Stable gravastars
– an alternative to black holes?

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Black holes are ubiquitous

EVIDENCE

- Binary systems, one partner $M \gtrsim 3M_\odot$ – most spectacular examples now from centre of our galaxy

- Energetic objects with accretion disks: proof that Kerr black holes exist – iron emission line redshift from within $r < 6M$, last stable orbit of Schwarzschild black hole [Dabrowski et al, Mon. Not. R. Soc. 288, L11 (1997)]

- Gamma ray bursters

- Active Galactic Nuclei...
But are they really black holes?

- The evidence is from the black hole exterior
- Need to show that an event horizon exists
- Difference between a completely absorbing surface and “something else” difficult to prove beyond all reasonable doubt [Abramowicz, Kluzniak and Lasota, Astron. Astrophys. 396, L31 (2002)]

ALTERNATIVES
- Boson condensate stars
- … (many crazy ideas)
- Gravitational vacuum condensate stars (gravastars)
Quantum gravity + black holes $\Rightarrow$ problems

BLACK HOLE INFORMATION PARADOX

- Hawking 1973: in presence of quantum fields black holes radiate with (almost) black body spectrum.

- Heat capacity negative: rate $\propto M^{-1}$, runs away as $M \to 0$

- If it evaporates completely, information is lost

WAYS OUT

- Change quantum mechanics to allow unitarity violation

- Stable remnant black hole remains

- Quantum gravity intervenes near horizon scale; unitarity is preserved
Funny business at the event horizon?

In Schwarzschild geometry

\[ ds^2 = -A(r) \, dt^2 + A^{-1}(r) \, dr^2 + r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2), \quad A(r) = 1 - \frac{2M}{r}. \]

an infalling observer \textit{locally} measures energies of other infalling things to diverge \( P^0 = A(r)^{-1/2} E \to \infty. \)

Should we care?

- 't Hooft: black hole holography, brick wall model . . .
- many less respectable characters. . .
The Mazur-Mottola gravastar

The gravastar (GRAvitational VAcuum condensate STAR of Mazur and Mottola), gr-qc/0109035, is an onion-like construction, with 5 layers:

- An external Schwarzschild vacuum, with energy density, $\rho = 0$, and pressure, $P = 0$.

- A thin shell, with surface density $\sigma_+$ and surface tension $\vartheta_+$; with radius $r_+ \gtrsim 2M$.

- A (relatively thin) finite-thickness shell of stiff matter with equation of state $P = \rho$; straddling $r = 2M$ where the horizon would in normal circumstances have formed.

- A second thin shell; with radius $r_- \lesssim 2M$, and with surface density $\sigma_-$ and surface tension $\vartheta_-$.

- A de Sitter interior, with $P = -\rho$. 
The Mazur-Mottola gravastar

The two thin shells are used to “confine” the stiff matter in a transition layer straddling $r = 2M$, while the energy density in the de Sitter vacuum is chosen to satisfy

$$\frac{4\pi}{3} \rho (2M)^3 = M,$$

In the approximation where the transition layer is neglected, all of the mass of the resulting object can then be traced back to the energy density of the de Sitter vacuum.
The Mazur-Mottola gravastar

- Expect thermodynamic stability
- Solves black hole information paradox

BUT

- In limit $a_+ = 2M(1 + \epsilon)$, $\epsilon \to 0$, wouldn’t something blow up?
- Are there dynamically stable configurations?
New simplified gravastar

- Replace thick stiff matter shell by a thin shell
- Leave equation of state of thin shell free, but look for dynamically stable configurations

Our gravastar is a simple 3-layer model

- An external Schwarzschild vacuum, $\rho = 0 = p$.
- A single thin shell, with surface density $\sigma$ and surface tension $\vartheta$; with radius $a \gtrsim 2M$.
- A de Sitter interior, $P = -\rho$.
- To avoid forming an event horizon, we shall demand

$$\frac{4\pi}{3} \rho (2M)^3 \lesssim M.$$
The mathematical model

Consider the class of geometries

\[ ds^2 = - \left[ 1 - \frac{2m(r)}{r} \right] dt^2 + \frac{dr^2}{1 - 2m(r)/r} + r^2 (d\theta^2 + \sin^2 \theta \, d\phi^2). \]

Less general than the class of all static spherically symmetric geometries but sufficiently general to include both the Schwarzschild and the de Sitter geometries.

