.e. preserve or reserve the natural order of natural numbers, (excluding the both permutations).

Some semigroups of transformations have been studied since the sixties. In fact, presentations of the semigroup O_n of all isotone transformations and of the semigroup PO_n of all isotone partial transformations (excluding the permutation in both cases) were established by Aĭzenštat in 1962 and by Popova, respectively, in the same year. Some years later (1971), Howie studied some combinatorial and algebraic properties of O_n and, in 1992, Gomes and Howie established some more properties of O_n, namely its rank and idempotent rank. In recent years, it has been studied in different aspects by several authors. The monoid IO_n of all isotone partial injections of X_n has been the object of study since 1997 by Fernandes in various papers ([2,3,4]). It is the intersection of the semigroup of all partial injections with the semigroup of all order-preserving transformations on the set X_n. Ganyushkin and Mazorchuk studied in [5] some properties of IO_n. In particular, all irreducible systems of generators as well as all maximal subsemigroups are described. It happens that $2^n - 1$ such maximal subsemigroups. Beside the semigroup of all order preserving transformations, the semigroup of all monotone transformations on a finite set is well studied. Recently, the maximal subsemigroups of this semigroup were characterized by Gyudzhenov and Dimitrova ([6]). In this presentation, we want to describe the semigroup IM_n of all partial transformations on the finite set X_n which are monotone. It is the intersection of the semigroup of all partial injections with the semigroup of all monotone transformations on X_n. We start with the description of the ideals of IM_n. It happens that they form an $(n+1)$-element chain of two-sided principal ideals. Then we check that every Green‘s relation is a restriction of the corresponding Green‘s relation on the semigroup of all partial injections on X_n. Moreover, we present a description of all irreducible systems of generators in IM_n. Note that irreducible systems of generators in the semigroup of all monotone transformations on X_n are given in [1]. The main result of our presentation gives the characterization of all maximal subsemigroups of IM_n. It happens that $2^{n+1} - 3$ such semigroups splitted in three different types.

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The compression technique for solving the word problem

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Keywords: word problem for groups and semigroups, automorphism groups, efficient algorithms, text compression.

Extended Abstract

Since it was introduced by Dehn in 1910, the *word problem* for groups has emerged to a fundamental computational problem linking group theory, topology, mathematical logic, and computer science. The word problem for a finitely generated group G asks, whether a given word over the generators of G represents the identity of G. Dehn proved the decidability of the word problem for surface groups. On the other hand, 50 years after the appearance of Dehn's work, Novikov and independently Boone proved the existence of a finitely presented group with undecidable word problem, see [5] for references. However, many natural classes of groups with decidable word problem are known, as for instance finitely generated linear groups, automatic groups and one-relator groups. With the rise of computational complexity theory, also the complexity of the word problem became an active research area.

In order to prove upper bounds on the complexity of the word problem for a group G, a "compressed" variant of the word problem for G was introduced in [3, 4, 6]. In the *compressed word problem* for G, the input word over the generators is not given explicitly but succinctly via a *straight-line program* (SLP for short). This is a context free grammar that generates exactly one word. Since the length of this word may grow exponentially with the size (number of productions) of the SLP, SLPs can be seen indeed as a succinct string representation. SLPs turned out to be a very flexible compressed representation of strings, which are well suited for studying algorithms for compressed data. In [4, 6] it was shown that the word problem for the automorphism group $\mathrm{Aut}(G)$ of G can be reduced in polynomial time to the *compressed* word problem for G. In [3], it was shown that the compressed word problem for a finitely generated free group F can be solved in polynomial time. Hence, the word problem for $\mathrm{Aut}(F)$ turned out to be solvable in polynomial time [6], which solved an open problem from [2].

In the talk, I will present an overview on the compression technique for solving the word problem. I will discuss generalizations of the above mentioned result for free groups to right-angled Artin groups, nilpotent groups, and fundamental groups of graphs of groups with finite edge groups, see [1, 4] for more details. Applications of the compression method to the word problem in certain semidirect products and outer automorphism groups will be discussed as well. Finally, I will briefly discuss some results on the compressed word problem in semigroups. Already for free inverse semigroups the compressed word problem becomes intractable; more precisely, it becomes complete for the complexity class Π_2^p — the second level of the polynomial time hierarchy (this class contains NP as well as coNP). This result shows the limitations of the compression technique for solving word problems of semigroups.

Some of the presented results were obtained in joint work with Niko Haubold and Saul Schleimer.

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Complete rewriting systems and homotopy bases in semigroup theory

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Keywords: Complete rewriting systems; finite derivation type, homotopy bases; large ideals; Rees matrix semigroups; semilattices of semigroups; univocal factorizable semigroups; subgroups of groups.

Extended Abstract

The study of finiteness conditions for mathematical objects is one the most important topics for the mathematical community. Among them are finite and complete (that is, noetherian and confluent) rewriting systems which are used to solve word problems among other algebraic decision problems. Unfortunately, the property of having a finite and complete rewriting system is not invariable under monoid presentations. In the 90's, Squier [7] defined a new combinatorial property of rewriting systems called *finite derivation type* (FDT) and showed that a monoid defined by a finite and complete rewriting system has FDT. Roughly speaking, this property is defined as follows: given a presentation $\mathscr{P} = \langle A \mid R \rangle$ there is an associated 2-complex $\mathscr{D}(\mathscr{P})$, called its Squier complex, on which the free monoid A^* acts both on the left and on the right; the FDT property for $\mathscr{P}$ consists of requiring the existence of a finite set $\mathbf{X}$, called an homotopy base, of closed paths on $\mathscr{D}(\mathscr{P})$ that trivializes it, meaning, that the new 2-complex obtained from adding to $\mathscr{D}(\mathscr{P})$ the set of closed paths $A^* \cdot \mathbf{X} \cdot A^*$ has trivial fundamental groups. This FDT property is an invariant property of finite semigroup presentations and so it becomes an intrinsic property of the semigroup defined by such presentation.

Pride, in [8], proposes a research program to develop the calculus of homotopy bases (spherical pictures) for monoids. To study a semigroup it is natural to decompose it in terms of others semigroups from which it is easier to obtain certain information. Following this idea, Pride suggests the development of the calculus of homotopy bases for various standard monoid constructions.

In this talk we make a survey on the topic particularly showing our research in an attempt of understanding how these properties are preserved under well known semigroup constructions. In the case of complete rewriting systems we have considered semigroup constructions as submonoids generated by codes and univocal factorizable semigroups. Concerning the FDT property we have studied large ideals of semigroups, Rees matrix semigroups, semilattices of semigroups and subgroups of semigroups.

Some of the present results were obtained in collaboration with R. Gray.

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On Maximal Subgroups of Free Idempotent Generated Semigroups

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(This is a joint work with Mark Brittenham and John Meakin)

Keywords: Biordered set, idempotent generated semigroup, two-complex.

Extended Abstract

Let S be a semigroup with set $E(S)$ of idempotents, and let $\langle E(S)\rangle$ denote the subsemigroup of S generated by $E(S)$. We say that S is an *idempotent generated* semigroup if $S = \langle E(S)\rangle$. Idempotent generated semigroups have received considerable attention in the literature. For example, an early result of J. A. Erdös [4] proves that the idempotent generated part of the semigroup of $n \times n$ matrices over a field consists of the identity matrix and all singular matrices. J. M. Howie [5] proved a similar result for the full transformation monoid on a finite set and also showed that every semigroup may be embedded in an idempotent generated semigroup. This result has been extended in many different ways, and many authors have studied the structure of idempotent generated semigroups. Recently, Putcha [11] gave necessary and sufficient conditions for a reductive linear algebraic monoid to have the property that every non-unit is a product of idempotents, significantly generalizing the results of J.A. Erdös mentioned above.

In 1979 K.S.S. Nambooripad [8] published an influential paper about the structure of (von Neumann) regular semigroups. Nambooripad observed that the set $E(S)$ of idempotents of a semigroup carries the structure of a "biordered set", or a "regular biordered set" in the case of regular semigroups. The biorder on $E(S)$ is obtained by restricting multiplication in S to pairs of idempotents $e, f \in E(S)$ such that either $ef = e$ or $ef = f$ or $fe = e$ or $fe = f$. Equivalently, multiplication is restricted to pairs of elements that are comparable in one of Green's relation $\mathscr{R}$ or $\mathscr{L}$. Nambooripad also provided an axiomatic characterization of regular biordered sets in his paper. Later, Easdown [3] showed that arbitrary (not necessarily regular) axiomatically defined biordered sets are also exactly the biordered sets of semigroups. Although the axioms for a biordered set are quite complicated, they do occur in mathematics. For example, Putcha [10] showed that if G is a reductive algebraic group, or a finite group of Lie type, then the set of pairs of opposite parabolic subgroups of G has a natural structure of a regular biordered set, where products are defined via the projections of Tits [12].

If E is a regular biordered set, then there is a free object, which we will denote by $RIG(E)$, in the category of regular idempotent generated semigroups with biordered set E. Nambooripad showed how to study $RIG(E)$ via an associated groupoid $\mathscr{N}(E)$. There is also a free object, which we will denote by $IG(E)$, in the category of idempotent generated semigroups with biordered set E for an arbitrary (not necessarily regular) biordered set E.

In this talk we provide a topological approach to Nambooripad's theory by associating a 2-complex $K(E)$ to each regular biordered set E. The fundamental groupoid of the 2-complex $K(E)$ is Nambooripad's groupoid $\mathscr{N}(E)$. Our concern in this talk is in analyzing the structure of the maximal subgroups of $IG(E)$ and $RIG(E)$ when E is a regular biordered set. This is enough to understand the global structure of $RIG(E)$, since the biorder E encodes the rest of the structure of this semigroup. Since every regular idempotent generated semigroup with biorder E is an idempotent separating image of $RIG(E)$, we have a tool to understand this important class of semigroups.

It has been conjectured that these subgroups are free [6], and indeed there are several papers in the literature (see for example, [9], [7], [6]) that prove that the maximal subgroups are free for certain classes of biordered sets. The main result of this paper is to use these topological tools to give the first examples of non-free maximal subgroups in free idempotent generated semigroups over a biordered set. One example is associated with a certain geometry over the field with 2 elements. The second studies biordered sets arising from full matrix monoids over fields and shows that the multiplicative group of a field (or in fact any division ring) is the maximal subgroup of a free idempotent generated semigroup. Some of the work reported upon in this paper has appeared very recently in [1] and the rest will appear in [2].

