Astrometric Reference Frames in the solar system and beyond

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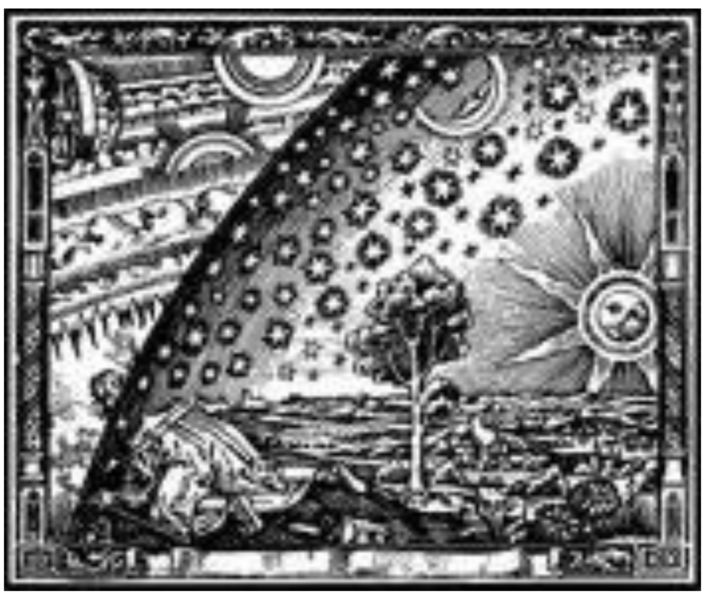
Why to bother about the “beyond” ?

Curiosity is as old as humankind. Ever since humans exist, they want to explore the world they are living in. They want to reach the “Edge of the World” to see what’s beyond it, even if it puts their lives in great danger (Maggelan’s circumnavigation of the Earth)



Outline:

- Definitions and Resolutions
- Role of boundary conditions
- Galactocentric reference system
- Examples
- Local Group Standard of Rest (LGSR)
- Cosmological reference frame
- Gravitational potential, Gdot, Pioneer,…
- Concusions

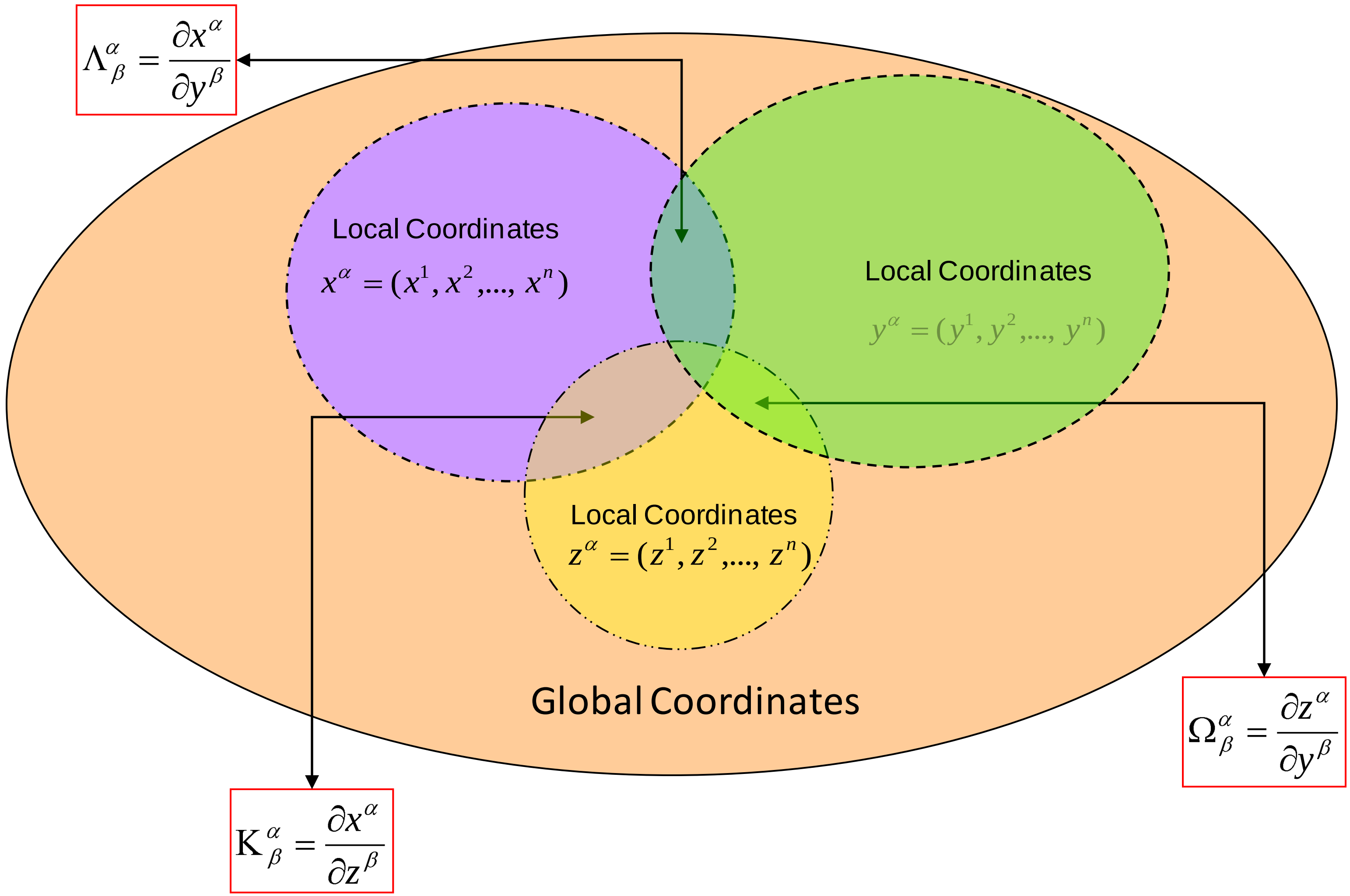
Basic definitions

- A reference system is the complete specification of how a (celestial) coordinate system is to be formed. The specification consists of a set of rules, resolutions, recommendations, etc. which are based on a solid theory.

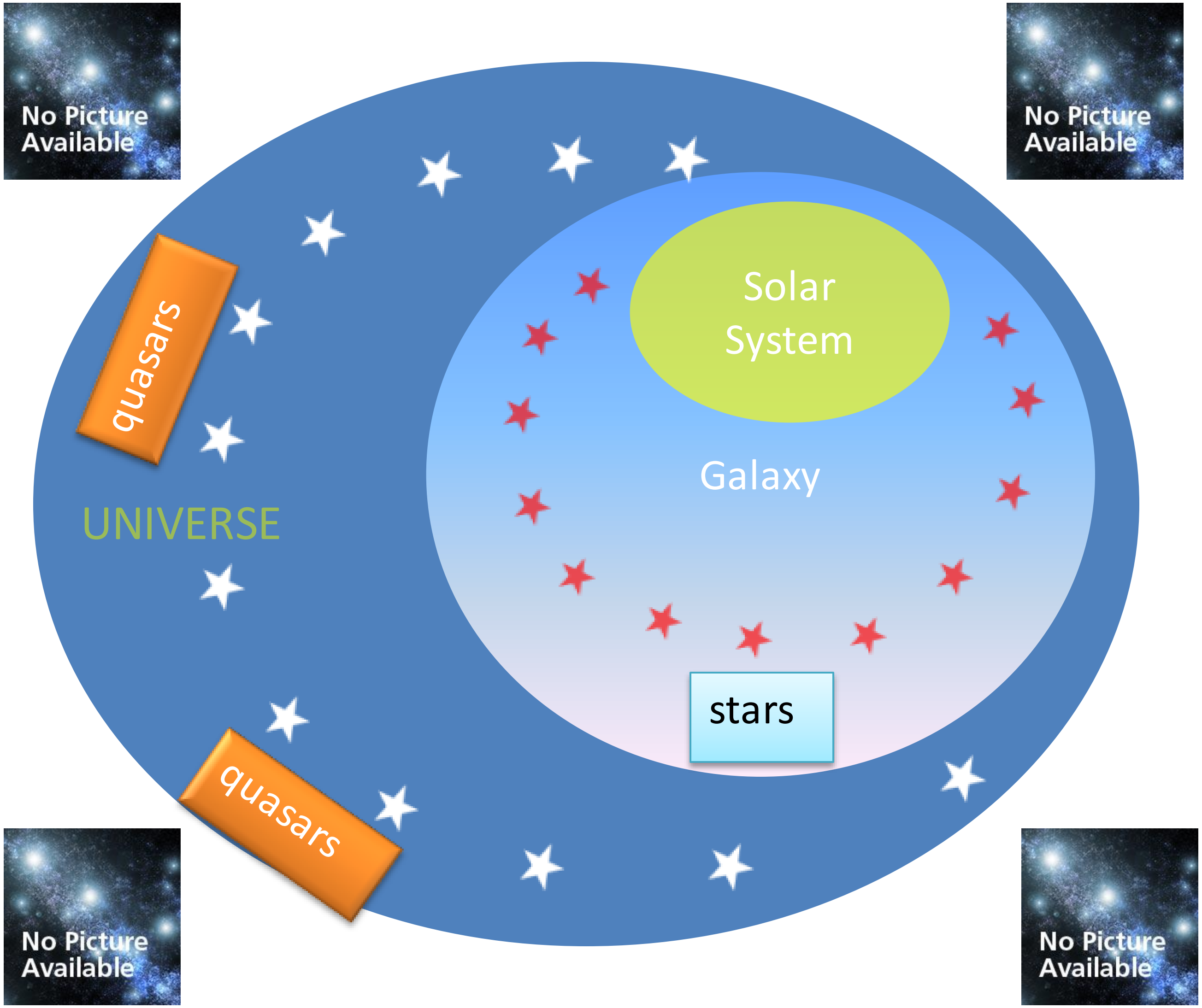
- A reference frame consists of a set of identifiable fiducial points (objects) on the sky along with their coordinates, which serve as the practical realization of a reference system.

- The current standard is the celestial reference system adopted by the IAU. Its origin is at the barycenter of the solar system, through *appropriate* modeling of VLBI observations in the framework of GR, with axes that are *intended* to be *fixed* with respect to *space* (time-independent,kinematically-nonrotaing orientation)

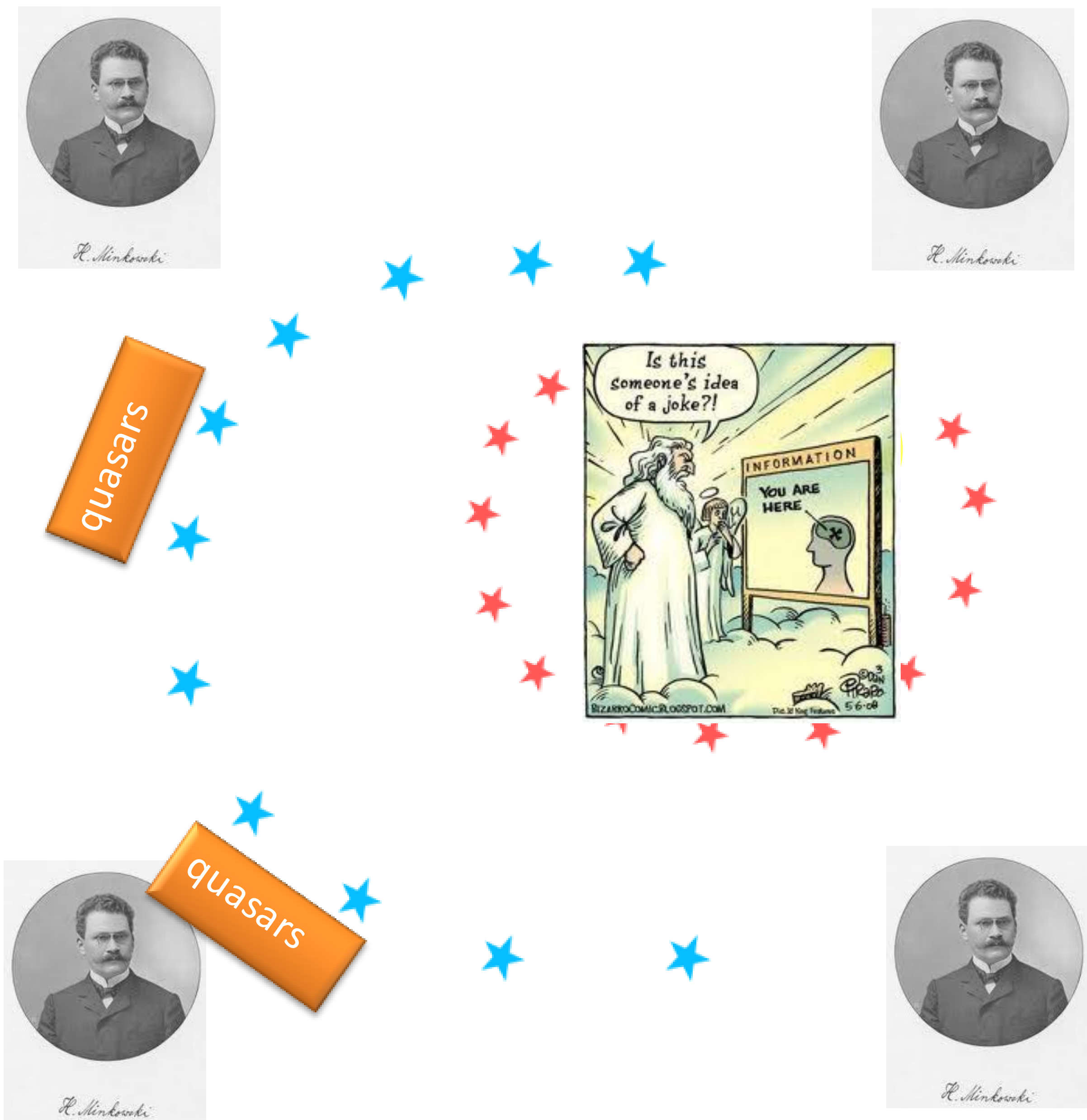
Reference System Specification in GR:
Manifold + Metric + Connection + Curvature



Field Equations + Boundary Conditions



Cosmological and the galactic relativity effects are rather large but almost static. Difficult to measure but important for astrometry and gravitational physics.