Connect two geometries of this type are connected along a timelike hypersurface at \( r = a(t) \), (normal \( n^a \))

\[ d\tau^2 = \left[ 1 - \frac{2m(a(t))}{a(t)} \right] dt^2 - \frac{1}{1 - 2m(a(t))/a(t)} \left[ \frac{da(t)}{dt} \right]^2 dt^2, \]
Israel–Lanczos–Sen thin-shell formalism

Induced metric on the shell, \((\tau\) proper time),

\[ h_{ab} = g_{ab} - n_a n_b, \]

\[ h_{ab} \, dx^a \, dx^b = -d\tau^2 + a(\tau)^2 \left( d\theta^2 + \sin^2\theta \, d\phi^2 \right). \]

Extrinsic curvature

\[ K_{ab} = h_a^c h_b^d \nabla c n_d \]

Junction conditions relate discontinuity in extrinsic curvature to the surface stress-energy, \(S_{ab}:\)

\[ [[K_{ab}]] = -8\pi \left[ S_{ab} - \frac{1}{2} S h_{ab} \right]; \quad [[K_{ab} - K \, h_{ab}]] = -8\pi \, S_{ab}. \]

\(\ldots [[X]] \) denotes discontinuity in \(X\) across the shell.
Dynamical analysis

We find

\[ \sigma = -\frac{1}{4\pi a} \left[ \sqrt{1 - 2m(a)/a + \dot{a}^2} \right], \tag{1} \]

\[ \vartheta = -\frac{1}{8\pi a} \left[ \frac{1 - m/a + m'/a + \dot{a}^2 + a\ddot{a}}{\sqrt{1 - 2m/a + \dot{a}^2}} \right]. \tag{2} \]

where \( \dot{a} \equiv \frac{da}{d\tau} \) and \( m'(a) \equiv \frac{dm}{da} \) etc.

In fact (2) follows from (1) by energy-momentum conservation

\[ \frac{d}{d\tau}(\sigma a^2) = \vartheta \frac{d}{d\tau}(a^2). \]
Dynamic master equation can be rewritten in a form of an “energy equation” for a non-relativistic particle,

\[ \frac{1}{2} \dot{a}^2 + V(a) = E, \]

with “potential”

\[ V(a) = \frac{1}{2} \left\{ 1 + \frac{4m_+(a)m_-(a)}{m_s^2(a)} - \left[ \frac{m_s(a)}{2a} + \frac{(m_+(a) + m_-(a))}{m_s(a)} \right]^2 \right\} \]

\( m_-(a) \) = “mass function” for interior geometry;
\( m_+(a) \) = “mass function” for exterior geometry;
\( m_s = 4\pi \sigma a^2 \) = mass of thin shell;
and “energy” \( E = 0 \).
Stability

∃ strictly stable solution for the shell (against spherically symmetric radial oscillations) iff ∃ some $m_s(a)$ and some $a_0$ such that we simultaneously have

$$V(a_0) = 0; \quad V'(a_0) = 0; \quad V''(a_0) > 0.$$ 

Quirk: $E \equiv 0$, the situation where $V(a) \equiv 0$, which in non-relativistic mechanics corresponds to neutral equilibrium, is now converted to a situation of stable equilibrium in this general relativity calculation. (Since now, because one is not free to increase the “energy” $E$, one has $\dot{a} \equiv 0.$)
Stability

- Less stringent notion of stability, “bounded excursion”, also useful. Suppose we have $a_2 > a_1$ such that

$$V(a_1) = 0; \quad V'(a_1) \leq 0; \quad V(a_2) = 0; \quad V'(a_2) \geq 0;$$

with $V(a) < 0$ for $a \in (a_1, a_2)$.

- In this situation the motion of the shell remains bounded by the interval $(a_1, a_2)$. Although not strictly stable, since the shell does in fact move, this notion of “bounded excursion” more accurately reflects some of the aspects of stability naturally arising in non-relativistic mechanics.

- Adding a small negative offset to a strictly stable potential converts it to one exhibiting “bounded excursion”

$$V(a) \rightarrow V(a) - \epsilon^2$$
Inverting the potential

Assume $V(a)$, $m_-(a)$ and $m_+(a)$ given, and invert to find $m_s(a)$ or $\sigma(a)$:

$$\sigma(a) = -\frac{1}{4\pi a} \left\{ \sqrt{1 - 2V(a) - \frac{2m_+(a)}{a}} - \sqrt{1 - 2V(a) - \frac{2m_-(a)}{a}} \right\}$$

For our case $m_+(a) = M/a$ (Schwarzschild), and $m_-(a) = (4\pi/3)\rho a^3 \equiv ka^3$ (de Sitter), so that

$$\sigma(a) \equiv \frac{1}{4\pi a} \left\{ \sqrt{1 - 2V(a) - 2ka^2} - \sqrt{1 - 2V(a) - \frac{2M}{a}} \right\},$$
Inverting the potential

The surface tension $\vartheta(a)$ is found as a result

$$
\vartheta(a) \equiv \frac{1}{8\pi a} \left\{ \frac{1 - 2V(a) - a V'(a) - 4ka^2}{\sqrt{1 - 2V(a) - 2ka^2}} - \frac{1 - 2V(a) - a V'(a) - M/a}{\sqrt{1 - 2V(a) - 2M/a}} \right\}.
$$

Cases of interest

- $V(a) \equiv 0$, a degenerate, but physically important case of static shell $\dot{a} \equiv 0$.