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A matrix construction for certain inverse semigroups with application in universal algebra

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(This is a joint work with Kalle Kaarli)

Keywords: Inverse monoid, factorizable inverse monoid, natural order, bi-congruence, categorical equivalence, arithmetical variety.

Extended Abstract

We provide an abstract characterization of inverse monoids that appear as monoids of bi-congruences of finite minimal algebras generating arithmetical varieties. As a tool, a matrix construction is introduced which might be of independent interest in inverse semigroup theory. Using this construction, we embed a certain kind of inverse monoids into a factorizable inverse monoid of the same kind.

More detailed: By using an unusual matrix construction, we prove:

Theorem. *For every finite E-reflexive complete distributive inverse monoid S (for these notions see [6]; 'distributive' means the finitary version of 'infinitely distributive' in [6]) there exist a factorizable finite E-reflexive complete distributive inverse monoid T and an idempotent e of T such that S is isomorphic to the subsemigroup* $\{x \in T \mid xe = x = ex\}$ *of T.*

Using earlier results of Kaarli [2], [3], this result allows to prove:

Theorem. *Given a finite inverse monoid S with zero element, the following are equivalent:*

(1) there exists a finite minimal algebra A generating an arithmetical variety such that S is isomorphic to the inverse monoid of bi-congruences of A,

(2) the monoid S is complete, E-reflexive, and distributive.

Explanation of the background:

This result, which closes the investigation started in [2] and [3], is motivated by the classification problem of arithmetical affine complete varieties of finite type. All the necessary information about such varieties can be found in [4]. Following the terminology introduced in [5], an algebra is called *minimal* if it has no proper subalgebras. It is known that an affine complete variety is locally finite iff it is generated by a finite minimal algebra iff it is term equivalent to a variety of finite type. On the other hand, every finite minimal algebra of an arithmetical variety generates an affine complete variety.

Algebras A and B are called *categorically equivalent* if there is an equivalence between the varieties Var A and Var B considered as categories under which the algebras A and B correspond to each other. It was proved in [3], using more general results of C. Bergman [1], that finite minimal algebras generating arithmetical varieties are categorically equivalent iff their monoids of bi-congruences are isomorphic. This raised a natural problem: describe inverse monoids that appear in such a situation. In [3] the problem was solved in the important special case when the algebra A is so-called *weakly diagonal*, which corresponds to the inverse monoid of bi-congruences being factorizable. Here we present a complete solution of the problem.

Given a set A and subsets $X, Y \subseteq A \times A$, the relational product of X and Y is denoted by $X \circ Y$. The converse of X is denoted by $X^{\smile}$. The symbols Δ_A and ∇_A denote the diagonal of A (the equality relation on A) and $A \times A$, respectively. A subset $X \subseteq A \times A$ is called a *bi-equivalence* of A if it satisfies the following conditions:

$$\Delta_A \subseteq X^{\smile} \circ X, \ \Delta_A \subseteq X \circ X^{\smile}, \ X = X \circ X^{\smile} \circ X.$$

A *bi-congruence* of an algebra A is a subalgebra of $A \times A$ which is also a bi-equivalence on A. It is known (see [6]) that the set of all bi-congruences of a congruence permutable algebra is an inverse monoid with respect to the operation of relational product, with identity Δ_A and zero element ∇_A. Moreover, if A is a minimal congruence permutable algebra then all subalgebras of $A \times A$ are bi-congruences of A.

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Limit algebras

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Keywords: Limit groups, discriminating, universal theories, Gromov-Hausdorff-Grigorhuck topology, hyperbolic groups, free solvable groups.

Extended Abstract

Fully residually free (limit) groups play an important part in the recent solution of Tarski's problems about free groups. What are limits of other groups? Why are they interesting? What common properties do they have? These are the main questions I am going to discuss in the talk. I will touch on the general theory of limit algebras (groups, semigroups and rings) and discuss some particular examples. The classical examples are limits of the fields $\mathbb{C}, \mathbb{R}, \mathbb{Q}_p$ and the ring $\mathbb{Z}$ - these are well-known, but from a different view-point. Examples of another type are limits of free and torsion-free hyperbolic groups, an interesting theory is emerging here (see, for example, [1, 3, 2] for basic results). The last class of groups, termed rigid solvable groups, that I would like to discuss in the talk are related to limits of free solvable groups (see [4]). Rigid solvable groups have many amazing properties that are lacking in solvable groups in general (Malcev's type completions, robust theory of Krull dimension, good algorithmic properties). At the end of the talk, if time permits, I will discuss some open problems concerning limits of groups and semigroups.

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The word problem for ω-terms over DA

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Keywords: finite semigroups; pseudovariety DA; word problem; omega-term.

Extended Abstract

The pseudovariety DA, the class of finite semigroups whose regular $\mathscr{D}$-classes are aperiodic semigroups, has been the subject of recent studies due to its various applications. It is known that languages whose syntactic monoid lies in DA have important algebraic, combinatorial, automata-theoretical and logical caracterizations that enable us to solve problems in computational and complexity theory (see Tesson and Thérien [4]).

In the 1960's, Krohn and Rhodes showed that any finite semigroup divides a wreath product whose factors are, alternately, finite aperiodic semigroups and finite simple groups. To determine the Krohn-Rhodes group complexity of a finite semigroup became one of the most important problems in finite semigroup theory. Several results obtained in this theory were motivated by this problem.

In an attempt to establish the decidability of semidirect products of pseudovarieties of semigroups, Almeida and Steinberg [2] introduced the notion of tameness, property that they thought might lead to a substantial reduction of the Krohn-Rhodes problem. To prove the ω-tameness of a pseudovariety we have to solve two problems: the ω-word problem and to prove that, if a system of equations with rational constraints has a solution in any semigroup of the pseudovariety, then it has a uniform solution given by suitable ω-terms satisfying the same constraints.

In this talk, we solve the word problem for ω-terms over DA. For that purpose, using the central basic factorization of an implicit operation on DA (see Almeida [1]) and extending to DA some techniques developed by Almeida and Zeitoun [3], we obtained a representation of the implicit operations on DA by trees and by automata. We proved that two implicit operations are equal over DA if and only if they have the same DA-tree. A similar result holds for the associated DA-automata. Considering certain types of factors of an implicit operation on DA, we can prove that a pseudoword on DA is an ω-term if and only if the associated minimal DA-automaton is finite. Finally, we complete the result by effectively computing in polynomial-time the minimal DA-automaton associated to an ω-term.

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Synchronizing Groups

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Keywords: Finite permutation groups; finite transformation semigroups; finite automata; reset words; synchronizing semigroups; synchronizing groups.

Extended Abstract

Synchronizing groups are certain finite permutation groups recently identified by Benjamin Steinberg and by João Araújo as being of considerable interest for their application to an old problem in automata theory (see [1] and [2]). Their study quickly reduces to a problem about primitive permutation groups, and the first steps in an analysis of the relevant groups is to be found in [4]. Since January 2008 a number of mathematicians in various parts of the world have been collaborating by email to study them, and a rich theory, with connections to graph theory (see [3]), finite geometry, representation theory of finite groups, and other parts of mathematics, is emerging [5]. This lecture is conceived as an introduction to and survey of the subject.

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Ends lemma based semihypergroups of certain transformation operators

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Keywords: ordered semigroup, quasi-ordered semigroup, semihypergroup, quasi-automaton, multiautomaton.

Extended Abstract

Chvalina in his book [3] gave the following construction (now called *Ends lemma*) of hyperstructures from ordered structures. (The notation $\mathscr{P}'(S)$ stands for the set of all non-empty subsets of S, for concepts of hyperstructure theory cf. [1].)

Ends lemma. [Theorem 1.3, p. 146] *Let* $(S, \cdot, \leq)$ *be an ordered semigroup. Binary hyperoperation* $* : S \times S \to \mathscr{P}'(S)$ *defined by*

$$a * b = [a \cdot b)_{\leq} = \{x \in S; a \cdot b \leq x\}$$

is associative. The hyperoperation $*$ *is commutative if and only if* $(S, \cdot)$ *is a commutative semigroup.*

This lemma can be extended in two directions. First, the semigroup need not be an ordered one as the lemma equally holds if the relation $\leq$ is quasi-ordering. The second extension studies hyperstructures based on ordered (or quasi-ordered) groups.

Even though a similar idea (in order to study relations) was touched upon by Vougioklis in [8], the *Ends lemma* has been used mainly by Czech authors such as Hort, Hošková, Chvalina, Chvalinová, Moučka, Novák, Račková and others. The impact of this construction is extensive – examples range from the study of differential equations (e.g. [6]) or matrix theory (e.g [7]) to applications of preference relations in microeconomics (joint works by Chvalina, Moučka, Novák).

In [5] there were introduced and studied transformation operators $T(\lambda, F, \varphi) : \mathbb{C}^{\Omega} \to \mathbb{C}^{\Omega}$, where $\lambda \in \mathbb{C}$ and functions $F, \varphi \in \mathbb{C}^{\Omega}$, defined by

$$T(\lambda, F, \varphi)(f(z)) = \lambda F(z) f(z) + \varphi(z)$$

for any function $f \in \mathbb{C}^{\Omega}$ and any $z \in \mathbb{C}$. Notice that $\mathbb{C}^{\Omega} = \{f : \Omega \to \mathbb{C}\}$, where $\emptyset \neq \Omega \subseteq \mathbb{C}$. Some properties of certain semigroups and *Ends lemma* based semihypergroups constructed on the set of all such transformation operators have been studied in [5] and some other works by Chvalina, Moučka and Novák.

In the talk there will be included constructions of semihypergroups which will apply the *Ends lemma* on the set of the above defined transformation operators. Some properties of these semihypergroups will be studied – especially their identities, i.e. neutral elements, but also other properties defined in [1]. The talk will also deal with the study of a certain hyperstructure generalization of the concept of a quasi-automaton (cf. [2], p. 11) called a multiautomaton, which has already been used in this respect e.g. in [4] (albeit applied on a different hyperstructure) – an example of a multiautomaton with the input semihypergroup constructed in the above described way will be included.

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Algebras, groups and monoids determined by set theoretic solutions of the Yang-Baxter equation

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Keywords: Yang-Baxter equation, set theoretic solution, finitely presented monoid, permutation group, torsion free abelian by finite group, Noetherian PI-algebra.