The IAU 2000 paradigm:
Space-time is flat at ***infinity***.
The only source of gravity is a matter of the solar system.

This authoritative book presents the theoretical development of gravitational physics as it applies to the dynamics of celestial bodies and the analysis of precise astronomical observations. The authors work at the University of Missouri and the United States Naval Observatory, which is one of the premier institutions in the world for expertise in astrometry, celestial mechanics, and timekeeping. The initial chapters review the fundamental principles of celestial mechanics and of special and general relativity. This background material is the foundation for understanding relativistic celestial mechanics, astrometry, and geodesy which is treated in the main part of the book. The text is based on recent recommendations from the International Astronomical Union.

From the contents:

- Newtonian celestial mechanics
- Introduction to Special Relativity
- General Relativity
- Relativistic Reference Frames
- Post-Newtonian Coordinate Transformations
- Relativistic Celestial Mechanics
- Relativistic Astrometry
- Relativistic Geodesy
- Relativity in IAU Resolutions

__Sergei Kopeikin__ studied general relativity at the Department of Astronomy of Moscow State University, Russia. He obtained his PhD in relativistic astrophysics from Moscow State University in 1986, where he was then employed as an associate professor. In 1993, he moved to Japan to teach astronomy at Hitotsubashi University, Tokyo. He was an adjunct staff member and thereafter visiting professor at the National Astronomical Observatory of Japan. In 1997, Professor Kopeikin moved to Germany and worked at the Institute of Theoretical Physics of the Friedrich Schiller University, Jena. Three years later he accepted the position of a professor of physics at the University of Missouri, Columbia, USA.

__Michael Efroimsky__ received his PhD from the University of Oxford in 1995. He then worked at Tufts, Harvard, and the University of Minnesota. Since 2002, he has been working as a staff astronomer at the US Naval Observatory in Washington, D.C. His current research interests are centered around celestial mechanics of the solar system. Dr. Efroimsky served as the Chair of the Division on Dynamical Astronomy of the American Astronomical Society, and is currently a member of several commissions of the International Astronomical Union.

__George Kaplan__ was a staff astronomer at the U.S. Naval Observatory in Washington, D.C., from 1971 to 2007, and now works as an independent consultant. He received his PhD degree from the University of Maryland, USA, in 1985. His professional interests focus on the field of positional astronomy, both its observational and theoretical aspects. His work includes publications in astrometry, celestial reference systems, solar system ephemerides, Earth rotation, navigation algorithms, and astronomical software. Dr. Kaplan is currently the president of Commission 4 (Ephemerides) of the International Astronomical Union. The minor planet 16074 is named in his honor.

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Kopeikin · Efroimsky · Kaplan

Relativistic Celestial Mechanics of the Solar System

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Michael Efroimsky, George Kaplan

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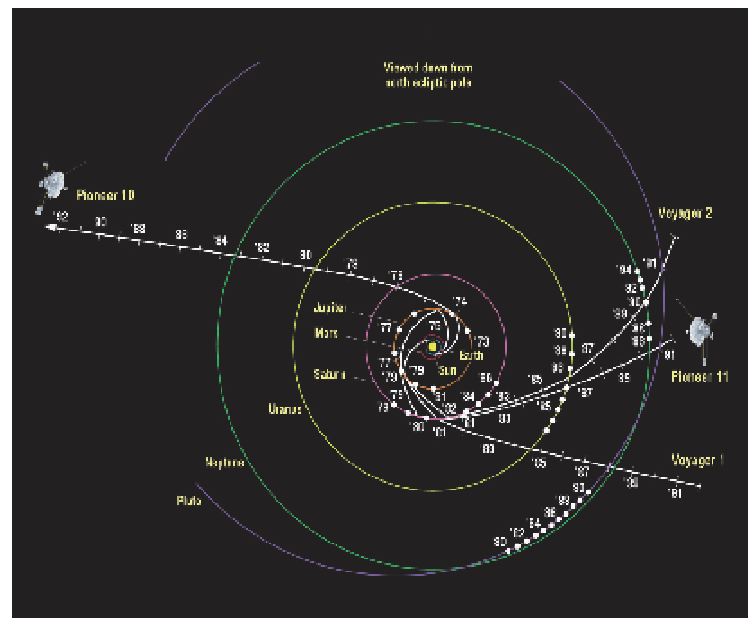
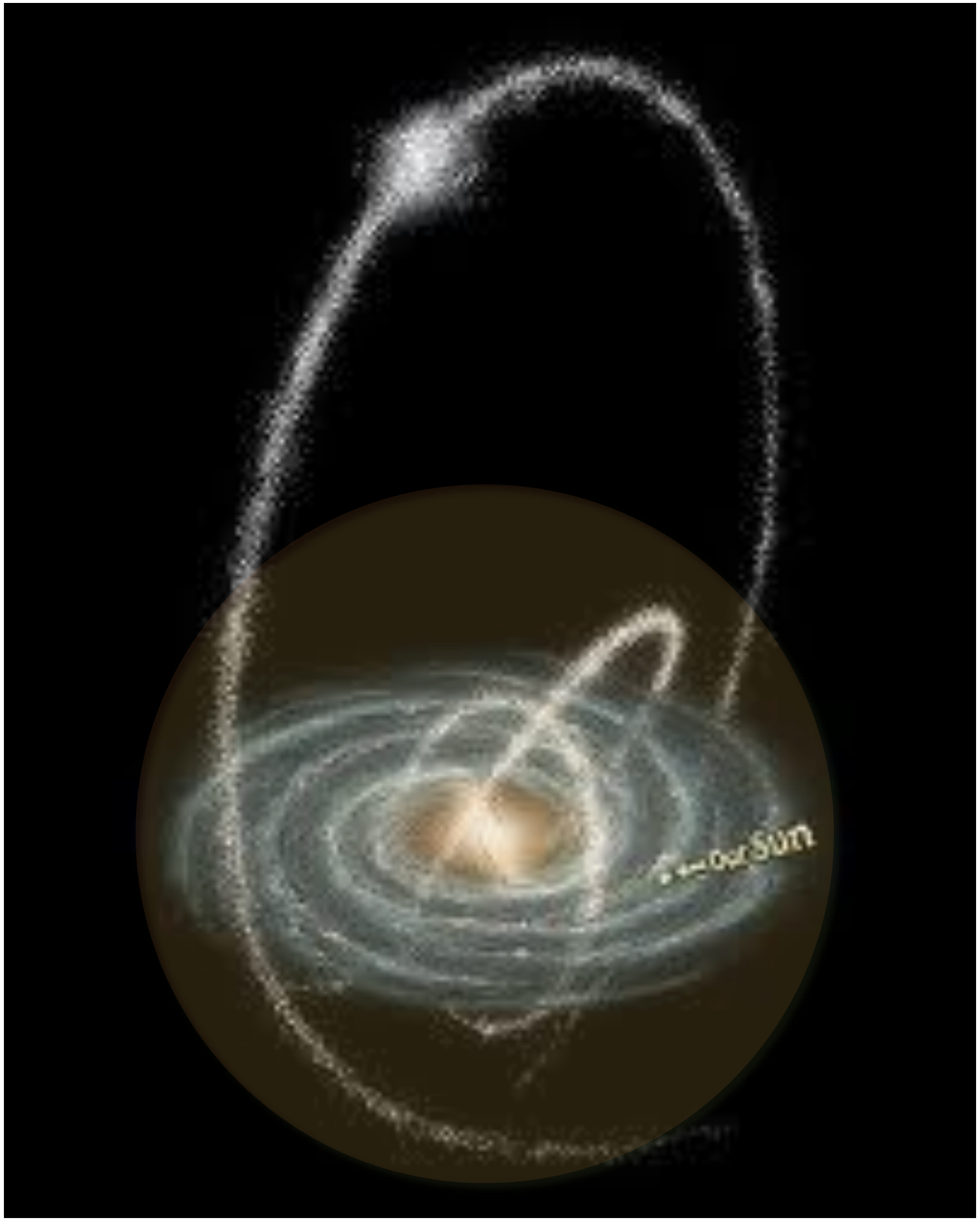




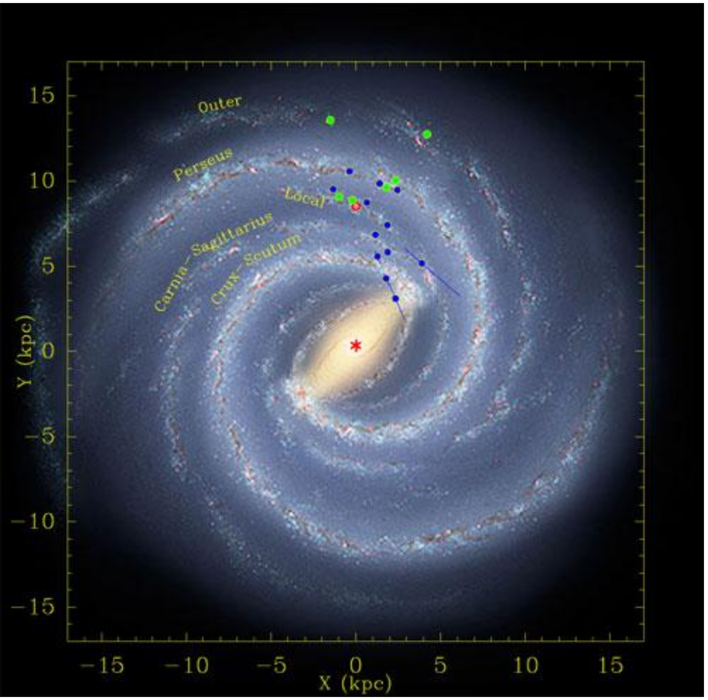
Galactic Dynamics:

- *stars*
- *globular clusters*
- *dark matter halo*
- *structure of the disc, bulge,…*

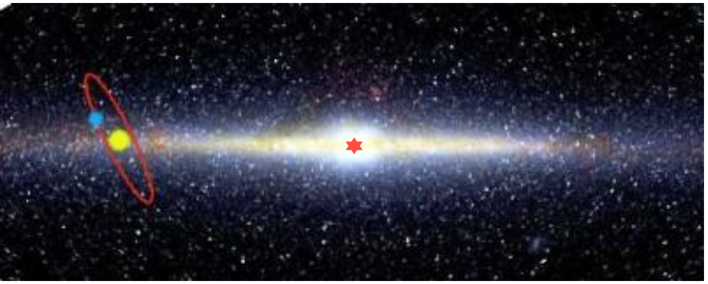
- *propagation of light*

- *fundamental gravitational physics (the central BH, MOND, gravitational waves)*

Galactocentric Reference System



M. J. Reid et al., NRAO eNews: March 2009, **2**, Issue 3



- A rotational transformation of the Barycentric Coordinate Reference System supplemented by a translation of the BCRS origin to the galactic center.

- The X-Y plane is the transverse plane of the disc of the Milky Way (the galactic plane).

- A "small" uncertainty in the distance to the galactic centre prevents this system from being widely used.

- Galactic coordinates (no translation to the center of the galaxy) are much more common.