- $V(a) = \frac{1}{2} (a - a_0)^2 f(a)$, where $f(a)$ is an arbitrary positive function which is regular at $a_0$. Trivial: master equation has unique solution at $a = a_0$ and $\dot{a} = 0$, and all possibility of motion is excluded by fiat.
Inverting the potential

- \( V(a) = \frac{1}{2} (a - a_0)^2 \ f(a) - \epsilon^2 \) gives models stable under “bounded excursion”.

We consider just the \( V(a) \equiv 0 \) (purely static shell) in what follows.

- Stable solutions with a shell satisfying the Dominant Energy Condition exist if \( 0 < kM^2 < \lambda_{cr} \), where
  \[
  0 < kM^2 < \lambda_{cr}, \text{ where } \lambda_{cr} \approx 0.0243045493773
  \]
  \[
  (400000000 \ \lambda_{cr}^4 - 1054320000 \ \lambda_{cr}^3
  
  + 257041039 \lambda_{cr}^2 - 19516500 \ \lambda_{cr} + 337500) = 0
  \]
  i.e., \( \lambda_{cr} \approx 0.0243045493773 \ldots \)

- For parameter values \( 0 < kM^2 < \lambda_{cr} \), there will be a range of values \( a_1 < a < a_2 \) over which the dominant energy condition is satisfied.
Surface energy density and tension

Surface energy density $\sigma$ (in units $M^{-1}$), as a function of radius, $a$ (in units $M$). ($kM^2 = 1/18$; $V(a) \equiv 0$.)

Surface tension, $\theta$ (in units $M^{-1}$), as a function of radius, $a$ (in units $M$). ($kM^2 = 1/18$; $V(a) \equiv 0$.)
Equation of state: Case 1

Example: $kM^2 = 1/18; V(a) \equiv 0$.

Surface energy density as a function of surface tension. Right hand panel shows an enlargement of central region. The dominant energy condition is violated always.
Example: $kM^2 = 1/72$; $V(a) \equiv 0$. Parameter values for which the dominant energy condition is violated are shown by a thin line, and parameter values for which the dominant energy condition is satisfied, viz., $2.124319 \, M < a < 3 \, M$, are shown by a thick line.
Special geometries: Mazur-Mottola 1

- \( k \rightarrow 1/(8M^2) \) gives the “Mazur–Mottola limit”
  \( k(2M)^3 = M \).

- To understand the nature of this limit it is convenient to write
  \[
  k = \frac{1}{8M^2(1 + \epsilon)^2}, \quad \epsilon \gtrsim 0.
  \]

- Energy density and surface tension are both real for
  \( a \in (2M, 2M[1 + \epsilon]) \).

- On kinematic grounds, we have a severely restricted range of possible motions for the shell.

- In limit \( \epsilon \rightarrow 0 \), \( \sigma \rightarrow 0 \) and \( \vartheta \rightarrow -\infty \). OUCH!
Stiff shell gravastar

We propose a new stable 3-layer limit of Mazur-Mottola model

- Forget about equating exterior mass with that of de Sitter vacuum

- Take a thin shell at \( a > 2M \) with stiff equation of state
  \[ P = \rho \text{ or } \vartheta = -\sigma. \]

- For \( kM^2 < 0.0243045493773 \ldots \) there are two values of \( a \)
  at which we can place a stiff shell in stable gravastars,
  the lower value, \( a_1 \), being in the range
  \[ 2M < a_1 < 2.3005600972496 \ M. \]

- The inner stiff shell case is certainly so close to the putative horizon that any test to distinguish such an object from a true Schwarzschild black hole would be extremely difficult in astrophysical contexts.
Do I buy it?

PERSONAL PREJUDICES

Something funny ought to happen at the horizon, but we should preserve the “holey” properties of black holes.

Hawking evaporation is a process which has more to do with quantum field theory than quantum gravity per se.

The fundamental quantum dynamics which explains black hole entropy remain to be found (despite much work).

As a quantum positivist I advocate the view that classical space should not exist inside a black hole: we want a sum over all possible interior geometries consistent with the surface boundary data.

BH holographic principle would be consistent with such a view; but gravastars not ostensibly so.
Conclusion

Gravastars are

- interesting
- better than we expected at the outset
- maybe good enough to convince a number of people
- would have to be firmly placed in a quantum gravity context (why is a de Sitter fluid natural?) to convince me