Extended Abstract

Abstract. Set theoretic solutions of the Yang-Baxter equation lead to fascinating classes of finitely presented monoids, groups and associative algebras over a field. All of them are determined by presentations of the form $\langle x_1, \dots, x_n | R \rangle$, where R is a set of $n(n-1)/2$ relations, each of the form $x_i x_j = x_k x_m$. On the other hand, these monoids and groups can be characterized as special submonoids of the semidirect product of the free abelian group Fa_n of rank n and the symmetric group Sym_n, with the natural action. The purpose of the talk is to present the main known results on such algebras, groups and monoids, exhibiting both their structural and combinatorial properties. These results, as well as some of the recent research activity in this area and some background can be found in the references provided below. This includes the role of the Yang-Baxter equation as one of the basic equations in mathematical physics that lies in the foundations of the theory of quantum groups. Recent solutions of some of the fundamental problems concerning set theoretic solutions, obtained in a joint work with F.Cedo and E.Jespers, as well as other remaining open problems, will be also presented.

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Actions of maximal growth

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Extended Abstract

This is a joint work with Yu. A.Bahturin [1] . We study acts and modules of maximal growth over finitely generated free monoids and free associative algebras as well as free groups and free group algebras. The maximality of the growth implies some other specific properties of these acts and modules that makes them close to the free ones; at the same time, we show that being a strong "infiniteness" condition, the maximality of the growth can still be combined with various finiteness conditions, which would normally make finitely generated acts finite and finitely generated modules finite-dimensional.

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Logically perfect and homogeneous algebras

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Keywords: Logically perfect algebras, homogeneous algebras.

Abstract

We study algebras in an arbitrary fixed variety of algebras Θ. The notion of homogeneouty of groups was introduced by Ph. Hall, while for the models it goes back to A. Tarski. We use this notion in the Universal algebraic geometry (UAG), relating it with another important notion of logically perfect algebra. We consider conditions when these notions coincide and when they are different.

An algebra H is called *homogeneous*, if for any isomorphism $\eta : A \to B$ of its finitely generated subalgebras this isomorphism is induced by some automorphism of the algebra H.

The definition of logical perfectness is associated with a logically-geometrical approach in UAG. The same approach gives rise to the notion of a logically separable algebra, i.e., an algebra which can be separated from all other algebras from Θ in the logic of types. Recall that the notion of type is one of the central notions in the model theory.

We consider also logically noetherian algebras. We also give examples how the general notions defined above work in the case of semigroups. The details can be found in [1].

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Decompositions and complexity of linear automata

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(This is a joint work with Boris Plotkin)

Keywords: linear automaton, semigroup, triangular product, wreath product, complexity.

Extended Abstract

The main goal of the talk is to introduce the notion of linear automaton complexity. Let K be a commutative ring with a unit, $Mod-K$ a category of modules over K.

Definition 0.1 *A linear automaton in $Mod-K$ is a three-sorted algebraic structure, a triple (A, Γ, B), where A and B are the modules from $Mod-K$, and Γ is a semigroup which acts in A and B and from A to B. The arising operations of actions $\circ$, $\cdot$ and $*$ should satisfy the conditions*

$$a \circ \gamma_1\gamma_2 = (a \circ \gamma_1) \circ \gamma_2; \qquad b \cdot \gamma_1\gamma_2 = (b \cdot \gamma_1) \cdot \gamma_2;$$

$$a * \gamma_1\gamma_2 = (a \circ \gamma_1) * \gamma_2 + (a * \gamma_1) \cdot \gamma_2, \qquad a \in A, b \in B, \gamma_1, \gamma_2 \in \Gamma.$$

Recall that pure semigroup automaton is a three-sorted algebraic structure of the form (A, Γ, B), where Γ is a semigroup, A, B are the sets, and the axioms $a \circ \gamma_1\gamma_2 = (a \circ \gamma_1) \circ \gamma_2$, $a * \gamma_1\gamma_2 = (a \circ \gamma_1) * \gamma_2$ are fulfilled. The complexity theory of such automata is well known (see, for example [2], [5]). The basis of this theory constitutes the famous Krohn-Rhodes decomposition theory of pure (semigroup) automata (see [3],[4], [1], etc). Suppose we have a (finite) semigroup automaton $\mathscr{A}$. The Krohn-Rhodes theory basically says that any semigroup automaton $\mathscr{A}$ can be built up by cascading simple group automata, which divide $\mathscr{A}$, with certain trivial automata, the so-called "flips-flops". We would prefer to say that the Krohn-Rhodes Theorem allows to decompose any finite semigroup automaton into indecomposable (irreducible) bricks, via the construction of cascade connection of automata. The cascade connection of semigroup automata $\mathscr{A}_1$ and $\mathscr{A}_2$ is tightly related to wreath product of automata. It can be seen that every cascade connection of the automata is embedded into their wreath product. The wreath product construction leads to the decomposition of pure automata and to the definition of Krohn-Rhodes complexity (KR-complexity) of the automaton.

The decomposition theory and, correspondingly, the complexity theory for linear automata require additional constructions.We use the following operations (see [6],[7] for definitions):

1. Triangular product of linear automata.
2. Wreath product of a linear automaton with a pure one.
3. Wreath product of pure automata.

Keeping in mind an arbitrary finite semigroup Γ, we use a compressing operation of an irreducible automaton (A,Γ) to an irreducible automaton (A,Σ) with 0-simple semigroup Σ. The following decomposition theorem gives the way to define the complexity of a linear automaton

Theorem 0.1 *An arbitrary finitely dimension linear automaton $\mathscr{A}$ with a finite 0-simple semigroup Γ can be decomposed into indecomposable automata using the operations 1–3.*

Definition 0.2 *Define the complexity of a linear automaton $\mathscr{A}$ to be the minimal number of the operations 1—3 used in the decomposition of the automaton and the semigroup into indecomposable components.*

Let us outline the general decomposition strategy. A given linear automaton $\mathscr{A} = (A,\Gamma,B)$ with a finite semigroup Γ and finitely dimension A and B can be decomposed into indecomposable with respect to the triangular product semi-automata of the form (A,Γ). The number of such automata is determined by the lengthes of composition series in respect to Γ in A and B. It is the sum of such lengthes, say, n. The corresponding number of operations ∇ is $n-1$.

Now apply the compressing operation to every irreducible representation (A,Γ) and pass to (B,Σ) with 0-simple Σ. We add n operations and the total number of operations will be $n-1+n=2n-1$. For every such Σ take the corresponding group G (arising from the matrix representation $(X,G,[X,Y],Y)$ of Σ) and a corresponding set $A_1 \subset A$. Then, irreducible (A_1,G) is a divisor of (B,Σ). We have $(A,\Gamma)|(A_1,G)\ wr\ (Y,Y^r)$. We add $2n-1$ wreath products, which sums up to $4n-2$ operations.

Further we decompose these irreducible (sic!) representations (A,G) by taking composition series $G \supset H_1 \supset \ldots \supset H_{k-1} \supset 1$ in G. All factors here are simple groups. We have $(A,G)|(A,H_{k-1})wr(X,\Phi)$ where (X,Φ) is $k-1$ pure automata – wreath products of the form (X_i,G_i) with all G_i being simple groups. So, for each of $2n-1$ representations there are added corresponding $k-1$ pure indecomposable wreath products.The total number of the used operations is the complexity of $\mathscr{A}$.

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Operators on varieties of monoids related to polynomial operators on classes of regular languages

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(This is a joint work with Ondřej Klíma)

Keywords: Positive varieties of languages, polynomial operator, varieties of ordered monoids.

Extended Abstract

One can assign to each positive variety $\mathcal{V}$ of regular languages and a fixed natural number k the class of all (positive) boolean combinations of the restricted polynomials, i.e. the languages of the form $L_0 a_1 L_1 a_2 \dots a_\ell L_\ell$, where $\ell \leq k$, $a_1, \dots, a_\ell$ are letters and $L_0, \dots, L_\ell$ are languages from the variety $\mathcal{V}$. The resulting classes $\mathsf{PPol}_k\mathcal{V}$ and $\mathsf{BPol}_k\mathcal{V}$ are positive and boolean varieties, respectively.

In the case that the pseudovariety $\mathbf{V}$ of finite ordered monoids corresponding to $\mathcal{V}$ consists of finite members of a variety $\mathsf{Var}\,\mathcal{V}$ of ordered monoids (that is, $\mathsf{Var}\,\mathcal{V} = \mathsf{HSP}\,\mathbf{V}$ and $\mathbf{V} = \mathsf{Fin}(\mathsf{Var}\,\mathcal{V})$) which is locally finite, we can effectively describe the fully invariant compatible quasiorder on X^* (where $X = \{x_1, x_2, \dots\}$ are variables) for $\mathsf{Var}(\mathsf{PPol}_k\mathcal{V})$ and $\mathsf{Var}(\mathsf{BPol}_k\mathcal{V})$ in terms of that for $\mathbf{V}$.

The paper [2] deals with the operator on the class $\mathcal{V}_0$ of languages where $\mathcal{V}_0(A) = \{\emptyset, A^*\}$ for each finite alphabet A. They form two natural hierarchies within piecewise testable languages and the boolean case has been studied in papers by Simon [5], Blanchet-Sadri [1], Volkov [6] and others. The main issues were the existence of finite bases of identities for the corresponding pseudovarieties of monoids and generating monoids for these pseudovarieties. In [2] we studied mainly the positive case and even finite unions of $A^* a_1 A^* a_2 A^* \dots A^* a_\ell A^*$, where $a_1, \dots, a_\ell \in A$, $\ell \leq k$.

In [3] we rather considered the operators on classes of languages and the corresponding operator on classes of ordered monoids in a general case. We characterized in various terms varieties which are generated by a finite number of languages.

In [4] we study four hierarchies of languages which result by applying the restricted positive or boolean polynomial operator to the positive varieties where the class $\mathcal{V}(A)$ equals to finite unions of B^*, $B \subseteq A$ or $\mathcal{V}(A)$ equals to finite unions of $\overline{B}$, $B \subseteq A$ where $\overline{B}$ is the set of all words over A containing exactly the letters from B. Our basic questions are to explore all the inclusions among our varieties and we start to discuss the existence of finite bases for corresponding pseudovarieties of (ordered) monoids. Hopefully our results bring a bit more light into the complexity of the structure of (positive) subvarieties of the second level of the Straubing-Thérien hierarchy.

In our talk we are going to concentrate on the monoid side, we will mainly discuss the identities for our varieties, relatively free monoids. and a possibility of a generating by a single (ordered) monoid.