References to the galactocentric coordinates

- Classic works by Lindblad, Oort, Ogorodnikoff, Milne
- The **Hipparcos Space Astrometry Mission Team**
- R_0, Θ_0, ω_0 measurements by Reid, Ghez et al., Genzel et al.
- Relativistic aspects:
  - Damour & Taylor (1991) "*On the orbital period change of the binary pulsar PSR 1913+16*", ApJ, **366**, 501-511
  - Brumberg & Kopeikin (1990) "*Relativistic time scales in the solar system*", CMDA, **48**, 23-44
  - Kopeikin (1994) "*Supplementary parameters in the parameterized post-Keplerian formalism*", **434**, 67-70
  - Kopeikin & Ozernoy (1999) "*Post-Newtonian Theory for Precision Doppler Measurements of Binary Star Orbits*", ApJ, **523**, 771-785
  - Kopeikin & Makarov (2007) "*Astrometric Effects of Secular Aberration*", AJ, **131**, 1471-1478

Galactocentric Reference System

t - the barycentric coordinate time (T CB)

$\vec{x}$ - the barycentric spatial coordinates (TCB coordinates))

T - the galactocentric dynamical time (TDG)

$\vec{X}$ - the galactocentric spatial coordinat es (TDG coordinates)

Spacetime at infinity is asymptotically flat. The metric tensor:

$g_{\alpha\beta}(t, \vec{x})$ - the barycentric coordinates of th e solar system

$G_{\alpha\beta}(T, \vec{X})$ - the galactocentric coordinates of the Milky Way

The metric tensors are found by solving the gravity field equations of Einstein's theory of general relativity or by a valid alternative theory.

Spacetime transformation

$$G_{\alpha\beta}(T,\vec{X}) = g_{\mu\nu}(t,\vec{x})\frac{\partial x^{\mu}}{\partial X^{\alpha}}\frac{\partial x^{\nu}}{\partial X^{\beta}}$$

matching the two metrics

annual variation ~ 0.37 s

$\sim 8.4\times10^{-7}$

$\sim 2\times10^{-15}\,\mathrm{yr}^{-1}$

$$t = T - \frac{1}{c^2}\left[B(T) - \vec{V}_{\odot}\cdot(\vec{X} - \vec{X}_{\odot})\right]$$

$$B(T) = B_0 + \dot{B}_0 (T - T_0) + \frac{1}{2}\ddot{B}_0 (T - T_0)^2 + \ldots$$

$$x^i = \Lambda^{ij}(X^j - X^j_{\odot})$$

$$\Lambda^{ij} = \left[1 + \frac{U_G}{c^2}\right]\delta^{ij} + \frac{1}{c^2}\left\{\frac{1}{2}V^i_{\odot}V^j_{\odot} + F^{[ij]}\right\}$$

$$\frac{dB}{dT} = \frac{1}{2}\vec{V}^2_{\odot} + U_G$$

time delay of TCB versus TDG

$$\frac{dF^{[ij]}}{dT} = \frac{3}{2}\left(V^i_{\odot}\partial^j U_G - V^j_{\odot}\partial^i U_G\right)$$

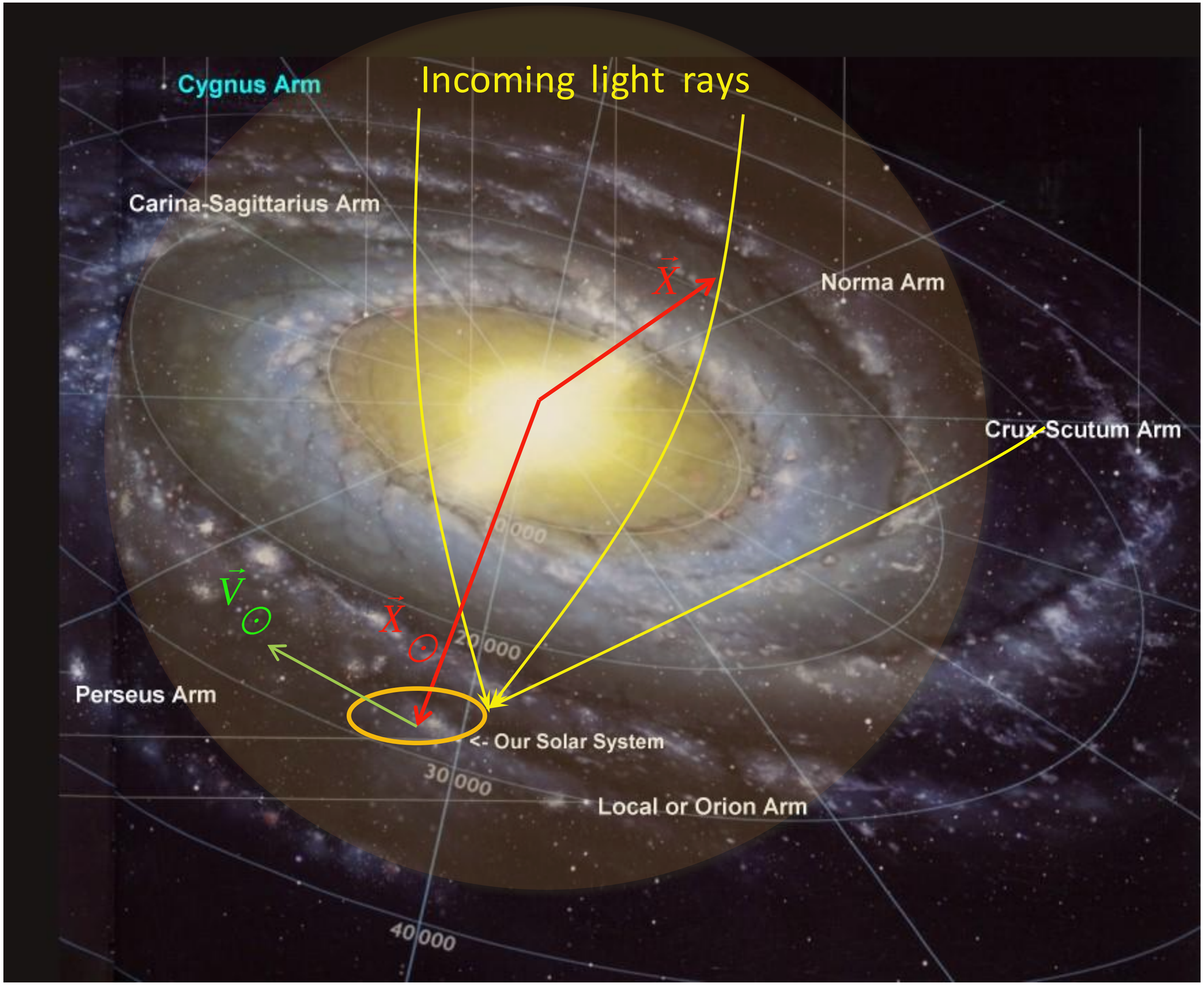
the de Sitter precession of Laplace's invariable plane

The precession rate is $\sim 0.004\ \mu\mathrm{as}\cdot\mathrm{yr}^{-1}$

$$\vec{X}_{\odot}(T) = \vec{X}_{\odot}(T_0) + \vec{V}_{\odot}(T_0)(T - T_0) + \frac{1}{2}\vec{A}_{\odot}(T_0)(T - T_0)^2 + \ldots$$

$$\vec{V}_{\odot}(T) = \vec{V}_{\odot}(T_0) + \vec{A}_{\odot}(T_0)(T - T_0) + \ldots$$

Light propagation



The astrometric equation

$$\vec{\mathbf{K}} = \vec{\mathbf{s}} - \vec{\boldsymbol{\alpha}} - \vec{\boldsymbol{\alpha}}_{\pi} - \vec{\boldsymbol{\alpha}}_{\mu} - \vec{\boldsymbol{\alpha}}_{A} - \vec{\boldsymbol{\alpha}}_{E} - O\ \vec{\boldsymbol{\alpha}}^{2}$$

catalogue

observed

the solar system corrections

kinematic parallax

Intrinsic proper motion

aberration

gravitational deflection (macrolensing)

non-linear terms

$$\vec{\boldsymbol{\alpha}}_{\mu} = \vec{\boldsymbol{\mu}}(T - T_0) + \ldots$$

$$\vec{\boldsymbol{\alpha}}_{\pi} = \vec{\boldsymbol{\alpha}}_{\pi}(T_0) + \dot{\vec{\boldsymbol{\alpha}}}_{\pi}(T - T_0) + \ldots$$

$$\vec{\boldsymbol{\alpha}}_{A} = \vec{\boldsymbol{\alpha}}_{A}(T_0) + \dot{\vec{\boldsymbol{\alpha}}}_{A}(T - T_0) + \ldots$$

$$\vec{\boldsymbol{\alpha}}_{E} = \vec{\boldsymbol{\alpha}}_{E}(T_0) + \dot{\vec{\boldsymbol{\alpha}}}_{E}(T - T_0) + \ldots$$

$$\vec{\boldsymbol{\mu}}_{\mathrm{obs}} = \vec{\boldsymbol{\mu}} + \dot{\vec{\boldsymbol{\alpha}}}_{\pi} + \dot{\vec{\boldsymbol{\alpha}}}_{A} + \dot{\vec{\boldsymbol{\alpha}}}_{E}$$

Numerical Estimates

$$\dot{\vec{\boldsymbol{\alpha}}}_{\pi} = \frac{\mathbf{K}\times\ \mathbf{V}_{\odot}\times\mathbf{K}}{R} = 51\ \text{mas}\cdot\text{yr}^{-1}\left[\frac{1\ \text{kpc}}{R}\right]$$

→ Distance to Sgr A*

$$= 51\ \mu\text{as}\cdot\text{yr}^{-1}\left[\frac{1\ \text{Mpc}}{R}\right]$$

→ Distance to the Local Group

$$\dot{\vec{\boldsymbol{\alpha}}}_{A} = \frac{\mathbf{K}\times\ \mathbf{A}_{\odot}\times\mathbf{K}}{c} = 4.2\ \mu\text{as}\cdot\text{yr}^{-1}$$

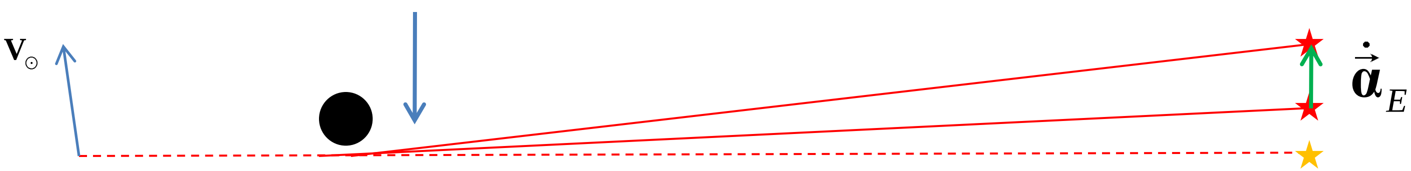
→ The galactocentric acceleration of the solar system

$$\dot{\vec{\boldsymbol{\alpha}}}_{E} = \frac{4GM(d)}{c^{2}d}\,\frac{\mathbf{K}\times\ \mathbf{V}_{\odot}\times\mathbf{K}}{d} = 10\ \mu\text{as}\cdot\text{yr}^{-1}\left[\frac{M(d)}{10^{6}M_{\odot}}\right]\left[\frac{1\ \text{pc}}{d}\right]^{2}$$

→ Macrolensing – F. Mignard;

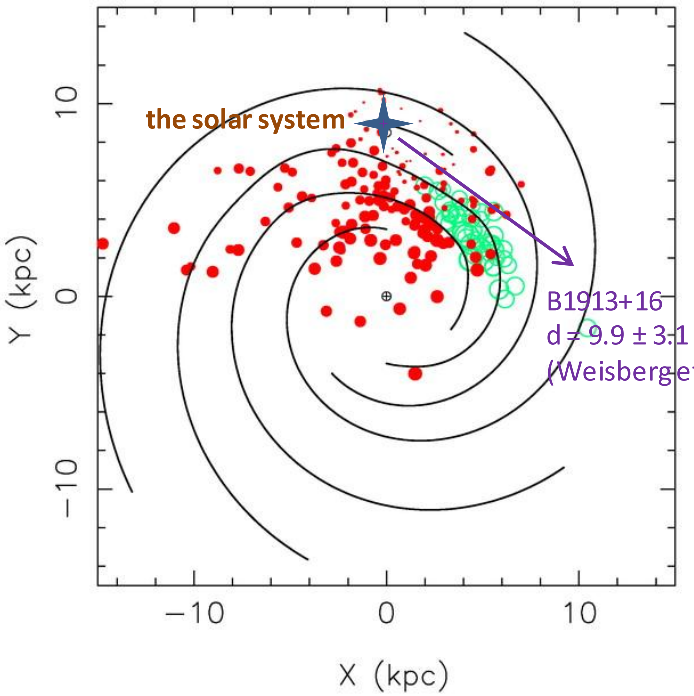
100 µas – Ian Browne

$$\boldsymbol{\alpha}_{\text{Sgr A}^*} = -6.379\pm 0.026\ \text{mas}\cdot yr^{-1}\quad (\text{Reid and Brunthaler 2004})$$



Probability of a macrolensing event – Francois Finet P=0.006

Pulsar Timing of PSR B1913+16



The distribution of the dispersion measures at the locations of pulsars. The spiral arm locations are adopted from the Cordes-Lazio (2002) model (Figure from Ramesh Bhat et al. , 2004)

Observed value of the orbital period

$$P_{\mathrm{b,\ obs}} = P_{\mathrm{b}} \frac{1 - \frac{V_r}{c}}{\sqrt{1 - \frac{V^2}{c^2}}} \qquad \vec{\boldsymbol{V}} = \vec{\boldsymbol{k}} V_r + \vec{\boldsymbol{V}}_\mu$$

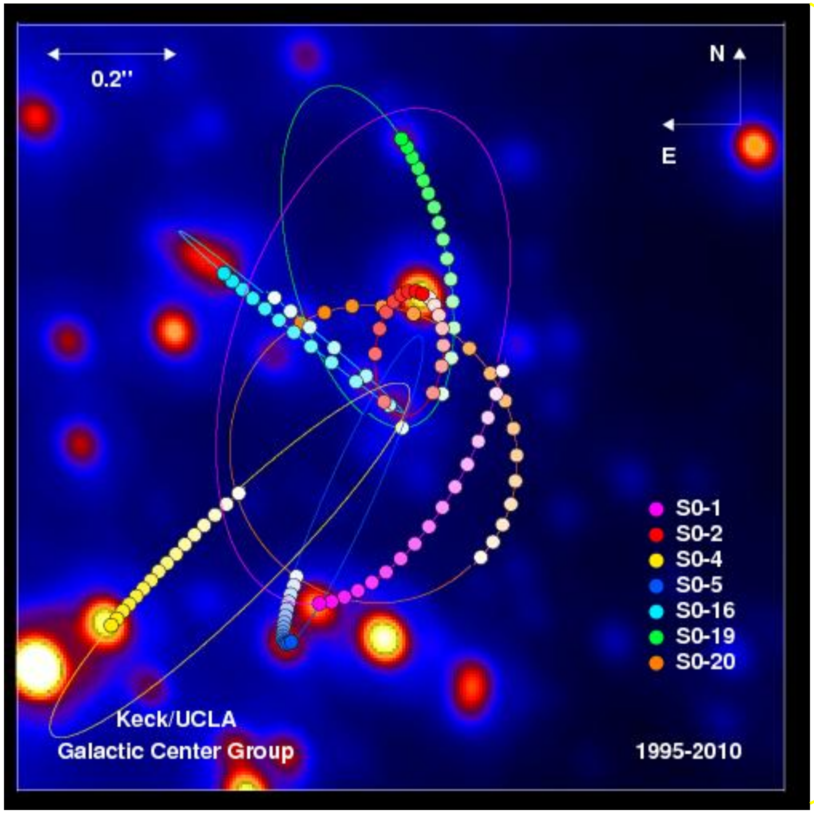
$$\frac{\dot{P}_{\mathrm{b,\ obs}}}{P_{\mathrm{b}}} = \frac{\dot{P}_{\mathrm{b,\ GR}}}{P_{\mathrm{b}}} - \frac{A_{\mathrm{rad}}}{c} - \frac{\mu^2 R}{c} - \dots$$

J.H. Taylor and collaborators

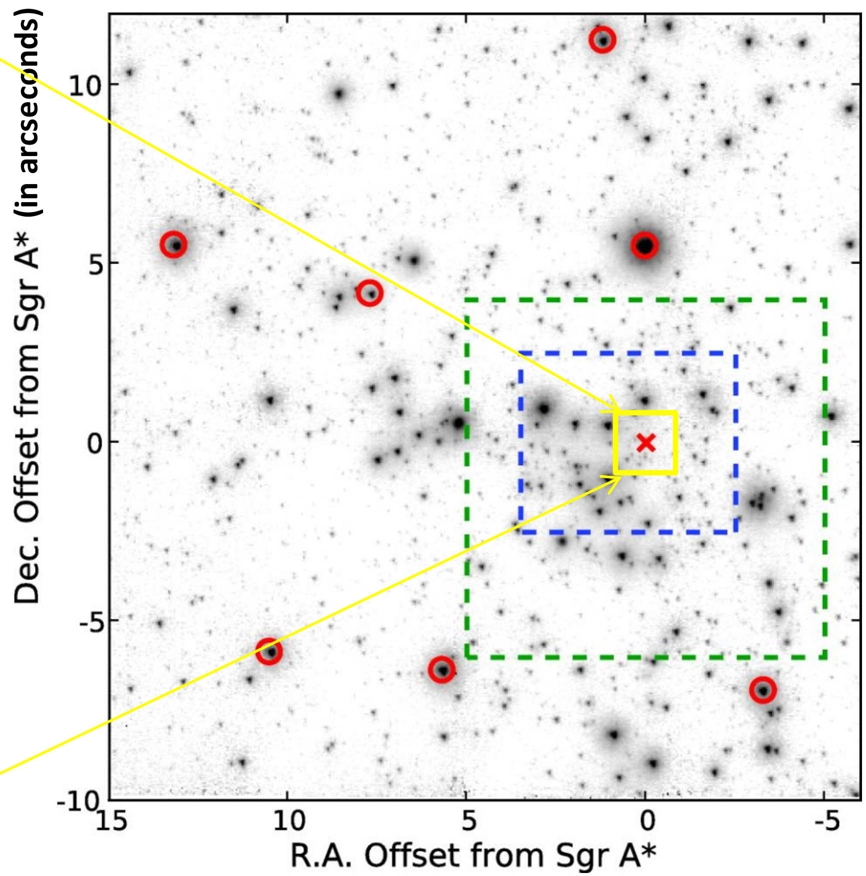
$$\frac{\dot{P}_{\mathrm{b}} \text{-} \dot{P}_{\mathrm{b,gal}}}{\dot{P}_{\mathrm{b,GR}}} = 1.0081 \pm 0.0098 \qquad \text{Damour \& Taylor, 1991}$$

$$\frac{\dot{P}_{\mathrm{b}} \text{-} \dot{P}_{\mathrm{b,gal}}}{\dot{P}_{\mathrm{b,GR}}} = 0.997 \pm 0.002 \qquad \text{Weisberg, Nice, \& Taylor, 2010}$$

Fundamental galactocentric parameters and the central BH

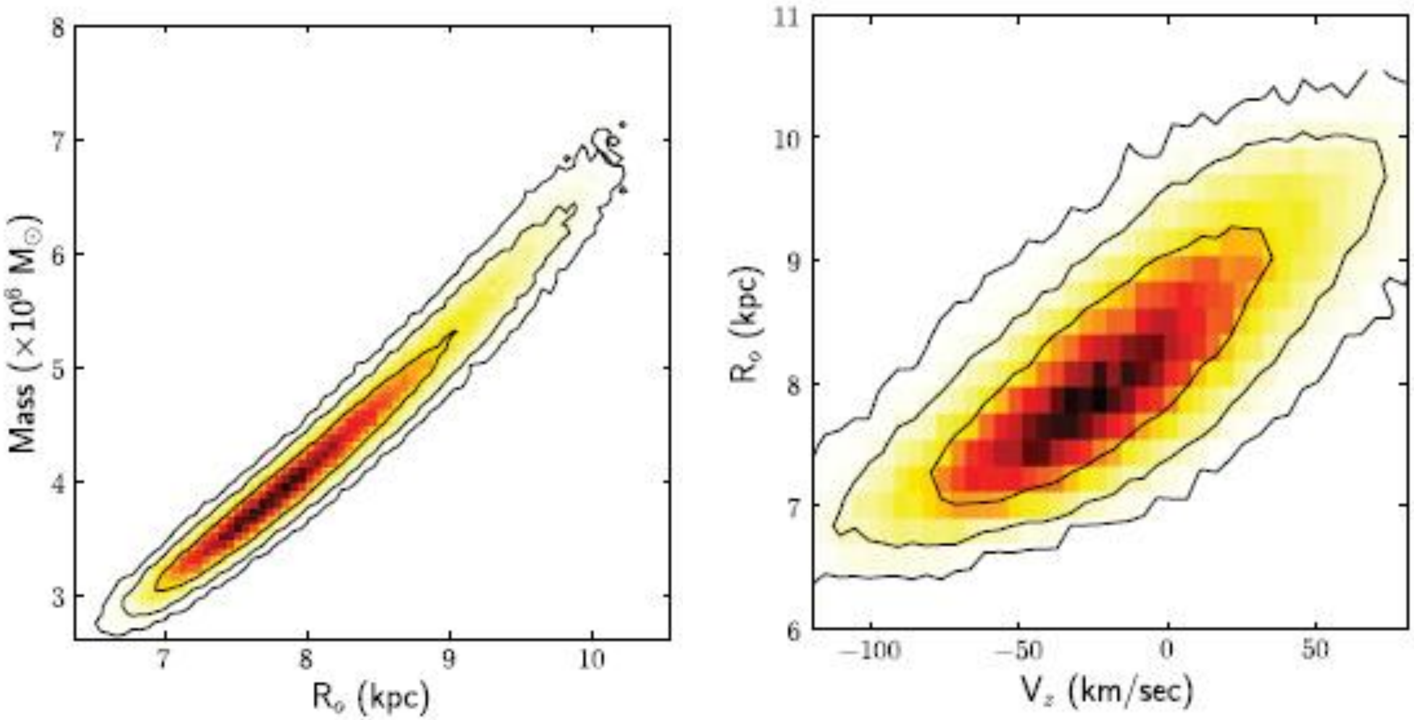


Orbits of stars orbiting the super-massive BH at the center of the Milky Way.



The seven masers (red giants), whose radio positions are well measured by Reid et al. (2007) and which are used to establish an absolute reference frame, are circled. Picture credit to: Ghez et al. ApJ (2008) **689,** 1044

Mass-distance and mass-velocity correlation relationships

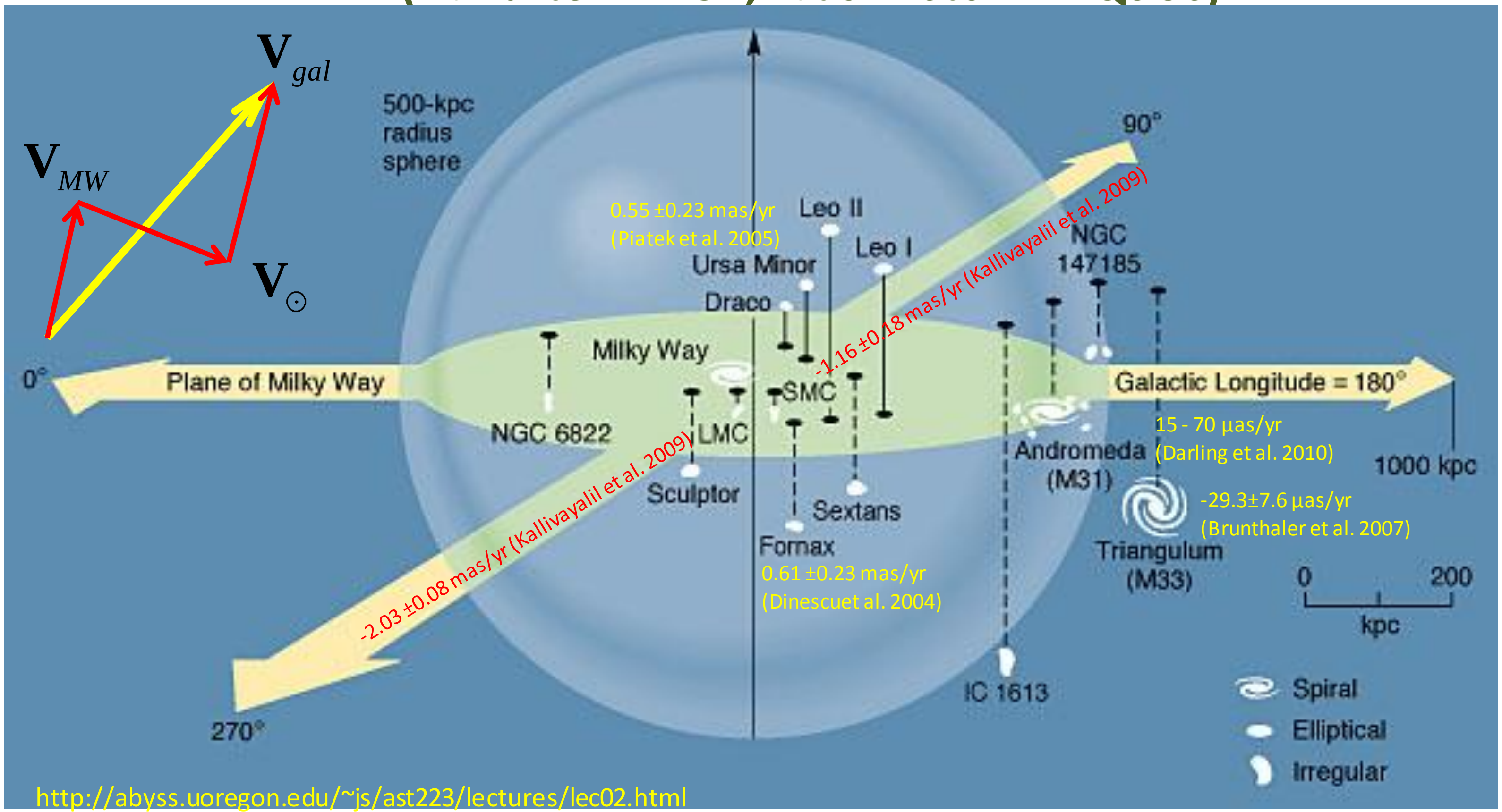


Correlation of the measured mass of the central BH with the distance to the center of the Milky Way: $M \sim R_0^{1.8}$ and velocity.