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Bilateral semidirect product decompositions of transformation monoids

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(This is a joint work with Vítor H. Fernandes)

Keywords: bilateral semidirect products, transformation semigroups, free monoids, presentations.

Extended Abstract

Let S and T be two semigroups. Let

$$\begin{array}{rcccccc} \delta: & T & \longrightarrow & \mathcal{T}(S) & & & \\ & u & \longmapsto & \delta_u: & S & \longrightarrow & S \\ & & & & s & \longmapsto & u\cdot s \end{array} \qquad \text{and} \qquad \begin{array}{rcccccc} \varphi: & S & \rightarrow & \mathcal{T}(T) & & & \\ & s & \longmapsto & \varphi_s: & T & \longrightarrow & T \\ & & & & u & \longmapsto & u^s \end{array}$$

be an anti-homomorphism of semigroups and a homomorphism of semigroups, respectively, such that

(SPR) $(uv)^s = u^{v\cdot s}v^s$, for $s \in S$ and $u, v \in T$ (*Sequential Processing Rule*) and

(SCR) $u\cdot(sr) = (u\cdot s)(u^s\cdot r)$, for $s, r \in S$ and $u \in T$ (*Serial Composition Rule*).

Within these conditions, the set $S \times T$ is a semigroup with respect to the multiplication

$$(s,u)(r,v) = (s(u\cdot r), u^r v),$$

for $s, r \in S$ and $u, v \in T$. We denote this semigroup by $S \circledK T$ and call it the *(Kunze) bilateral semidirect product* of S and T (associated with δ and φ). If S and T are monoids and δ and φ are *monoidal* (i.e. $u\cdot 1 = 1$, for $u \in T$, and $1^s = 1$, for $s \in S$) and preserve the identity, then $S \circledK T$ is a monoid with identity $(1,1)$.

This notion was introduced and studied by Kunze in [1] and was strongly motivated by automata theoretic ideas (see [2, 3]). In [4] Kunze proved that the full transformation semigroup on a finite set X is a quotient of a bilateral semidirect product of the symmetric group on X and the semigroup of all order preserving full transformations on X, for some linear order on X. On the other hand, in the same paper, Kunze showed that the semigroup of all order preserving full transformations on a finite chain is a quotient of a bilateral semidirect product of two of its remarkable subsemigroups.

In this talk we will also present decompositions of certain monoids of transformations by means of bilateral semidirect products and quotients [5]. Our strategy to construct bilateral semidirect product decompositions is quite different from Kunze techniques. In fact, we first develop a general method which consists in the construction of a bilateral semidirect product of two free monoids that, under certain conditions, induces a bilateral semidirect product of two monoids defined by presentations associated to these free monoids. After, we apply this method to some monoids of transformations that preserve or reverse the order or the orientation on a finite chain. In particular, we give a simpler, shorter and transparent proof of Kunze's result [4] on the semigroup of all order preserving full transformations on a finite chain.

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Geometric Semigroup Theory with Applications to Upper Bounds for Complexity

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(This is a joint work with Jon McCammond and Benjamin Steinberg)

Keywords: Geometric Semigroup Theory, Complexity of Finite Semigroups.

Extended Abstract

I am reporting on joint work with Jon McCammond and Benjamin Steinberg. I will talk about results in Geometric Semigroup Theory, which uses the geometric and topological properties of the associated automata to study finitely-generated semigroups. This theory includes expansions of finite semigroups preserving finiteness, leading to properties closely related to the Burnside-McCammond Automata.

I hope to make some remarks at the end of the talk on how Geometric Semigroup Theory yields upper bounds to Complexity.

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Modular numerical semigroups with embedding dimension equal to three

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(This is a joint work with J.C. Rosales)

Keywords: Proportionally modular numerical semigroups, modular numerical semigroups, Diophantine inequalities.

Extended Abstract

Let $\mathbb{N}$ be the set of nonnegative integer numbers. A *numerical semigroup* is a subset S of $\mathbb{N}$ such that it is closed under addition, $0 \in S$ and $\mathbb{N} \setminus S$ is finite. If $A \subseteq \mathbb{N}$, we denote by $\langle A \rangle$ the submonoid of $(\mathbb{N},+)$ generated by A, this is,

$$\langle A \rangle = \{\lambda_1 a_1 + \ldots + \lambda_n a_n \mid n \in \mathbb{N} \setminus \{0\}, a_1, \ldots, a_n \in A \text{ and } \lambda_1, \ldots, \lambda_n \in \mathbb{N}\}.$$

It is well known (see [2]) that $\langle A \rangle$ is a numerical semigroup if and only if $\gcd\{A\} = 1$, where gcd means *greatest common divisor*.

Let S be a numerical semigroup and let X be a subset of S. We say that X is a *system of generators* of S if $S = \langle X \rangle$. In addition, if no proper subset of X generates S, then we say that X is a *minimal system of generators* of S. Every numerical semigroup admits a unique minimal system of generators and, moreover, such system has finitely many elements (see [1, 2]). The cardinal of this system is known as the *embedding dimension* of S and it is denoted by $\mathrm{e}(S)$. On the other hand, if $X = \{n_1 < n_2 < \ldots < n_e\}$ is a minimal system of generators of S, then n_1, n_2 are known as the *multiplicity* and the *ratio* of S.

Let m, n be integers such that $n \neq 0$. We denote by $m \bmod n$ the remainder of the division of m by n. Following the notation of [3], we say that a *proportionally modular Diophantine inequality* is an expression of the form

$$ax \bmod b \leq cx \tag{1}$$

where a, b, c are positive integers. We call $a, b,$ and c the *factor*, the *modulus*, and the *proportion* of the inequality, respectively. Let $\mathrm{S}(a,b,c)$ be the set of integer solutions of (1). Then $\mathrm{S}(a,b,c)$ is a numerical semigroup (see [3]) that we call *proportionally modular numerical semigroup* (PM-*semigroup*).

As a consequence of [5, Theorem 31] (see its proof and [5, Corollary 18]) we have an easy characterization for PM-semigroups: a numerical semigroup S is a PM-semigroup if and only if there exists a convex arrangement $n_1, n_2, \ldots, n_e$ of its set of minimal generators that satisfies the following conditions

1. $\gcd\{n_i, n_{i+1}\} = 1$ for all $i \in \{1, \ldots, e-1\}$.
2. $(n_{i-1} + n_{i+1}) \equiv 0 \bmod n_i$ for all $i \in \{2, \ldots, e-1\}$.

A *modular Diophantine inequality* (see [4]) is an expression of the form

$$ax \bmod b \leq x, \tag{2}$$

this is, it is a proportionally modular Diophantine inequality with proportion equal to one. A numerical semigroup is a *modular numerical semigroup* (M-*semigroup*) if it is the set of integer solutions of a modular Diophantine inequality. Therefore, every M-semigroup is a PM-semigroup, but the reciprocal is false. In effect, from [3, Example 26], we have that the numerical semigroup $\langle 3,8,10\rangle$ is a PM-semigroup, but is not an M-semigroup.

Let us observe that it is easy to determine whether or not a numerical semigroup is a PM-semigroup via the previous characterization. However, this question is more complicated for M-semigroups. In [4] there is an algorithm to give the answer to this problem, but we have not got a good characterization for M-semigroups.

As a first step to give an answer to this question, our aim is to show the explicit descriptions of all M-semigroups with embedding dimension equal to three. In order to do it, we consider two ideas. First, the relation between proportionally modular numerical semigroups and numerical semigroups associated with an interval (see [3]). Secondly, the description of a proportionally modular numerical semigroup with embedding dimension equal to three when we fix the multiplicity and the ratio.

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Rewriting Generators for Semigroups

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Keywords: Subsemigroup, subgroup, generators, index, presentation, automatic structure.

Extended Abstract

Suppose we are interested in a property of semigroups which has to do with generators. Examples of such properties would be: finite generation, finite presentability, automaticity, existence of complete rewriting systems, various homological finiteness conditions, etc. Suppose further we are interested as to under which circumstances this property is preserved under taking subsemigroups. The first step in such an analysis is to take a semigroup S with a generating set A, and a subsemigroup T of S, and try to find a 'nice' generating set B for T. Proving that B generates T usually consists in describing a 'process' which takes a word over A which happens to represent an element of T and 'rewrites' it into a word over B representing the same element. The nicer this 'rewriting process' the easier the subsequent analysis of various properties is.

A classical example of this situation is Schreier's Theorem for groups, giving a generating set for a subgroup in terms of a generating set for the group and coset representatives. As more or less immediate corollaries one is then able to prove, for a group G and a subgroup of finite index H results of the following type: G is finitely generated if and only if H is finitely generated; G is finitely presented if and only if H is finitely presented; G is automatic if and only if H is automatic; etc.

In my talk, I will start by describing Schreier's rewriting process, in it straightforward modification for subgroups of semigroups. An interesting variation of this is a rewriting process for Schützenberger groups. I will also describe a rewriting process for subsemigroups with finite Rees index (i.e. finite complement). We will observe that the method here is conceptually the same, but differs greatly from the subgroup case in technical detail.

Then I will present some new work (joint with Robert Gray) concerning the subsemigroups of finite Green index. This notion builds on relative Green's equivalences modulo a subsemigroup, and generalises the notions of finite group index and Rees index. I will describe the rewriting process peculiar to this index. It, of necessity, displays facets in common with the Schreier rewriting, and with the Rees index rewriting. But it also displays a curious new property: it rewrites a word twice over!

Throughout the talk I will be linking the examples of rewriting mappings with the 'higher level' corollaries they lead to, concerning finite generation, presentability, automaticity, etc. In particular, I will discuss the difficulties that the above mentioned behaviour of the Green index rewriting causes in attempts to treat finite presentability.

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Fixed points of endomorphisms over special confluent rewriting systems

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Keywords: fixed points, special rewriting systems, monomorphisms, free products of cyclic groups.

Extended Abstract

Gersten proved in the eighties that the fixed point subgroup of a free group automorphism φ is finitely generated [7]. Using a different approach, Cooper gave an alternative proof, proving also that the fixed points of the continuous extension of φ to the boundary of the free group is in some sense finitely generated [5]. Gersten's result was generalized to further classes of groups and endomorphisms in subsequent years. Goldstein and Turner extended it to monomorphisms of free groups [8], and later to arbitrary endomorphisms [9]. Collins and Turner extended it to automorphisms of free products of freely indecomposable groups [4], and recently Sykiotis to monomorphisms [13]. Gaboriau, Jaeger, Levitt and Lustig remarked in [6] that some of the results on infinite fixed points would hold for virtually free groups with some adaptations. The proofs of some of these results required the sophisticated theory of train tracks, developed by Bestvina and Handel in the context of algebraic geometry to bound the rank of fixed point subgroups of free group automorphisms [1].