A. Ghez et al. : $M = 4.53 \pm 0.55 \times 10^6 M_\odot$ for $R_0 = 8.36 \pm 0.44$ kpc

The Local Group

(N. Bartel – M81, K. Jonhston – 4 QSOs)



Measuring the proper motions and geometric distances of galaxies within the Local Group is very important for our understanding of the history formation, evolution, and gravitational potential of the Local Group. Measurements that yield astrometric accuracies of 10 micro-arcseconds make determination of proper motions and angular rotation rates of galaxies out to a distance of 1 Mpc feasible. Supplemented with radial velocity measurements it will allow us to reconstruct a three-dimensional gravitational potential and to set a stringent limits on the parameters of dark matter halo.

Panoramic view of the entire near-infrared sky reveals the distribution of galaxies beyond the Milky Way. Blue are the nearest sources ($z < 0.01$); green are at moderate distances ($0.01 < z < 0.04$) and red are the most distant sources that 2MASS resolves ($0.04 < z < 0.1$). The map is projected with an equal area Aitoff in the Galactic system (Milky Way at center).

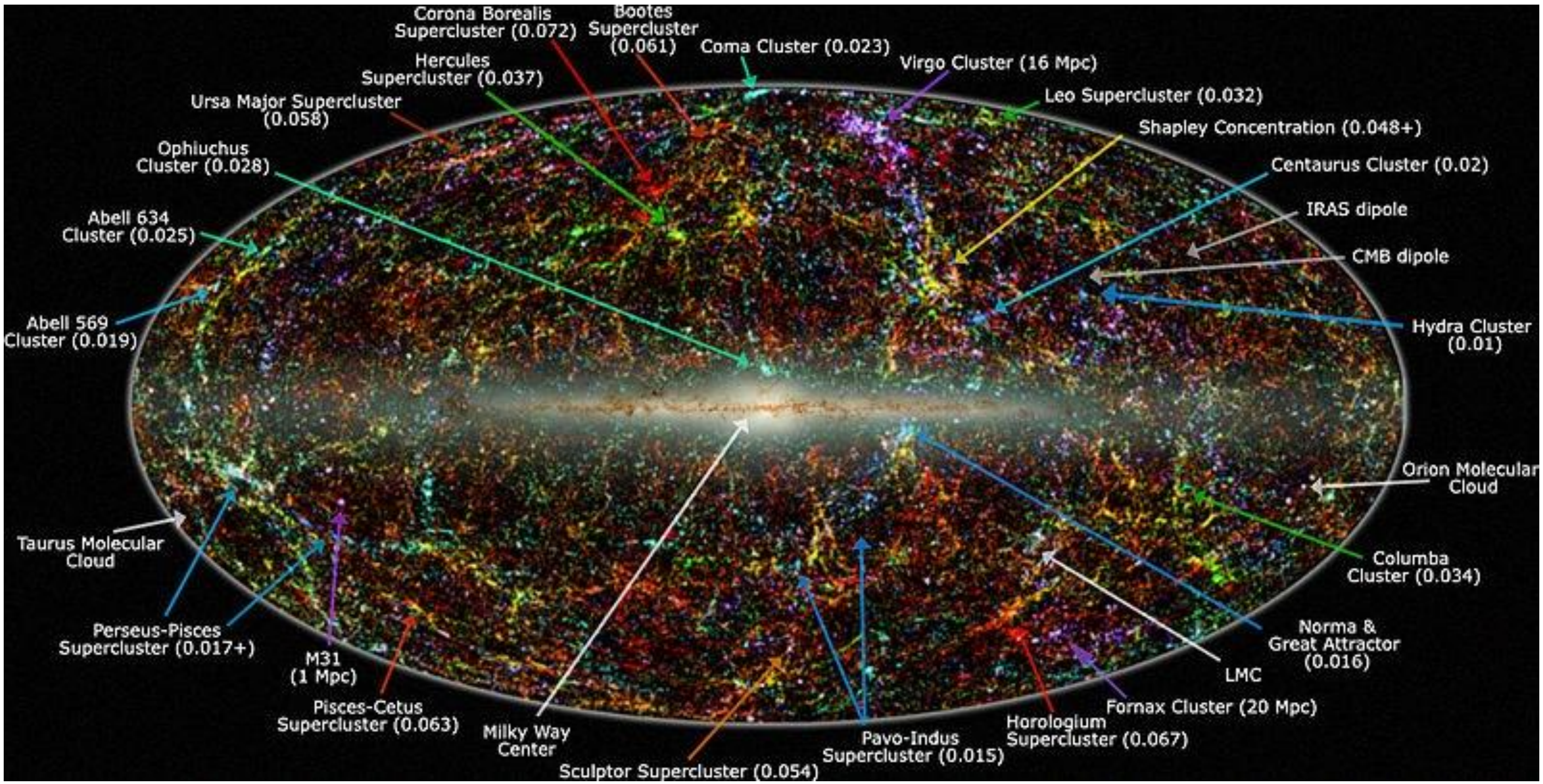


Image credit to: Thomas Jarrett

Theoretical aspects of astrometric cosmology

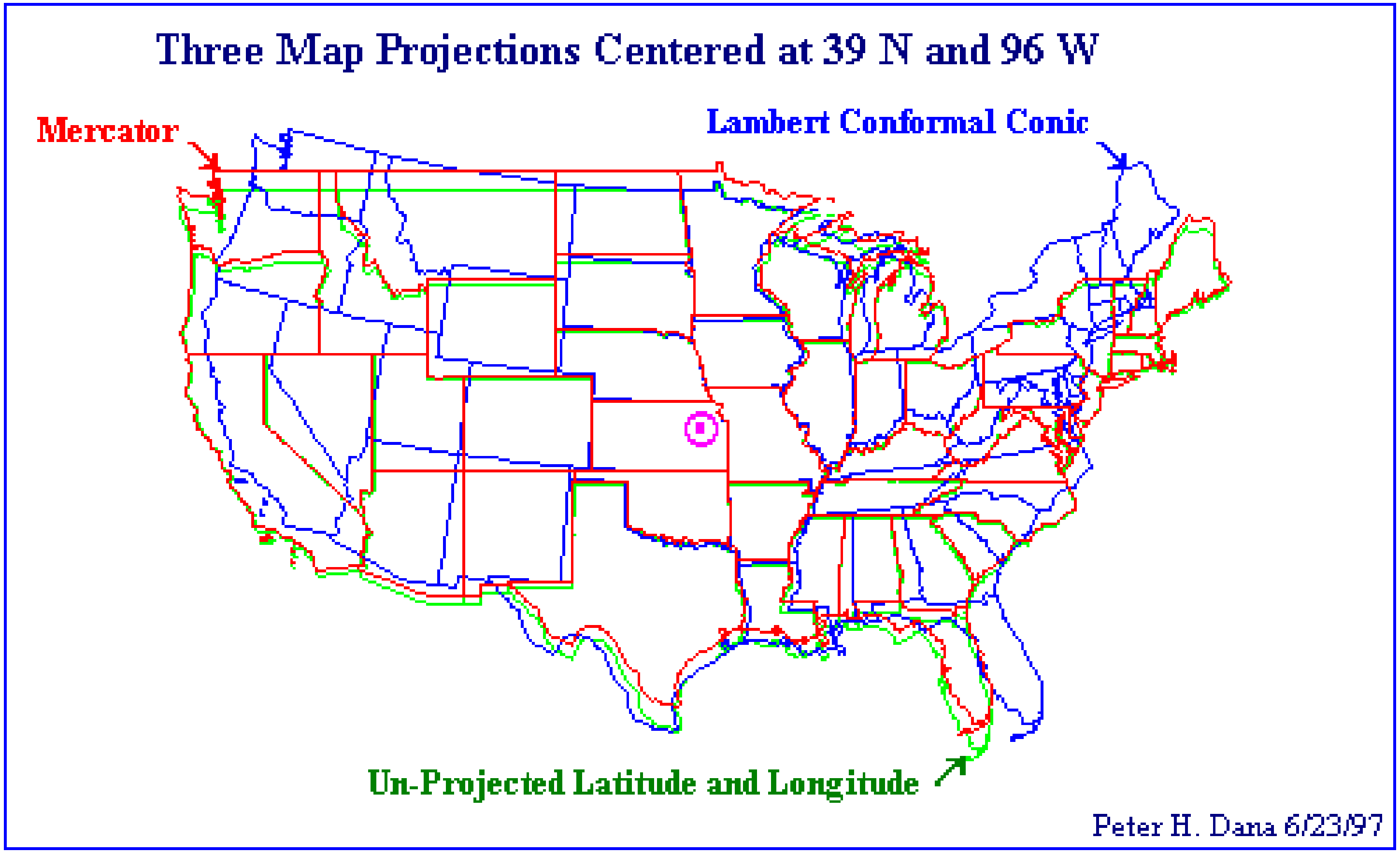
(other aspects, talk by M. Lattanzi)

The standard IAU resolutions state that the space-time is asymptotically flat at infinity.

However, we live in expanding universe, which is described by the cosmological metric of Friedmann-Robertson-Walker fully-symmetric and globally-curved four-dimensional space-time manifold.

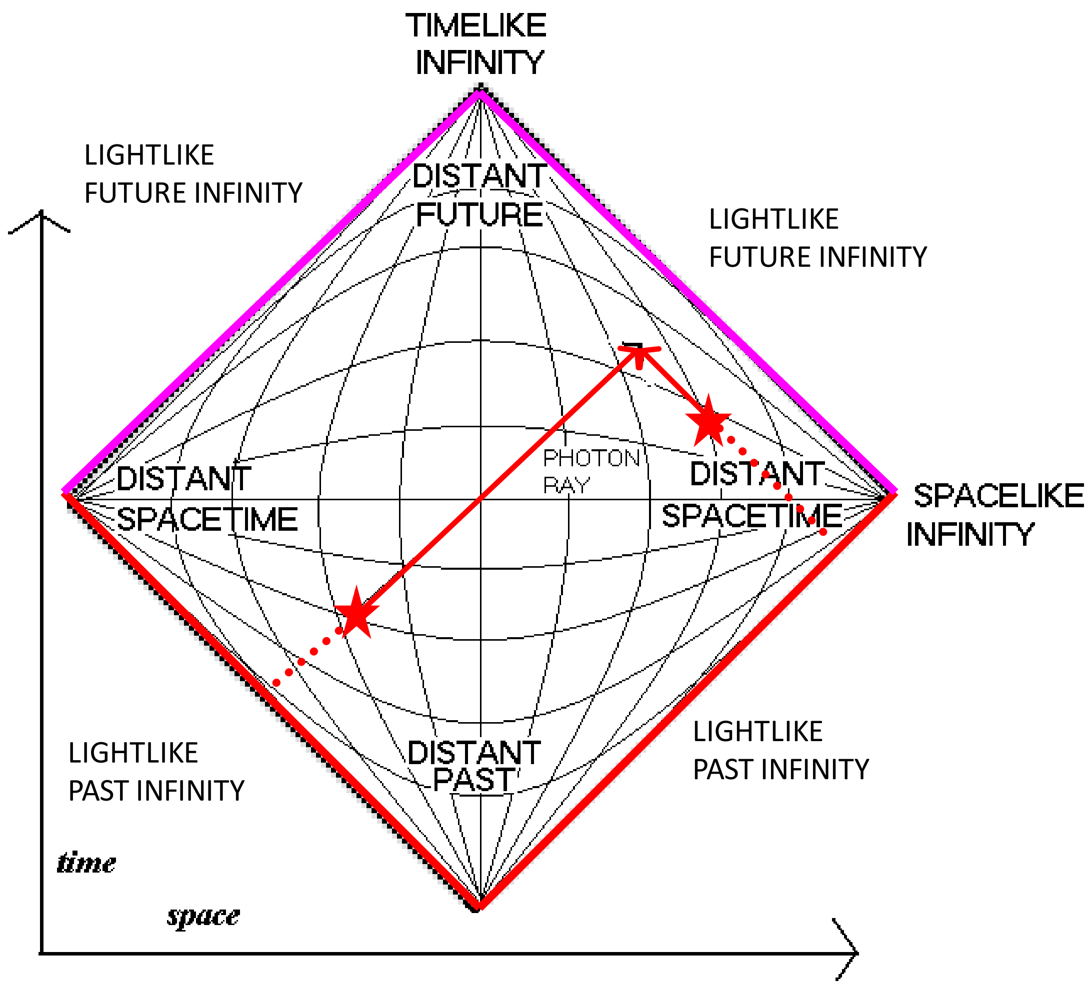
How to map cosmological space-time to the tangent space of a local astrometric observer?

Mapping a curved surface on a plane is not unique



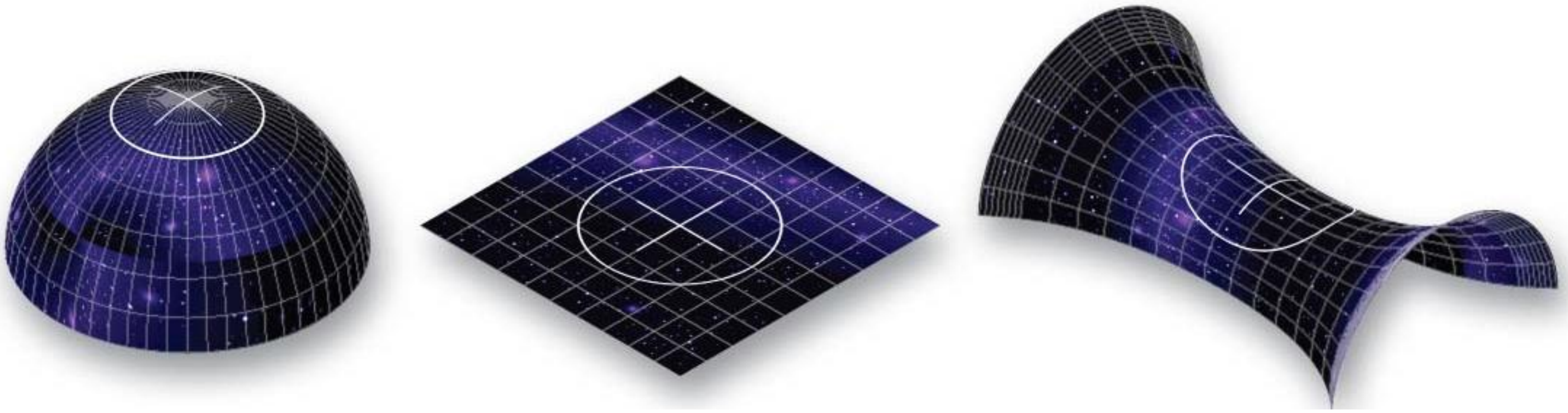
Conformal Infinity and Astrometric Catalogue

Astrometric catalogue (celestial reference frame) is defined on the celestial sphere by directions of light rays from a set of "non-moving" reference celestial objects located at ***conformal null-like past infinity*** of space-time (Penrose 1963)



$$ds^2 = -c^2 dt^2 + dr^2 + r^2 \underbrace{(d\vartheta^2 + \sin^2\vartheta d\varphi^2)}_{\text{metric tensor of a unit sphere}}$$

Space in the Universe



Riemann space

Example: sphere

Curvature of space is positive.
Parallel lines converge to a point.
The shortest distance between two points is a piece of a big circle.

Euclid space

Example: plane

Curvature of space is flat.
Parallel lines never converges.
The shortest distance between two points is a straight line.

Lobachevsky space

Example: pseudo-sphere

Curvature of space is negative.
Parallel lines always diverge . The shortest distance between two points is a piece of a conical section passing through the center of the pseudo-sphere.

Goals and targets of astrometric cosmology

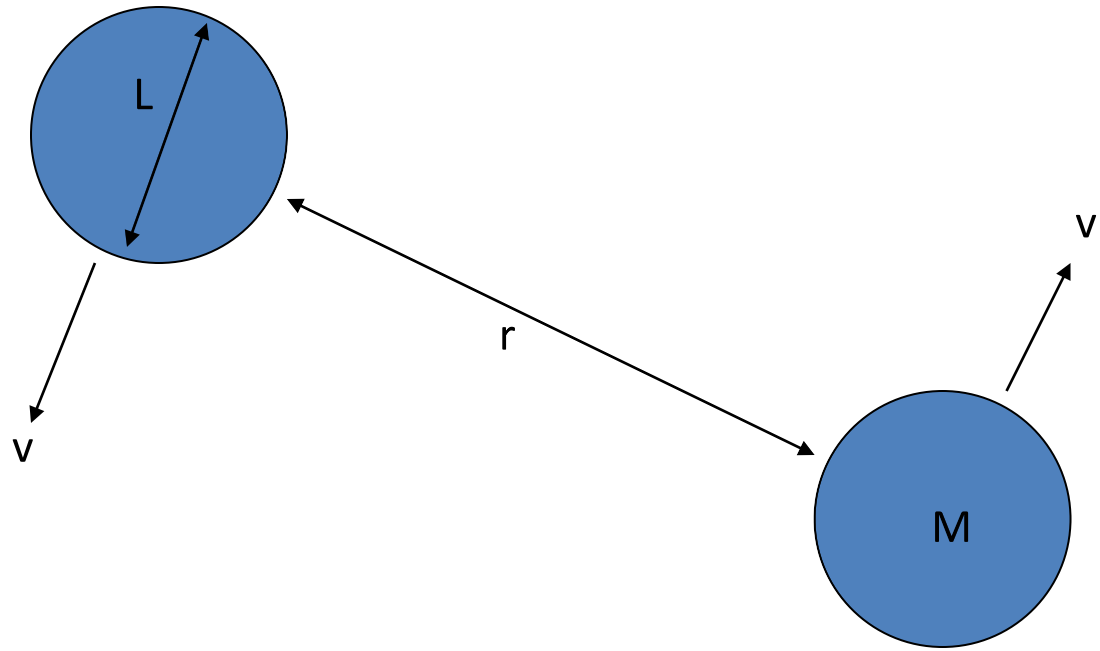
- Gravitational-wave astronomy of cosmological sources at early universe
- Cosmological gravitational lenses
- Dynamics of interacting galaxies and BBH
- Dark matter and dark energy
- Cosmological effects inside the solar system
- Cosmological PPN formalism and gravity tests

Mathematics to solve (challenging) :

- Interaction between internal gravity and cosmological expansion (Do planet orbits take part in the cosmological expansion?)
- Separation of matter and metric tensor perturbations from their background values
- Back-reaction of the perturbations on the background (non-linear approximations)
- Equations of motion and observables

Binary system in flat space-time

$$\eta \approx \frac{GM}{c^2 L}$$ characterizes the strength of gravity inside the body



$$\varepsilon \approx \frac{\mathrm{v}}{c} \cong \left(\frac{GM}{c^2 r}\right)^{1/2} \cong \frac{r}{\lambda}$$

Characterizes:

(1) the speed of relative motion between the bodies

(2) the speed of internal motion

(3) wavelength of gravitational radiation

Post-Minkowskian approximations in asymptotically-flat space-time

The Minkowski metric = diag(-1,+1,+1,+1)

The gravitational constant G

$$g_{\alpha\beta} = f_{\alpha\beta} + G h^{(1)}_{\alpha\beta} + G^2 h^{(2)}_{\alpha\beta} + \dots$$

The metric tensor

The metric tensor perturbations

Small parameter: $\eta \approx \dfrac{GM}{c^2 L}$

Post-Newtonian approximation: additional small parameter $\varepsilon \approx \dfrac{v}{c}$

Post-Friedmanian approximations in cosmology

$$\bar{g}_{\alpha\beta} = R^2(t) \quad f_{\alpha\beta}$$

FRW metric — the scale factor — Minkowski metric

$$f_{\alpha\beta} = \mathrm{diag}\ \ -1, +1, +1, +1$$

R(t) – the “radius” of universe

$$H = \frac{\dot{R}}{R}$$ - the Hubble constant

$H_0 = 73.8 \ \pm\ 2.4\ \mathrm{km/s/Mpc}$ (HST)

$H_0 = 72.6 \ \pm\ 3.1\ \mathrm{km/s/Mpc}$ (grav. lensing)

$H_0 = 71.0 \ \pm\ 2.5\ \mathrm{km/s/Mpc}$ (WMAP)

$$\bar{G}_{\alpha\beta} = \bar{T}_{\alpha\beta}$$

The Einstein tensor of the background geometry $\bar{g}_{\alpha\beta}$

The stress-energy tensor of the background matter

Post-Friedmanian approximation in cosmology

(Kopeikin et al. 2001; Ramirez & Kopeikin 2002; work in progress)

'radius' of the universe

The metric tensor of the standard IAU resolutions

$$g_{\alpha\beta}(t,\vec{x}) = R^2(t)\left[f_{\alpha\beta} + G h^{(1)}_{\alpha\beta}(t,\vec{x},R(t)) + G^2 h^{(2)}_{\alpha\beta}(t,\vec{x},R(t)) + \ldots \right]$$

The metric tensor

The PN metric tensor perturbations

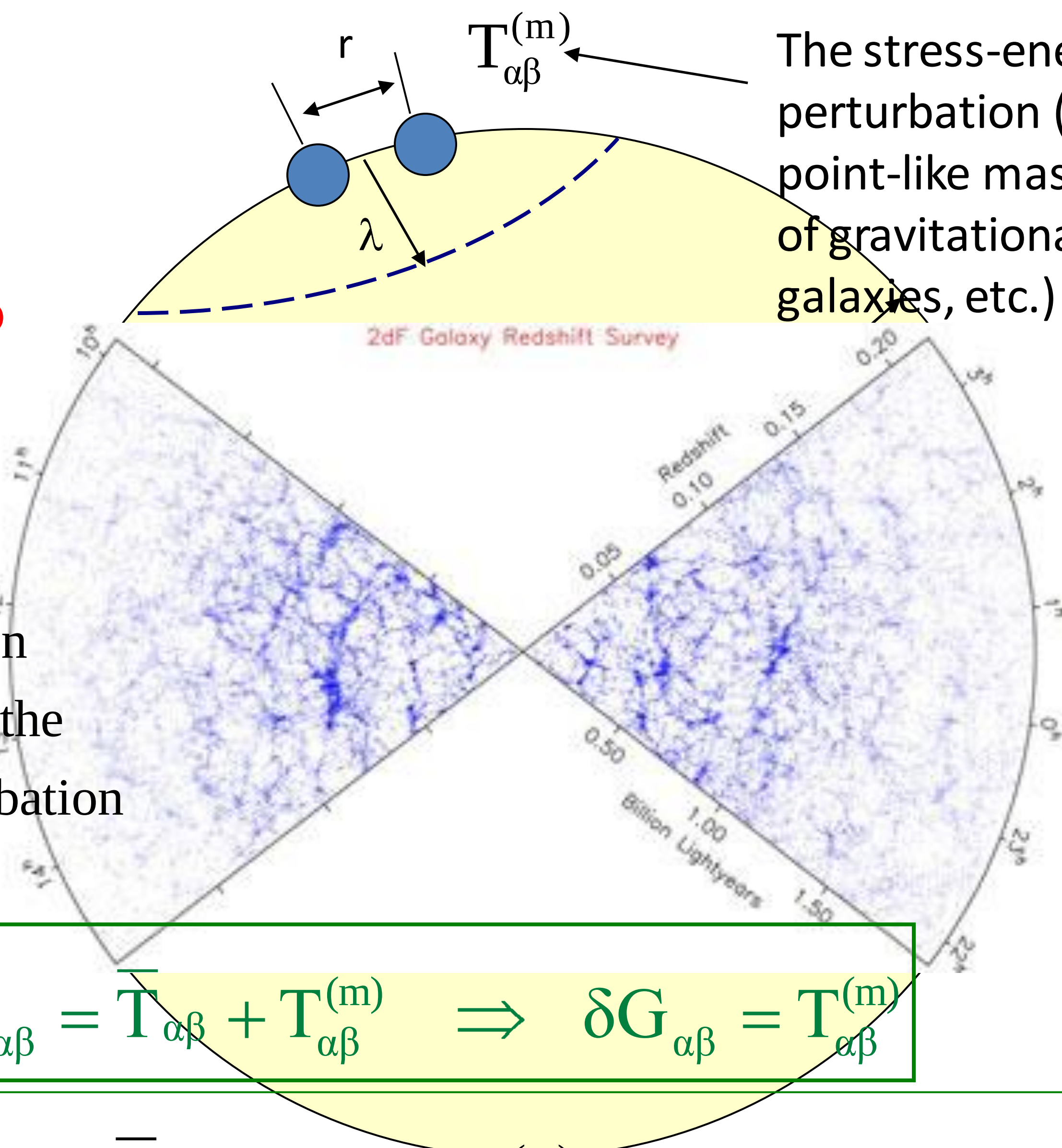
Three basic parameters: $\eta \approx \dfrac{GM}{c^2 L}$; $\sigma \approx \dfrac{r}{R} = \varepsilon\delta$; and $\dfrac{\Delta\rho}{\rho}$

where $\varepsilon \approx \dfrac{v}{c}$ and $\delta \approx \dfrac{\lambda}{R}$

Typical values of the parameters

	Binary pulsar	Solar system	Galaxy	Cluster of galaxies	Super-cluster
η	10^{-1}	2×10^{-6}	10^{-6}	10^{-7}	10^{-8}
ε	10^{-3}	10^{-4}	10^{-3}	10^{-3}	10^{-3}
σ	10^{-17}	10^{-13}	10^{-7}	10^{-5}	10^{-3}
$\left[\frac{\Delta\rho}{\rho}\right]_0$	10^{33}	10^{24}	10^{8}	10^{4}	10

How to perturb the manifold?