On the other hand, Cassaigne and the author developed in [3] an approach to the study of monoids defined by special confluent rewriting systems that preserves some of the features of the free group case and includes free products of cyclic groups as a particular case, as well as the partially reversible monoids introduced in [11]. In fact, the undirected Cayley graph of these monoids is hyperbolic and has a compact completion for the prefix metric. Uniformly continuous endomorphisms, algorithmically characterized in [3], admit a continuous extension to the boundary. In [2], the same authors used this approach to study the dynamics of infinite periodic points for two classes of endomorphisms of the monoids in question.

Quite recently [12], the author pursued this same approach to prove finite generation properties for both finite and infinite fixed points. This is achieved through a combination of automata-theoretic, combinatorial and topological techniques. Two classes of endomorphisms are studied: boundary-injective endomorphisms and endomorphisms with bounded length decrease.

The second class provides constructibility results that are reminiscent of those of Maslakova for the finite fixed points of free group automorphisms [10]. Moreover, both classes are recursive and algorithms to test the corresponding properties are provided.

If we restrict our attention to the group case, the first class provides new proofs for already known results for monomorphisms of free groups and more generally free products of cyclic groups, as well as a new result concerning the infinite fixed points for monomorphisms.

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Semigroup representation theory and the Černý conjecture

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Keywords: representation theory, Černý conjecture.

Extended Abstract

Representation theory has long been an important aspect of finite group theory, but for the most part has not had much impact on finite semigroup theory. We discuss a representation theoretic approach to the Černý conjecture on synchronizing automata and its generalization by Pin. Some of this work is joint with Jorge Almeida.

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Almost factorizable locally inverse semigroups

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Keywords: locally inverse semigroup, almost factorizability, Pastijn product.

Extended Abstract

A factorizable inverse monoid can be identified, up to isomorphism, by an inverse submonoid M of a symmetric inverse monoid $I(X)$ where each element of M is a restriction of a permutation of X belonging to M. So factorizable inverse monoids are natural objects, and appear in a number of branches of mathematics.

The notion of an almost factorizable inverse semigroup was introduced by Lawson [6] as the semigroup analogue of a factorizable inverse monoid. Among others, he established (see also McAlister [7] where the main ideas and some of the results were implicit) that

1. every almost factorizable inverse semigroup is obtained from a factorizable inverse monoid by deleting its group of units, and, consequently, each inverse semigroup is embeddable in an almost factorizable inverse semigroup;

2. the almost factorizable inverse semigroups are just the homomorphic images (or, equivalently, the idempotent separating homomorphic images) of semidirect products of semilattices by groups.

These results have been generalized in several directions: for straight locally inverse semigroups by Dombi [1], for orthodox semigroups by Hartmann [4, 5] and for right adequate and for weakly ample semigroups by El Qallali [2], and by Gomes and the speaker [3], respectively.

Here we introduce a notion of almost factorizability for the whole class of locally inverse semigroups, and generalize the results mentioned above.

The idea of addressing ourselves to this topic stemmed from a recent result by Pastijn and Oliveira [8] which provided an analogue of the monoid of permissible subsets of an inverse semigroup for any locally inverse semigroup. However, while having sought for an appropriate notion of an almost factorizable locally inverse semigroup, we wanted to obtain a natural class with the above properties containing all Pastijn products of normal bands by completely simple semigroups rather than to mimic the role of the monoid of permissible subsets in the definition of an almost factorizable inverse semigroups.

Let S be a locally inverse semigroup. Consider the semigroup $\mathscr{O}(S)$ which consists of all order ideals of S, and the multiplication is the forming of the usual set product. We say that S is *almost factorizable* if there exists a completely simple subsemigroup $\mathscr{U}$ in $\mathscr{O}(S)$ such that the following conditions are satisfied:

1. $\bigcup E(\mathscr{U}) = E(S)$,

2. for every $a \in S$, there exists $H \in \mathscr{U}$ with $a \in H$.

In particular, an inverse semigroup is almost factorizable in this sense if and only if it is in the usual sense. Furthermore, a generalized inverse semigroup is almost factorizable if and only if its greatest inverse semigroup homomorphic image is almost factorizable. Finally, a straight locally inverse semigroup which is almost factorizable in the sense of [1], is almost factorizable also in our sense.

A homomorphism $\phi\colon S \to T$ between locally inverse semigroups is called *locally idempotent separating* if the congruence induced by ϕ on S has the property that each class contaning an idempotent element is a completely simple subsemigroup of S.

Theorem 1 *For any locally inverse semigroup S, the following statements are equivalent:*

1. *S is almost factorizable,*
2. *S is a homomorphic image (or, equivalently, a locally idempotent separating homomorphic image) of a Pastijn product of a normal band by a completely simple semigroup.*

Theorem 2 *Each locally inverse semigroup is embeddable into an almost factorizable locally inverse semigroup.*

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The finite basis problem for finite semigroups revisited

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Keywords: finite semigroup, semigroup variety, finite identity basis, Tarski's problem, inherently non-finitely based semigroup, unary semigroup, completely 0-simple semigroup.

Extended Abstract

At the International Conference on Semigroups held in Braga in 1999, I presented a survey on the finite basis problem (FBP) for finite semigroups [1], see also [2] for an expanded journal version. In this talk I would like to overview the developments in the area over the last decade.

To start with, I shall report on the current status of 19 problems formulated in [2]. Some progress has been achieved in 8 cases; in particular, the FBP has been solved for all but one concrete families of finite semigroups that were suggested for investigation in [2]. The corresponding results are due to Goldberg, Jackson, Kaďourek, Lee, Olga Sapir and myself.

I shall briefly discuss a new approach to the FBP based on its relation to the complexity of the finite membership problem for finitely generated semigroup varieties. This interesting approach has been developed by Jackson and McKenzie.

I shall also describe the state-of-art in the FBP for finite unary semigroups (that is, semigroups endowed with an additional unary operation). There have been considerable advances in this direction; for instance, the FBP has been solved for many important unary semigroups such as the semigroup of all $n \times n$-matrices ($n \geq 2$) over a finite field endowed with transposition and various partition semigroups endowed with their natural involution. The results here are due to Auinger, Dolinka, Jackson, Kaďourek and myself; many of them have not been published yet.

Recent studies of the class of so-called Rees–Sushkevich varieties by Kublanovsky, Lee, Reilly and myself suggest that any further progress in the FBP may crucially depend on a better understanding of the restriction of the problem to the class of completely 0-simple semigroups. In conclusion, if time permits, I shall outline this direction of attack.

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On idempotent dimonoids

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Keywords: idempotent dimonoid, semilattice of rectangular dimonoids, rectangular dimonoid.

Extended Abstract

Jean-Louis Loday [2] introduced the notion of a dimonoid. A set D equipped with two associative operations $\prec$ and $\succ$ satisfying the following axioms:

$$(x \prec y) \prec z = x \prec (y \succ z),$$
$$(x \succ y) \prec z = x \succ (y \prec z),$$
$$(x \prec y) \succ z = x \succ (y \succ z),$$

for all $x, y, z \in D$, is called a dimonoid. If the operations $\prec$ and $\succ$ coincide, then the dimonoid becomes a semigroup.

We introduce the notion of a diband of dimonoids. A dimonoid $(D, \prec, \succ)$ is called an idempotent dimonoid or a diband if $x \prec x = x = x \succ x$ for all $x \in D$. Let J be some idempotent dimonoid. We call a dimonoid $(D, \prec, \succ)$ a diband of subdimonoids D_i $(i \in J)$ if $D = \bigcup_{i\in J} D_i$, $D_i \bigcap D_j = \varnothing$ for $i \neq j$ and $D_i \prec D_j \subseteq D_{i\prec j}$, $D_i \succ D_j \subseteq D_{i\succ j}$ for all $i, j \in J$. If J is a band (= idempotent semigroup), then we say that $(D, \prec, \succ)$ is a band J of subdimonoids D_i $(i \in J)$. If J is a commutative band, then we say that $(D, \prec, \succ)$ is a semilattice J of subdimonoids D_i $(i \in J)$.

We call a dimonoid $(D, \prec, \succ)$ rectangular if both semigroups $(D, \prec)$ and $(D, \succ)$ are rectangular bands.

Theorem. Every idempotent dimonoid $(D, \prec, \succ)$ is a semilattice Y of rectangular subdimonoids D_i, $i \in Y$.

This result is a generalization of Clifford's theorem [1] about the decomposition of idempotent semigroups into semilattices of rectangular bands.

In addition, we construct some examples of idempotent dimonoids and investigate the least semilattice congruence of these dimonoids.

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L- and R-cross-sections of the symmetric inverse 0-category

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Keywords: Green relation, cross-section, symmetric inverse 0-category.

Extended Abstract

Let $U(X)$ be the set of all nonempty subsets of a set X, $Map_b(A;B)$ be the set of all bijective mappings of A on a set B, and let $BSymX$ be the union of all sets $Map_b(A;B)$, where $A, B \in U(X)$. On the set $BSym^0X = BSymX \cup \{0\}$ we will define an operation $*$ by the rule: if $\varphi \neq 0 \neq \psi$ and $Im\varphi = Dom\psi$, then $\varphi * \psi = \varphi \circ \psi$, where $\circ$ is the ordinary composition of these mappings, otherwise $\varphi * \psi = 0$. Relative to such operation, the set $BSym^0X$ is a semigroup which is called the symmetric inverse 0-category on the set X. More general semigroups and some of their properties have been studied in [1, 2].

The Green relations L and R (see [3]) on $BSym^0X$ are described by the next

Lemma 1. *Let $\varphi, \psi \in BSym^0X$. Then:*
(i) $(\varphi;\psi) \in R \Leftrightarrow (\varphi = \psi = 0$ or $\varphi \neq 0 \neq \psi$, $Dom\varphi = Dom\psi)$;
(ii) $(\varphi;\psi) \in L \Leftrightarrow (\varphi = \psi = 0$ or $\varphi \neq 0 \neq \psi$, $Im\varphi = Im\psi)$.