The stress-energy tensor of a bare perturbation (isolated system, point-like mass, localized package of gravitational waves, a cluster of galaxies, etc.)

$\rho h_{\alpha\beta} \ll \Delta\rho$

clean separation of a bare from the induced perturbation

$$\overline{G}_{\alpha\beta} + \delta G_{\alpha\beta} = \overline{T}_{\alpha\beta} + T^{(m)}_{\alpha\beta} \quad \Rightarrow \quad \delta G_{\alpha\beta} = T^{(m)}_{\alpha\beta}$$

NO !

$$\overline{G}_{\alpha\beta} + \delta G_{\alpha\beta} = \overline{T}_{\alpha\beta} + \underbrace{\delta T_{\alpha\beta} + T^{(m)}_{\alpha\beta}} \quad \Rightarrow \quad \delta G_{\alpha\beta} - \delta T_{\alpha\beta} = T^{(m)}_{\alpha\beta}$$

YES !

Post-Friedmanian cosmological equations

$$\gamma^{\alpha\beta} \equiv h^{\alpha\beta} - \frac{1}{2} f^{\alpha\beta} h$$

$$\gamma^{0\beta}{}_{|\beta} = 2H\varphi; \qquad \gamma^{i\beta}{}_{|\beta} = 0$$

$$\gamma^{00} = \frac{w}{4c^2}; \quad \gamma^{0i} = -\frac{w^i}{4c^2}; \quad \gamma^{ij} = \frac{w^{ij}}{4c^2}; \quad \chi = w$$

$$\left(\vec{\nabla}^2 - \frac{1}{c^2}\partial_\tau^2\right)\chi - 2H\partial_\tau\chi + \frac{5}{2}H^2\chi = -4\pi G[T_{00}^{(m)} + T_{ii}^{(m)}]$$

$$\left(\vec{\nabla}^2 - \frac{1}{c^2}\partial_\tau^2\right)w - 2H\partial_\tau w = -4\pi G[T_{00}^{(m)} + T_{ii}^{(m)}]$$

$$\left(\vec{\nabla}^2 - \frac{1}{c^2}\partial_\tau^2\right)w^i - 2H\partial_\tau w^i + H^2 w^i = -4\pi G T_{(m)}^{oi}$$

$$\left(\vec{\nabla}^2 - \frac{1}{c^2}\partial_\tau^2\right)w^{ij} - 2H\partial_\tau w^{ij} = -4\pi G T_{(m)}^{ij}$$

Solving the cosmological equations with the retarded Green functions

$$\left(\Box - \frac{a}{\eta}\frac{\partial}{\partial\eta} - \frac{b}{\eta^2}\right) F(\eta, x) = 4\pi S(\eta, x)$$

$$\nu = \sqrt{\left(\frac{1-a}{2}\right)^2 - b}$$

1. Index $\nu = 3/2$

$$F(\eta, \boldsymbol{x}) = \frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\frac{\Psi}{\eta}\right),$$

$$\Psi(\eta, \boldsymbol{x}) = \int d^3\boldsymbol{x}'\left(1 - \frac{\eta}{|\boldsymbol{x}-\boldsymbol{x}'|}\right)\int_{\eta_0}^{\eta-|\boldsymbol{x}-\boldsymbol{x}'|} vS(v, \boldsymbol{x}')dv\,.$$

2. Index $\nu = 5/2$

$$F(\eta, \boldsymbol{x}) = \frac{\partial}{\partial\eta}\left[\frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\frac{\Psi}{\eta}\right)\right],$$

$$\Psi(\eta, \boldsymbol{x}) = \int d^3\boldsymbol{x}'\left(1 - \frac{\eta}{|\boldsymbol{x}-\boldsymbol{x}'|}\right)\int_{\eta_0}^{\eta-|\boldsymbol{x}-\boldsymbol{x}'|} vdv\int_{v_0}^{v} S(y, \boldsymbol{x}')dy\,.$$

Impact of cosmology on the Newtonian potential and time scale

$$\mathrm{h}_{00}(\eta,\vec{x}) = G\int_V \frac{\rho(\eta_{ret.},\vec{x})R^2(\eta_{ret.})}{|\vec{x}-\vec{x}\,'|}\left[1 - H(\eta_{ret.})\,|\vec{x}-\vec{x}\,'| + \frac{1}{4}H^2(\eta_{ret.})\,|\vec{x}-\vec{x}\,'|^2\right]d^3x'$$

$$= \mathrm{U}(\eta_{ret.},\vec{x}) + G\int_V \rho(\eta_{ret.},\vec{x})H(\eta_{ret.})R^2(\eta_{ret.})d^3x' + \frac{G}{4}\int_V \rho(\eta_{ret.},\vec{x})H^2(\eta_{ret.})R^2(\eta_{ret.})\,|\vec{x}-\vec{x}\,'|\,d^3x'$$

The retarded time : $\eta_{ret.} = \eta - \frac{1}{c}|\vec{x}-\vec{x}\,'|$

Baryonic rest mass : $M = \int_V \rho(\eta,\vec{x})R^3(\eta)d^3x' = \text{constant}$

$G_{\text{obs}} = \frac{G}{R(\eta)} \mapsto G_{\text{obs}} = G_0 - \dot{G}(t-t_0)$ where $\dot{G} = H_0 = \frac{\dot{R}}{R^2}$ $\Rightarrow$ **Dirac's Hypothesis**

$H_0 = 6\times10^{-11}\,yr^{-1} \geq$ current observational limit = $1\times10^{-12}\,yr^{-1}$ (**we don't see it. Why?**)

$TCB = t$ corresponds to the model with an asymptotically flat space-time that is the conformal time η in the cosmological model. In principle, one has to expect the quadratic divergence of the TCB time scale because of the cosmological expansion. Pioneer "anomaly" effect?

Conclusions

✓Theory of the Reference Frames in the solar system is well-understood

✓Extension beyond the IAU 2000 resolutions is required

✓Post-post-Newtonian approximation in the solar system

✓Post-Newtonian effects and/or systematic effects in the Milky Way

✓Local group standard of rest

✓Astrometric cosmology is under development.

✓More theoretical and educational efforts.

Thank You!

The Friedmann Universe

$$\bar{g}_{\alpha\beta} = a^2(\eta) f_{\alpha\beta}$$

$$\bar{g}^{\alpha\beta} = \frac{1}{a^2(\eta)} f^{\alpha\beta}$$

$$f_{\alpha\beta} = \mathrm{diag}\ -1, +1, +1, +1$$

$$\bar{G}_{\alpha\beta} \equiv \bar{R}_{\alpha\beta} - \frac{1}{2}\bar{g}_{\alpha\beta}\bar{R} = 8\pi \bar{T}_{\alpha\beta}$$

$$\bar{T}_{\alpha\beta} = \bar{\rho}\, \bar{u}_\alpha \bar{u}_\beta$$

$$\bar{u}_\alpha(\eta) = a^{-1}(\eta)\bar{g}_{0\alpha} = -a(\eta)\delta^0_\alpha$$

$$H(\eta) = \frac{\dot{a}(\eta)}{a^2(\eta)}$$

$$\bar{R}_{\alpha\beta} = \frac{\dot{H}}{a}\left(\bar{g}_{\alpha\beta} - 2\bar{u}_\alpha \bar{u}_\beta\right) + 3H^2 \bar{g}_{\alpha\beta}$$

$$\bar{R} = 6\left(\frac{\dot{H}}{a} + 2H^2\right)$$

$$a(\eta) = \frac{2\eta^2}{H_0}, \qquad H(\eta) = \frac{H_0}{\eta^3}, \qquad \rho(\eta) = \frac{3H_0^2}{8\pi\eta^6}, \qquad t = \frac{2\eta^3}{3H_0}$$

Basic Assumptions

$$\boxed{g_{\alpha\beta}(\eta, x) = \bar{g}_{\alpha\beta} + h_{\alpha\beta}(\eta, x)}$$

$$\boxed{\psi_{\alpha\beta} = h_{\alpha\beta} - \frac{1}{2}\,\bar{g}_{\alpha\beta}\,h} \qquad \boxed{B_\alpha \equiv \psi_\alpha{}^{\nu}{}_{|\nu}}$$

$$\boxed{G_{\alpha\beta} \equiv R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi T_{\alpha\beta}}$$

$$\boxed{G_{\alpha\beta} = \bar{G}_{\alpha\beta} + \delta G_{\alpha\beta}} \qquad \boxed{T_{\alpha\beta} = \bar{T}_{\alpha\beta} + \delta T_{\alpha\beta}}$$

$$\begin{aligned} \delta G_{\alpha\beta} &= -\frac{1}{2}\left(\psi_{\alpha\beta}{}^{|\nu}{}_{|\nu} + \bar{g}_{\alpha\beta} B^{\nu}{}_{|\nu} - B_{\alpha|\beta} - B_{\beta|\alpha}\right) \\ &+ 2\bar{R}^{\nu}{}_{(\alpha}\psi_{\beta)\nu} - \frac{2}{3}\bar{R}\psi_{\alpha\beta} - \frac{1}{2}\left(\bar{R}_{\alpha\beta} - \frac{1}{3}\bar{g}_{\alpha\beta}\bar{R}\right)\psi \end{aligned}$$

$$\boxed{\boxed{\delta G_{\alpha\beta} = 8\pi\ \delta T_{\alpha\beta}}}$$

Bianchi Identity and Gauge Invariance

$$\delta T_{\alpha\beta} = T^{(m)}_{\alpha\beta} + T^{(c)}_{\alpha\beta}$$

$$T^{(m)\beta}_{\alpha\quad|\beta} = 0$$

$$T^{(c)\nu}_{\alpha\quad|\nu} + \overline{T}^{\beta}_{\alpha}\delta\Gamma^{\nu}_{\beta\nu} - \overline{T}^{\beta}_{\nu}\delta\Gamma^{\nu}_{\alpha\beta} = 0$$

$$\delta\Gamma^{\nu}_{\alpha\beta} = \frac{1}{2}\left(h^{\nu}_{\alpha|\beta} + h^{\nu}_{\beta|\alpha} - h_{\alpha\beta}{}^{|\nu}\right)$$

$$x'^{\alpha} = x^{\alpha} - \xi^{\alpha}(\eta, x)$$

$$\delta G'_{\alpha\beta}(\eta, x) = \delta G_{\alpha\beta}(\eta, x) + \mathcal{L}_{\xi}\overline{G}_{\alpha\beta}(\eta)$$

$$\delta T'_{\alpha\beta}(\eta, x) = \delta T_{\alpha\beta}(\eta, x) + \mathcal{L}_{\xi}\overline{T}_{\alpha\beta}(\eta)$$

$$T'^{(m)}_{\alpha\beta} = T^{(m)}_{\alpha\beta}$$

$$T'^{(c)}_{\alpha\beta}(\eta, x) = T^{(c)}_{\alpha\beta}(\eta, x) + \mathcal{L}_{\xi}\overline{T}_{\alpha\beta}(\eta)$$

Gauge-invariant structure of the stress-energy tensor

$$T^{(c)}_{\alpha\beta} = T^{(\psi)}_{\alpha\beta} + T^{(\phi)}_{\alpha\beta}$$

$$T^{(\psi)}_{\alpha\beta} = \frac{1}{2}\,\overline{g}_{\alpha\beta}\left(\overline{T}^{\mu\nu}\,\psi_{\mu\nu} - \frac{1}{2}\overline{T}\,\psi\right)$$

$$T^{(\phi)}_{\alpha\beta} = \frac{1}{2}\,\overline{g}_{\alpha\beta}\,\overline{T}\,\phi + \frac{H}{8\pi}\left(\overline{u}_\alpha\,\phi_{|\beta} + \overline{u}_\beta\,\phi_{|\alpha} - \overline{g}_{\alpha\beta}\,\overline{u}^\mu\phi_{|\mu}\right)$$

$$\psi'_{\alpha\beta}(\eta,x) = \psi_{\alpha\beta}(\eta,x) + \xi_{\alpha|\beta} + \xi_{\beta|\alpha} - \overline{g}_{\alpha\beta}\xi^{\mu}{}_{|\mu}$$

$$\phi'(\eta,x) = \phi(\eta,x) + \overline{\phi}_{|\mu}\xi^\mu$$

$$\overline{\phi} = 3\ln a(\eta)$$

$$T'^{(c)}_{\alpha\beta}(\eta,x) = T^{(c)}_{\alpha\beta}(\eta,x) + \mathcal{L}_\xi\overline{T}_{\alpha\beta}(\eta)$$

Gauge-invariant equations for cosmological perturbations

$$\boxed{\delta G_{\alpha\beta}(\eta, x) - 8\pi T^{(c)}_{\alpha\beta}(\eta, x) = 8\pi T^{(m)}_{\alpha\beta}(\eta, x)}$$

$$\begin{aligned}
\psi_{\alpha\beta}{}^{|\nu}{}_{|\nu} &+ \bar{g}_{\alpha\beta} B^{\nu}{}_{|\nu} - B_{\alpha|\beta} - B_{\beta|\alpha} - 4\bar{R}^{\ \nu}_{(\alpha}\psi_{\beta)\nu} + \frac{4}{3}\,\bar{R}\,\psi_{\alpha\beta} \\
&+ \left(\bar{R}_{\alpha\beta} - \frac{1}{3}\bar{g}_{\alpha\beta}\bar{R}\right)\psi + \bar{g}_{\alpha\beta}\left(\bar{R}^{\mu\nu}\,\psi_{\mu\nu} - \bar{R}\,\phi\right) \\
&+ 2H\left(\bar{u}_\alpha\,\phi_{|\beta} + \bar{u}_\beta\,\phi_{|\alpha} - \bar{g}_{\alpha\beta}\,\bar{u}^{\mu}\phi_{|\mu}\right) = -16\pi\,T^{(m)}_{\alpha\beta}
\end{aligned}$$

$$\boxed{\phi^{|\alpha}{}_{|\alpha} - \frac{3}{2}H^2\left(3\phi + \frac{1}{2}\psi + 5\bar{u}^{\alpha}\bar{u}^{\beta}\psi_{\alpha\beta}\right) - 3H\bar{u}^{\alpha}B_{\alpha} = 0}$$

$$\boxed{\xi^{\alpha|\beta}{}_{|\beta} + \bar{R}^{\alpha}_{\ \beta}\,\xi^{\beta} = B'^{\alpha}(\eta, x) - B^{\alpha}(\eta, x)}$$

New Cosmological Gauge

$$\phi_{|\alpha} = \frac{3}{2}\mathrm{H}\left(\overline{\mathrm{u}}_\alpha\phi + \frac{1}{2}\overline{\mathrm{u}}^\beta\psi_{\alpha\beta}\right) + \mathrm{A}_\alpha$$

$$\mathrm{A}^\alpha{}_{|\alpha} = 0$$

$$\mathrm{B}_\alpha = 2\mathrm{H}\left(\overline{\mathrm{u}}_\alpha\phi - \overline{\mathrm{u}}^\beta\psi_{\alpha\beta}\right)$$

$$\psi_{\alpha\beta}(\eta,\mathrm{x}) = \mathrm{a}^2(\eta)\ \varphi_{\alpha\beta}(\eta,\mathrm{x})$$

$$\varphi(\eta,\mathrm{x}) = \mathrm{f}^{\alpha\beta}\varphi_{\alpha\beta}(\eta,\mathrm{x})$$

$$\mathrm{h}_{\alpha\beta} = \varphi_{\alpha\beta} - \frac{1}{2}\mathrm{f}_{\alpha\beta}\varphi$$

$$\mathcal{H} \equiv \frac{\dot{\mathrm{a}}}{\mathrm{a}}$$

$$\Box\varphi_{\alpha\beta} - 2\mathcal{H}\varphi_{\alpha\beta,0} + \mathcal{H}^2\left[\delta^0_\alpha\varphi_{\beta 0} + \delta^0_\beta\varphi_{\alpha 0} + \delta^0_\alpha\delta^0_\beta(\varphi - 2\phi)\right] = -16\pi\ \mathrm{T}^{(\mathrm{m})}_{\alpha\beta}$$

$$\Box\phi - 2\mathcal{H}\overline{\mathrm{u}}^\alpha\phi_{|\alpha} + \frac{3}{2}\mathcal{H}^2\left(\phi - \mathrm{h}_{00}\right) = 0$$

The Field Equations

$$
\begin{aligned}
\Box\chi - \frac{4}{\eta}\frac{\partial\chi}{\partial\eta} + \frac{10\chi}{\eta^2} &= -16\pi\left(T_{00}^{(m)} + \frac{1}{2}T^{(m)}\right) , \\
\Box h_{00} - \frac{4}{\eta}\frac{\partial h_{00}}{\partial\eta} &= -16\pi\left(T_{00}^{(m)} + \frac{1}{2}T^{(m)}\right) - \frac{4\chi}{\eta^2} , \\
\Box h_{0i} - \frac{4}{\eta}\frac{\partial h_{0i}}{\partial\eta} + \frac{4h_{0i}}{\eta^2} &= -16\pi T_{0i}^{(m)} , \\
\Box h_{ij} - \frac{4}{\eta}\frac{\partial h_{ij}}{\partial\eta} &= -16\pi\left(T_{ij}^{(m)} - \frac{1}{2}\delta_{ij}T^{(m)}\right) ,
\end{aligned}
$$

where $\chi \equiv h_{00} - \phi$.

The residual gauge freedom

$$\boxed{\Box\xi^0 - \frac{4}{\eta}\frac{\partial\xi^0}{\partial\eta} + \frac{4\xi^0}{\eta^2} = 0} \qquad \boxed{\Box\xi^i - \frac{4}{\eta}\frac{\partial\xi^i}{\partial\eta} = 0}$$

Further Developments: Arbitrary Cosmological Equation of State

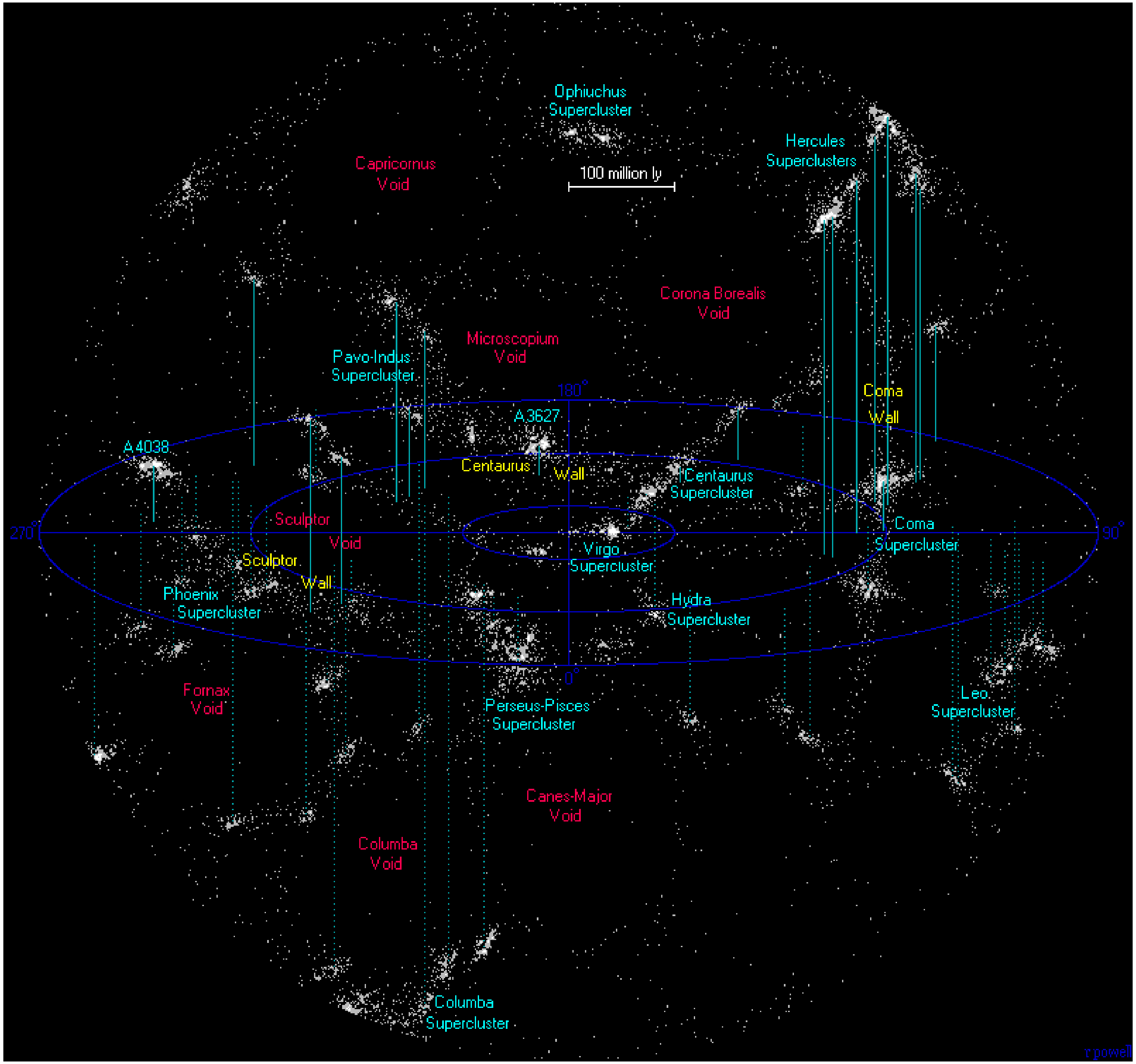
$$\bar{\mathrm{p}} = \alpha\,\bar{\rho}$$

$$\boxed{\left(\Box - \frac{\mathrm{a}}{\eta}\frac{\partial}{\partial\eta} - \frac{\mathrm{b}}{\eta^2}\right) \mathrm{F}(\eta,\mathrm{x}) = -4\pi \mathrm{S}(\eta,\mathrm{x})}$$

$$\boxed{\nu = \sqrt{\left(\frac{1-a}{2}\right)^2 - b}}$$

$$\nu_1 = \frac{3(1-\alpha)}{2(1+3\alpha)} \qquad \nu_2 = \frac{5+3\alpha}{2(1+3\alpha)}$$

	$\alpha = -1$	$\alpha = \frac{1}{3}$	$\alpha = 0$
ν_1	$-\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$
ν_2	$-\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$



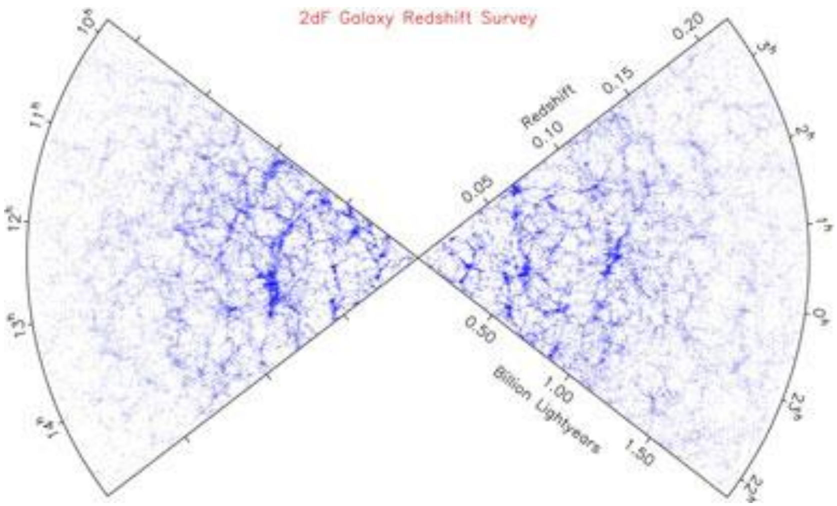
Map of voids and superclusters within 500 million light years from Milky May

Image credit to *Richard Powell*

Large-scale structure

Spatial distribution of around 100,000 galaxies, revealing large-scale structures and voids in the local Universe.

Credit: M. Colless



Galaxies are formed in the centers of dark-matter halos.

Credit: A. Kravtsov (University of Chicago)