Let ρ be an equivalence relation on a semigroup S. A subsemigroup T of S is called a ρ-cross-section if T contains exactly one element from every equivalence class that is T is ρ-shear. For every $i \in I = \{1, 2, ..., n\}$ we will put

$$S_i^0 = \{\varphi \in BSymX \,||Dom\varphi| = i\} \cup \{0\},$$

$$Q = \{\{\varphi, \psi\} \subseteq BSym^0X | \varphi^2 = 0 = \psi^2, \varphi \circ \psi \neq 0\}.$$

We will notice that cross-sections of some Green relations on symmetric semigroups have been studied by many authors (see e.g. [4, 5]).

Lemma 2. *For $A \subseteq S_i^0$, $i \in I$ the following statements are equivalent*:
(i) A is an R-cross-section of the semigroup S_i^0;
(ii) A is an R- shear and $\varphi \circ \psi \in \{\varphi, 0\}$ for all $\varphi, \psi \in A$;
(iii) A is an R- shear, $\varphi^2 \in \{\varphi, 0\}$ and $Y \cap A \neq Y$ for all $\varphi \in A$, $Y \in Q$.

Denote by $q(\rho^S)$ the quantity of all ρ-cross-sections of the semigroup S and by C_k^l the quantity of all l-element subsets of a k-element set.

Theorem. *Let X be a n-element set and $m = C_n^j, j \in I$.* Then

$$q(R^{BSym^0 X}) = \prod_{j=1}^{n} (\sum_{i=1}^{m} C_m^i \cdot (i \cdot j!)^{m-i}).$$

The quantity of L-cross-sections of the semigroup $BSym^0 X$ is described similarly.

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Other accepted Contributed Talks

(Of authors who could not attend the conference.)

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The fundamental structure of Lie groups and semigroups

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Keywords: Lie group, semigroup, Lie algebra, quotient space, transposition map.

Extended Abstract

Overview and history

In [4] H. Weyl described beautifully and completely the structure of complex semi-simple Lie algebras, developing the key element now known as Weyl basis. His investigation involved Lie's and Engel's classical theorems, E. Cartan's method for classifying real simple Lie algebras and root space decomposition. S. Helgason and particularly R. Richardson in [3] developed a new proof of the existence of a compact real form of a complex semi-simple Lie algebra. This proof removed the detailed structure description of Weyl's approach and thus simplified matters significantly. This method permits to get the existence of a compact real form of a semi-simple Lie algebra without having to deal with its intricate structure. Recently S. K. Donaldson [1] gave an even simpler version of Richardson's result. This version allows a generalization of this, so to speak, "basic compact form method", since Lie complex groups can now be attacked using just invariant geometric theory and differential geometry, avoiding the structure theory.

Semigroups and their representations were introduced by E. Hille in the fifties. Following Hille's stream of thought Langlands [2] proved some dense graph theorems of striking depth, and through them that there is a representation of a Lie algebra that is canonically associated with the semigroup.

We round up all these results and show that Lie groups and semigroups can be attacked using just what S. K. Donaldson calls "Weyl's Unitarian trick", that is to say, taking the compact form and reducing everything to it. We discuss the Hille-Langlands dense graph theorems and by linking them with a general argument in Riemannian geometry developed by S. K. Donaldson, we are able to prove that the representation canonically associated with a semigroup is actually a consequence of the fact that any subalgebra is symmetric with respect to the Euclidean structure if it is preserved by a transposition map: the key evidence for this is the following general principle of Riemannian geometry.

Take the representation

$$\rho : SL(V) \to SL(W)$$

given by the finite dimensional real vector spaces V and W. Set w to be a nonzero vector in W and set G_w to be the identity component of the stabilizer of w in $SL(V)$. We state that any compact subgroup of G_w is conjugate in G_w to a subgroup of $G_w \cap SO(V)$.

Define the quotient space $H = SL(V)/SO(V)$, the weighted flag (F, μ), composed of an strictly increasing sequence of vector subspaces

$$0 = F_0 \subset F_1 \subset F_2 \ldots \subset F_r = V$$

with weights $\mu_1 > \mu_2 \ldots > \mu_r$ and restrained to the conditions

$$n_i = dimF_i \; / \; F_{i-1}, \quad \sum n_i \; \mu_i = 0, \quad \sum n_i \; \mu_i^2 = 1,$$

define also the trace-free symmetric endomorphism S with Tr $S^2 = 1$, and the flag formed by

$$F_1 = E_1, \quad F_2 = E_1 \oplus E_2, \; \ldots$$

were μ_i are the eigenvalues of S with eigenspaces E_i.

It can now be seen precisely that the unit sphere $S_{[1]}$ in the tangent space of H at the base point [1] is identical with the set of all weighted flags.

Taking this principle and establishing the universal Lie algebra A, the associated representation $A(a)$ and the function $a \rightarrow A(a)$ defined on A with the property *

$$x \in D(A(a)) \cap D(A(b)) \rightarrow x \in D(A(sa+tb)) \rightarrow A(sa+tb)x = sA(a)x + tA(B)x$$

it can be seen that the influence of $SL(V)$ on the sphere at infinity in H coincides with the action under the defined flags, because ρ satisfies obviously the property * of the universal representation $A(a)$.

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On identities of indicator Burnside semigroups

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Keywords: semigroup, variety, lattice.

Extended Abstract

Following Kublanovsky [1], any subvariety of a periodic variety generated by completely 0-simple semigroups is referred to as a Rees–Sushkevich variety. One of the important results concerning Rees–Sushkevich varieties, due Hall et al. [2], is that for each integer $n \geq 1$, the variety RS_n generated by all completely 0-simple semigroups over groups of exponent dividing n is finitely based:

(I) $x^2 = x^{n+2}$
(II) $xyx = (xy)^{n+1}x$
(III) $(xhz)^n \cdot xyz = xyz \cdot (xhz)^n$

In [1] were constructed 13 finite semigroups and it was proven that a semigroup variety V is a Rees–Sushkevich variety if and only if it contains none of these semigroups:

(1) $A = [x, y|x = x^2; y^2 = 0; xy = yx]$
(2) $B = [x, y|x^2 = 0; y^2 = 0; xyx = yxy]$
(3) $C_\lambda = [x, y|x^2 = x^3; xy = y; yx^2 = 0; y^2 = 0]$
(4) $C_\rho = [x, y|x^2 = x^3; yx = y; x^2y = 0; y^2 = 0]$
(5) $N_3 = [x|x^3 = 0]$
(6) $D = [x, y|x^2 = 0; y = y^2; yxy = 0]$
(7) $K_n = [x, y|x^2 = 0; y^2 = y^{n+2}; yxy = 0; xy^qx = 0\ (q = \overline{2,n}); xyx = xy^{n+1}x]$
(8) $F_\lambda = [x, y|xy = xyx = xy^2; yx = yxy = yx^2; x^2 = x^2y = x^3; y^2 = y^2x = y^3]$
(9) $F_\rho = [x, y|xy = yxy = x^2y; yx = xyx = y^2x; x^2 = yx^2 = x^3; y^2 = xy^2 = y^3]$
(10) $W_\lambda = [a, x, y|a^2 = x^2 = y^2 = xy = yx = 0; ax = axax; ay = ayay;\ xay = xax; yax = yay]$
(11) $W_\rho = [a, x, y|a^2 = x^2 = y^2 = xy = yx = 0; xa = xaxa; ya = yaya;\ xay = yay; yax = xax]$
(12) $L_2^1 = [a, x, y|x = x^2; y = y^2; a = a^2; xy = x; yx = y; ax = xa = x; ay = ya = y]$
(13) $R_2^1 = [a, x, y|x = x^2; y = y^2; a = a^2; xy = y; ya = x; ax = xa = x; ay = ya = y]$

These semigroups are called indicator Burnside semigroup.

We provide a solution to the word problem and also provide a finite basis of identities for variety generated by indicator Burnside semigroups. For example, we have proven theorem that the finite basis of identities for variety generated by the semigroup K_n depends on n.

Theorem. If $n \geq 4$ and even, then the identities

$a^2 = a^{n+2}$
$a^2b^2 = b^2a^2$
$abc = ab^{n+1}c$

$abcd = acbd$
$ab^p a = a^{n+1} b^p a$, $(p, n) \neq 1$, $1 < p \leq n$
form a basis of identities for semigroup K_n.

If n is odd or $n = 2$, then identities
$a^2 = a^{n+2}$
$a^2 b^2 = b^2 a^2$
$abc = ab^{n+1} c$
$ab^p a = a^{n+1} b^p a$, $(p, n) \neq 1$, $1 < p \leq n$
form a basis of identities for semigroup K_n.

We have proven that only 10 of these semigroups generate small varieties and have described lattices of subvarieties. As a corollary we have shown that indicator Burnside semigroups generate hereditarily finitely based variety.

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Conjugacy Problems in Semigroups and Monoids

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Keywords: Semigroups, Conjugacy, Rewriting systems.

Extended Abstract

In semigroups and monoids, there are general different definitions of conjugacy. Let M be a monoid (or a semigroup) generated by $\sum$ and let u and v be two words in the free monoid $\sum^*$. Then the following questions can be asked:

1. Is there a word x in the free monoid $\sum^*$ such that $xv =_M ux$?
2. Is there a word y in the free monoid $\sum^*$ such that $vy =_M yu$?
3. Are there words x, y in the free monoid $\sum^*$ such that $vy =_M yu$ and $xv =_M ux$?

The first two relations are reflexive and transitive but not necessarily symmetric, while the third one is an equivalence relation, which we denote by $\equiv$.
A different generalization of conjugacy is:
(4) Are there words x, y in the free monoid such that $u =_M xy$ and $v =_M yx$?
This is called the **transposition problem**. This relation is reflexive and symmetric but not necessarily transitive. In general, if the answer to this question is positive then the answer to the above questions is also positive, that is $(4) \Rightarrow (3) \Rightarrow (1), (2)$.

In free monoids, it holds that $(1) \Leftrightarrow (2) \Leftrightarrow (3) \Leftrightarrow (4)$ [2]. Connections between these relations have been studied for free inverse semigroups [1] and for inverse semigroups with a special presentation [5].
In this work, we solve partially the conjugacy problems presented above using rewriting systems. In [3], the authors show that for each finite, length-reducing and complete rewriting system, the conjugacy relations (2) and (3) are decidable and in [4] they show that this does not hold for the transposition problem. Our approach is different, as it is an algorithmic one.
Let $M =< \sum \mid R >$ be a finitely presented monoid and let $\Re$ be a complete and reduced rewriting system for M. Let u be a word in $\sum^*$, we consider u and all its cyclic conjugates in $\sum^*$, $\{u_1 = u, u_2, .., u_k\}$, and we apply on any element u_i rules from $\Re$. We say that a word u is **cyclically irreducible** if u and all its cyclic conjugates are irreducible modulo $\Re$. If for some $1 \leq i \leq n$, u_i reduces to v, then we say that u **cyclically reduces to** v. A question which arises naturally is when u and all its cyclic conjugates cyclically reduce to a same cyclically irreducible element (up to cyclic conjugation in $\sum^*$), denoted by $\rho(u)$. The answer to this question gives a partial solution to the conjugacy problems presented above in the following way:

u and v are transposed $\Rightarrow \rho(u)$ and $\rho(v)$ are cyclic conjugates in $\sum^* \Rightarrow u \equiv v$
In order to solve this question, we define some new conditions on the rewriting system and we find, given a word u in $\sum^*$, a criteria that ensures the existence of a unique cyclically irreducible element $\rho(u)$.

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Tight Representations of Semilattices and Inverse Semigroups

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Keywords: Semi-lattices, tight representations, boolean inverse semigroups, continuous inverse semigroups.

Extended Abstract

We shall say that an inverse semigroup S is a *Boolean inverse semigroup*, if $E(S)$, the semilattice of idempotents of S, admits the structure of a Boolean algebra whose order coincides with the usual order on $E(S)$.

Boolean inverse semigroups are quite common, a well known example being the semigroup $\mathscr{I}(X)$ of all partially defined bijections on X. The semilattice of idempotents of $\mathscr{I}(X)$ coincides with the Boolean algebra $\mathscr{P}(X)$ of all subsets of X, this being the reason why $\mathscr{I}(X)$ is a indeed a Boolean inverse semigroup.

Given an inverse semigroup S one might like to study how far it is from being a Boolean inverse semigroup by considering homomorphisms

$$\sigma : S \to \mathscr{B},$$

into some Boolean inverse semigroup $\mathscr{B}$. Simply requiring σ to be a semigroup homomorphism completely sidesteps the issue since, in case S itself happens to be a Boolean inverse semigroup, a mere semigroup homomorphism has no reason to respect the Boolean algebra structures involved.

To deal with this situation we propose to consider a special class of homomorphisms called *tight representations*, which applies to every inverse semigroup with zero. In case S is a Boolean inverse semigroup we prove that tight representations are precisely those which restrict to a homomorphism $\sigma : E(S) \to E(\mathscr{B})$ in the category of Boolean algebras.

One of the most important homomorphisms from an inverse semigroup S to a Boolean inverse semigroup is the so called Vagner–Preston map

$$\gamma : S \to \mathscr{I}(X),$$

which shows, among other things, that every inverse semigroup is a subsemigroup of some $\mathscr{I}(X)$. However γ is never a tight representation, even in case S is a Boolean inverse semigroup. For example $\gamma(0)$ is never equal to the zero of $\mathscr{I}(X)$, namely the empty function. In fact this is not the only flaw presented by γ from the point of view of tight representations, as explained below.

It is the main purpose of this work to introduce a canonical tight representation

$$\lambda : S \to \mathscr{I}(\Omega),$$

where Ω is a certain compact topological space which we call the *regular representation*. Precisely speaking, Ω is the closure of the set of all ultrafilters within the semi-character space of $E(S)$.

Contrary to the Vagner–Preston representation, the regular representation is not always faithful, but under a certain *continuity* hypothesis we are able to precisely describe when is $\lambda(s) = \lambda(t)$, for a given pair of elements $s, t \in S$.

The issue boils down to the following situation: let $e \leq f$ be idempotents in $E(S)$ and suppose that there is no nonzero idempotent $d \leq f$ such that $d \perp e$ (meaning that $de = 0$). Very roughly speaking this means that the space between e and f is empty, in which case we say that e is *dense* in f. Notice however that when $e \neq f$, this will never happen in a Boolean inverse semigroup, since $d := f \wedge \neg e \neq 0$.

It turns out that when e is dense in f one has that $\lambda(e) = \lambda(f)$, even when $e \neq f$. In case e is not necessarily less than f, but ef is dense in both e and f, we will consequently also have that $\lambda(e) = \lambda(ef) = \lambda(f)$.

The impossibility of distinguishing between idempotents clearly has consequences for other elements. Suppose for example that $s, t \in S$ are such that $\lambda(s^* s) = \lambda(t^* t)$. Suppose moreover that $st^* t = ts^* s$. Then a simple computation shows that $\lambda(s) = \lambda(t)$, so we get another instance on non-faithfulness.

Fortunately we are able to prove that these well understood situations are the only ones allowing for $\lambda(s) = \lambda(t)$. Another consequence is that when the regular representation is unable to separate between two elements of S, then no tight representation can possibly do it.

As already hinted upon, this result requires that S be *continuous*, a concept we introduce in our work. To explain what this means let us say that two elements $s, t \in S$ *essentially coincide with each other*, in symbols $s \equiv t$, if $s^* s = t^* t$, and for every nonzero idempotent $f \leq s^* s$, there exists a nonzero idempotent $e \leq f$, such that $se = te$. Very roughly this means that s and t coincide on a dense set, although this idea may be made quite precise when we are speaking of *localizations* in the sense of Kumjian (“On localizations and simple C*-algebras", *Pacific J. Math.,* **112** (1984), 141–192).

Recalling that when two continuous functions agree on a dense set of their common domain they must coincide everywhere, we say that S is *continuous* if $s \equiv t$ implies that $s = t$. Localizations are continuous and so are E^*-unitary inverse semigroups.

The use of the continuity hypothesis in the Theorem referred to above naturally raises the question of whether or not this hypothesis is really needed. To resolve this issue, in the final section of this work we describe a general construction which leads to a non-continuous Boolean inverse semigroup S for which the conclusions of the Theorem does fail.

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On a new class of unique product monoids and its applications to ring theory

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Keywords: Artinian narrow unique product monoid; unique product monoid; skew generalized power series ring; reduced ring; Armendariz ring.

Extended Abstract

A monoid S is called a unique product monoid (or a u.p. monoid) if for any two nonempty finite subsets A, B of S there exists an element $s \in AB$ that is uniquely presented in the form $s = ab$ where $a \in A$ and $b \in B$. Unique product monoids and groups play an important role in ring theory, for example providing a positive case in the zero-divisor problem for group rings, and their structural properties have been extensively studied (e.g. see [1], or [7] and references therein). The class of u.p. monoids includes the right and the left totally ordered monoids, submonoids of a free group, and torsion-free nilpotent groups.

The aim of the talk is to present a new class of unique product monoids, called artinian narrow unique product monoids, recently introduced in [4]. Our interest in the new unique product condition was motivated by the fact that this condition enables to characterize various ring-theoretic properties of the skew generalized power series ring construction. This construction, defined in [6], embraces a wide range of classical ring-theoretic extensions, including skew polynomial rings, skew power series rings, skew Laurent polynomial rings, skew group rings, Mal'cev-Neumann Laurent series rings, and the "untwisted" versions of all of these. The construction is parallel to the well-known construction of the skew group ring $R[G, \omega]$, but instead of functions from a group G to a ring R having finite support, it uses functions from a strictly ordered monoid $(S, \leq)$ to the ring R whose support is artinian and narrow. Also the new class of unique product monoids, presented in the talk, is defined with respect to artinian and narrow subsets (similarly as u.p. monoids are defined with respect to finite subsets), and thus it is in order to recall what artinian and narrow sets are.

In what follows "an order" on a set means "a partial order". An ordered set $(S, \leq)$ is called *artinian* if every strictly decreasing sequence of elements of S is finite, and $(S, \leq)$ is called *narrow* if every subset of pairwise order-incomparable elements of S is finite. It is easy to see that $(S, \leq)$ is artinian and narrow if and only if every nonempty subset of S contains at least one, but only a finite number, of minimal elements. Sets with this property are also called *well-partially-ordered* or *well-quasi-ordered*, and the theory of such sets is rich and well developed (see [3]). Applications of this theory include, for example, formal languages and computational complexity (e.g. see [2]).

A monoid S (written multiplicatively) equipped with an order $\leq$ is called an *ordered monoid* if for any $s_1, s_2, t \in S$, $s_1 \leq s_2$ implies $s_1 t \leq s_2 t$ and $t s_1 \leq t s_2$. Moreover, if $s_1 < s_2$ implies $s_1 t < s_2 t$ and $t s_1 < t s_2$, then $(S, \leq)$ is said to be *strictly ordered*.

Now we are ready to state the definition of the new class of unique product monoids announced in the title. Let $(S, \leq)$ be an ordered monoid. We say that $(S, \leq)$ is an *artinian narrow unique product monoid* (or an *a.n.u.p.*

monoid) if for every two artinian and narrow subsets A and B of S there exists an element $s \in AB$ that is uniquely expressible in the form $s = ab$ with $a \in A$ and $b \in B$.

Clearly, every finite subset of an ordered set is artinian and narrow, and thus all a.n.u.p. monoids are indeed u.p. monoids. It is also clear that u.p. monoids are exactly a.n.u.p. monoids with respect to the trivial order, i.e. the order with respect to which any two distinct elements are incomparable. In the talk, a quite general method for constructing a.n.u.p. monoids will be presented, and further logical relationships between a.n.u.p. monoids and other significant classes of monoids will be explicated with several examples.

As an application, some results on skew generalized power series rings with exponents in a.n.u.p. monoids will be presented. A skew generalized power series ring $R[[S,\omega]]$ consists of all functions from a strictly ordered monoid $(S,\leq)$ to a coefficient ring R whose support is artinian and narrow, with pointwise addition, and with multiplication given by convolution twisted by an action ω of the monoid S on the ring R. It will be shown that the new class of a.n.u.p. monoids provides the appropriate setting for obtaining results on reduced rings and domains of skew generalized power series, and on analogues of Armendariz rings (see [5]).

This is a joint work with Greg Marks and Michał Ziembowski.

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On applications of semigroups of nonstandard words to free profinite semigroups

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Keywords: free profinite semigroup, nonstandard analysis.

Extended Abstract

The nonstandard approach to the theory of pseudovarieties was elaborated in the paper [4]. These results make it possible to characterize pseudovarieties in terms of nonstandard identities with the help of nonstandard words over a finite alphabet.

For a finite alphabet $A = \{x_1, \dots, x_m\}$, denote $W = W(A)$ the semigroup of all words over the alphabet A equipped with the concatenation. By principles of the nonstandard analysis [1] the nonstandard extension ${}^*W$ is the semigroup of all nonstandard words over the alphabet A under the concatenation. A word $w \in {}^*W$ is called finite if $w \in W$ and infinite otherwise. A word $u \in {}^*W$ is called a subword of $w \in {}^*W$ if $w = xuy$ for some $x, y \in {}^*W^1$.

In this paper we investigate algebraic properties of the semigroup of nonstandard words ${}^*W$ and consider applications of these results to the free profinite semigroup over the alphabet A [2].

For a word $w \in {}^*W$, denote $F(w)$ the set of all finite subwords of w. A word $w \in {}^*W$ is called recurrent, if for any $u \in F(w)$ there exists a word v such that $uvu \in F(w)$, and uniformly recurrent if $F(w) = F(u)$, for any infinite subword u of w.

Theorem 1. For any infinite words $w, v \in {}^*W$ the statements hold: (i) $F(v) \subset F(w)$ implies $w = xuy$, for some infinite prefix u of v; (ii) $w \leq v(\mathscr{J})$ implies $X(v) \subset X(w)$; (iii) if w is recurrent then $w = xuyuz$ for some infinite prefix u of w and $x, y, z \in {}^*W^1$.

Theorem 2. If uniformly recurrent words $w, v \in {}^*W$ satisfy $F(v) \subset F(w)$ then there exists a subword u of w such that $u \leq_{\mathscr{R}} p$ and $u \leq_{\mathscr{L}} q$, for some infinite prefix p and infinite suffix q of v.

For a word $w \in W$, denote (w) the principal ideal generating by w in the semigroup W. The set of all principal ideals of W satisfies the finite intersection property and by principles of the nonstandard analysis the set $\mathscr{M} = \bigcap\{{}^*(w) : w \in W\}$ is an ideal of the semigroup ${}^*W$. A word $u \in {}^*W$ satisfies $u \in \mathscr{M}$ if and only if every finite word $w \in W$ is a factor of u.

Theorem 4. If a sequence $u_1, \dots, u_n \in {}^*W$, for $1 < n \leq m$, is not a permutation of the alphabet A and satisfies $F(u_i) \neq W$, for every $1 \leq i \leq n$, then for any word $v = v(x_1, \dots, x_n)$ from the set ${}^*W$ the word $w = v(u_1, \dots, u_n)$ satisfies $F(w) \neq W$.

For words $v(x_1, \dots, x_n), u_1, \dots, u_n$ from the set ${}^*W$, the word $v(u_1, \dots, u_m)$ is said to be obtained by taking a substitution of the words $u_1, \dots, u_n$ into the word $v(x_1, \dots, x_n)$. The substitution is called trivial if $u_1, \dots, u_n$ is a permutation of the alphabet A and nontrivial otherwise. Hence we can define for any words $u_i = u_i(x_1, \dots, x_n)$, $v_i = v_i(x_1, \dots, x_n)$ $(i = \overline{1, n})$ the composition $(u_1, \dots, u_n) \cdot (v_1, \dots, v_n) = (u_1(v_1, \dots, v_n), \dots, u_n(v_1, \dots, v_n))$. It permits us to construct new words with the help of iterated compositions $(u_1,\dots,u_n)^k$ for arbitrary numbers $k \in {}^*\mathbf{N}$. A word $w \in {}^*W$ is said to be obtained by taking an iteration if it is a component of a n-tuple $(u_1,\dots,u_n)^k$ for some $u_1,\dots,u_n \in {}^*W$ and $k \in {}^*\mathbf{N}$.

Theorem 5. The complement $\mathscr{C}$ of the ideal $\mathscr{M}$ is closed under taking finite products, powers by arbitrary exponents, nontrivial substitutions and arbitrary iterations.

It follows from [4] that the semigroups of nonstandard words and their factor-semigroups encode in their structure common algebraic-combinatorial properties of the members of corresponding semigroup pseudovarieties. In particular, for the pseudovariety of all finite semigroups **Sg**, the free object over the set A is the quotient of the semigroup ${}^*W$ by the congruence

$$\varepsilon = \bigcap\{\ker {}^*f : \ f \text{ is a homomorphim of } W \text{ to a finite semigroup } S\}.$$

It was proved in [4] that the factor-semigroup $F(A) = {}^*W/\varepsilon$ is a nonstandard interpretation of the free profinite semigroup $\overline{\Omega}_m\mathbf{Sg}$ of all m-ary implicit operations on the pseudovariety **Sg**, i.e. it is a topologically A-generated compact Hausdorff topological semigroup and, for any $S \in \mathbf{Sg}$ and a mapping $\theta : A \to S$, there exists a uniformly continuous homomorphism $\varphi : F(A) \to S$ such that $\varphi \circ i_A = \theta$ for the canonical mapping $i_A : A \to F(A)$.

Theorem 6. The kernel of the free profinite semigroup $F(A)$ is equal to the factor-semigroup $\mathscr{M}/\varepsilon$.

In particular, these theorems give us a very short and elementary combinatorial proof of properties of the free profinite semigroup obtained early by J.Almeida and M.Volkov [2,3].

Corollary. The free profinite semigroup $F(A)$ satisfies the following properties:

1) the complement of the kernel of the semigroup $F(A)$ is closed under taking finite products, powers by arbitrary exponents and all nontrivial substitutions as well as under taking arbitrary iterations of any substitutions;

2) a word $w \in {}^*W$ is uniformly recurrent iff w is a $\mathscr{J}$-maximal element in the set $F(A) \setminus W$;

3) uniformly recurrent words determine in the semigroup $F(A)$ $\mathscr{J}$-classes consisting of regular elements;

4) if $w \in {}^*W$ is a uniformly recurrent word then, for any $v \in \mathscr{J}(w)$, there exists a subword u of w such that $u \equiv v(\mathscr{H})$.

5) for an infinite word $w \in {}^*W$, the set $F(w)$ is rational iff w is a uniformly recurrent periodic word, i.e. $w \equiv u^n(\mathscr{J})$ for some $u \in W$ and $n \in {}^*\mathbf{N}$.

The theorems are applied also to the theory of symbolic dynamics [3].

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Polynomial bound for inverting automorphisms of a free group

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(This is a joint work with P. Silva and M. Ladra)

Keywords: Free group, automorphisms, automata, polynomial bounds, uniform membership problem.

Extended Abstract

Let F_n be the free group (of rank n) with basis $\{a_1,\ldots,a_n\}$. For any automorphism $\varphi \in Aut(F_n)$, the length of φ is defined as $\|\varphi\| = \max\{|a_i\varphi| \mid i = 1,\ldots,n\}$; this value, up to multiplicative constants, is independent from the chosen basis $\{a_1,\ldots,a_n\}$.

Originated by cryptographic motivations, A. Myasnikov asked the following question: "is the length of the inverse of an automorphism of a free group bounded above by a polynomial function of the length of the automorphism itself?"; more precisely, "does there exists a polynomial $p(x)$ (depending only on the ambient rank n) such that $\|\varphi^{-1}\| \leqslant p(\|\varphi\|)$ for every $\varphi \in Aut(F_n)$?"

We consider this problem and find a graphical way of understanding it; then, using automata techniques, we give the following partial answer:

For the case of rank $n = 2$, we completely solve the problem i.e. we prove the existence of such polynomial. A variation of the same method allows us to study the general case; however, the bound obtained is less satisfactory and so, for $n \geqslant 3$, we can only prove the existence of a subexponential function $f(x)$ (depending only on the ambient rank n) such that $\|\varphi^{-1}\| \leqslant f(\|\varphi\|)$ for every $\varphi \in Aut(F_n)$.

In the talk, a sketch of the proofs of these results will be given, as well as some discussion on the difficulties that arise when trying to improve the subexponential bound to a polynomial one for the general case.

These results have several applications. For example, a full affirmative answer to Myasnikov's question should immediately imply that the uniform membership search problem for finitely generated subgroups of a free groupF_n is solvable in polynomial time (this problem is the following: given $w, w_1,\ldots,w_t \in F_n$ decide whether $w \in \langle w_1,\ldots,w_t\rangle$ and, in the affirmative case, write w as a word in $w_1,\ldots,w_t$).

Free subgroups of $\mathscr{U}(\mathbb{Z}G)$ generated by alternating units

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(This is a joint work with Jairo Z. Gonçalves)

Keywords: Group rings, free groups.

Extended Abstract

Let G be a finite group, and let $\mathbb{Z}G$ be the integral group ring of G. We denote by $\mathscr{U}(\mathbb{Z}G)$ the unit group of $\mathbb{Z}G$.

In this work, we search for free subgroups in $\mathscr{U}(\mathbb{Z}G)$, using first an alternating unit and a bicyclic unit, and then a pair of alternating units.

In a similar investigation [1], Gonçalves and del Rio show that in the integral group ring $\mathbb{Z}G$, with G a non-abelian group with order coprime with 6, there always exists a pair formed by a Bass cyclic unit and a bicyclic unit, such that the subgroup they generate is not 2-related.

Let G be a group containing $x \in G$ an element of odd order n, and $c \in \mathbb{N}$, $1 \leq c < 2n$ such that $(c, 2n) = 1$. Then, according to [4, Lemma 10.6], the element

$$g_c(x) := \frac{x^c + 1}{x + 1} = 1 - x + x^2 - \ldots + x^{c-1} = \sum_{i=0}^{c-1} (-x)^i = 1 - x + x^2 - \ldots + x^{c-1} \in \mathbb{Z}G$$

is a unit in $\mathbb{Z}G$, called an *alternating unit*.

Let now g be another element of G, of order $m > 1$, and suppose that $y \notin N_G(\langle g \rangle)$, the normalizer of $\langle g \rangle$ in G. Set $\widehat{g} = \sum_{i=0}^{n-1} g^i$. Then the element $\tau = (1-g)y\widehat{g} \in \mathbb{Z}G$ has square 0, but $\tau \neq 0$. As usual, we call $u = 1 + \tau$ a *bicyclic unit*.

Just like some other types of units in group rings [2], [3], alternating units defined in a homomorphic image of $\mathbb{Z}G$ may be lifted to alternating units in $\mathbb{Z}G$. So the research technique must involve studying the behavior of pairs formed by an alternating unit and a bicyclic unit, and pairs of alternating units in group rings $\mathbb{Z}H$, with H minimal groups that could be counter-examples to the result. As a partial result, we classify such groups as well.

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