Astronomical relativistic reference systems and their application for astrometry

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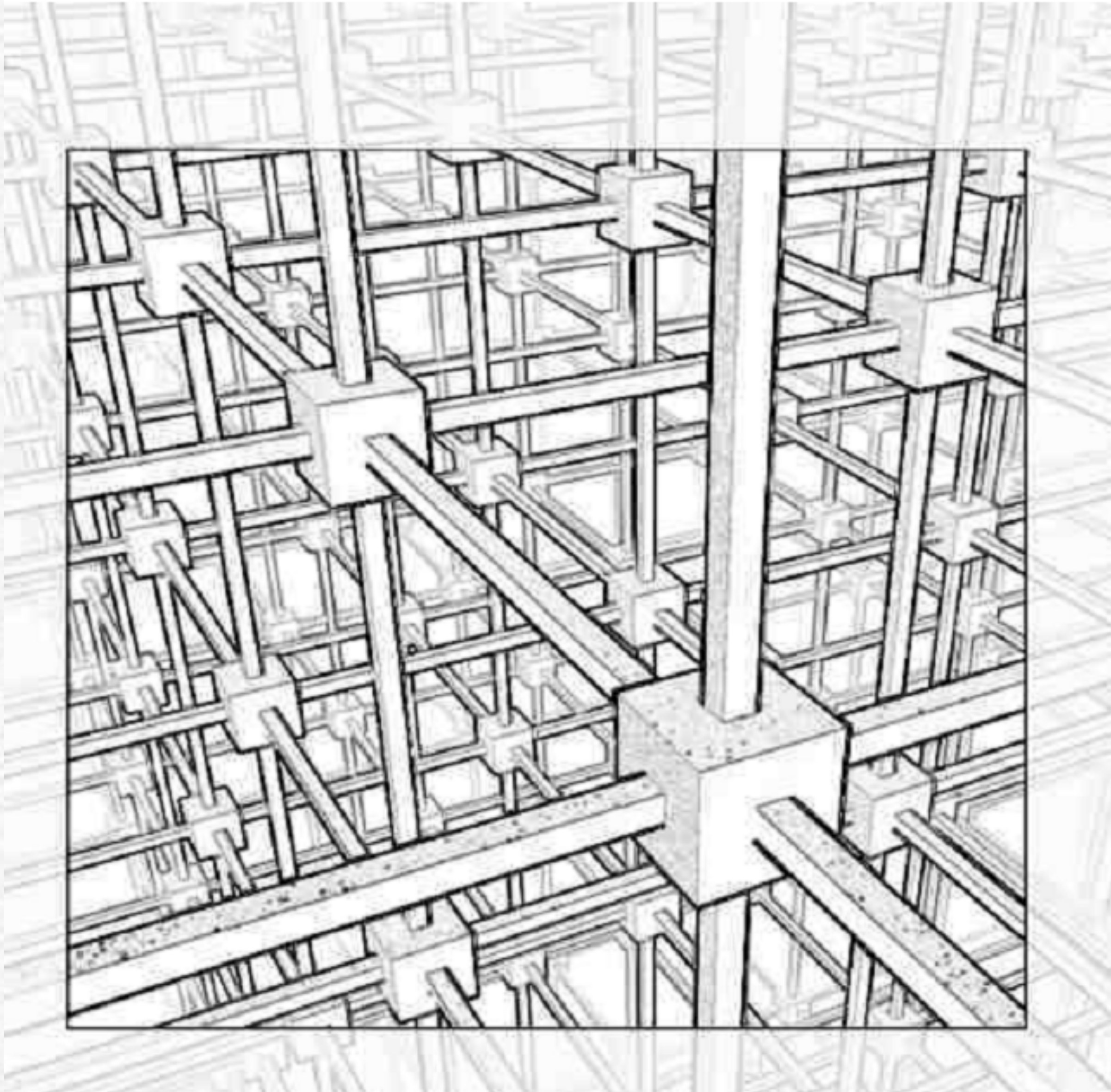
Modelling of positional observations in Newtonian physics

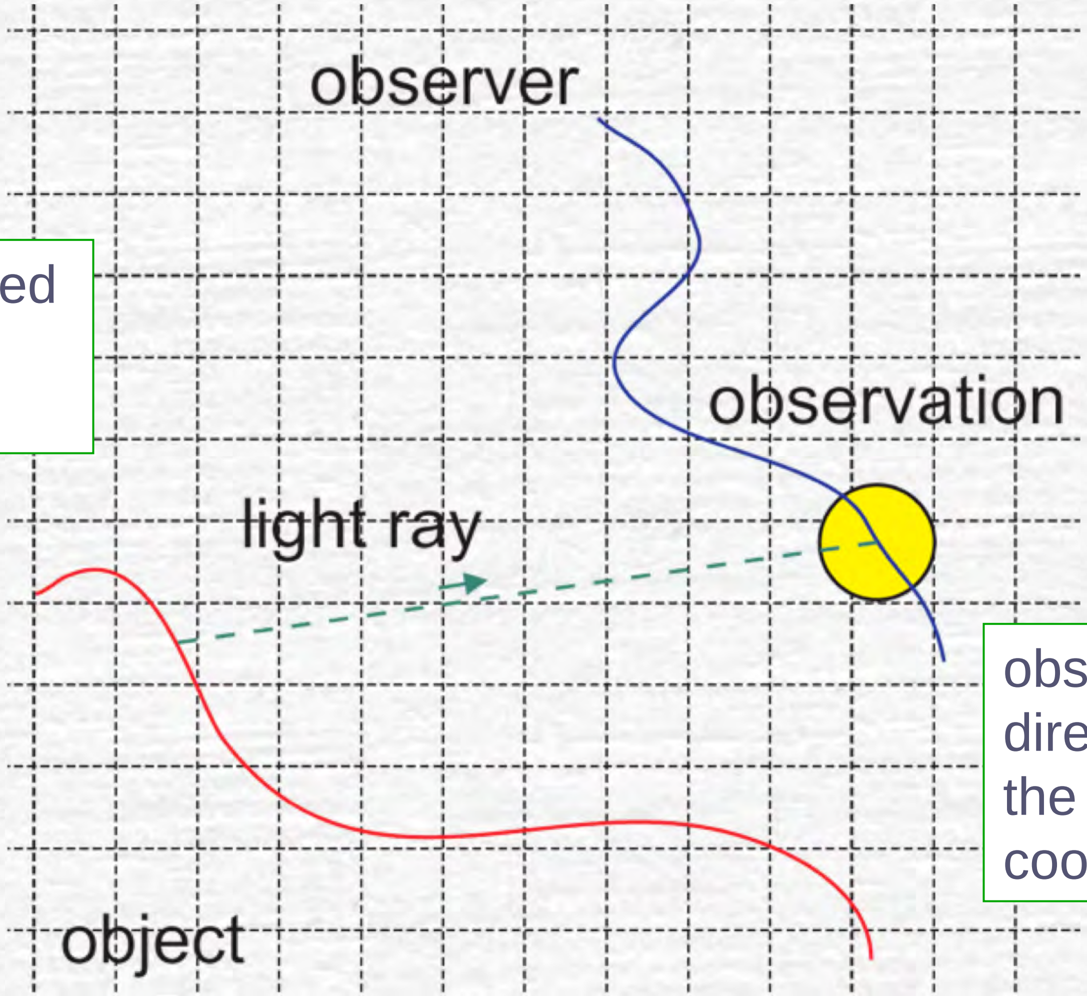


M. C. Escher

Cubic space division, 1952

Astronomical observation

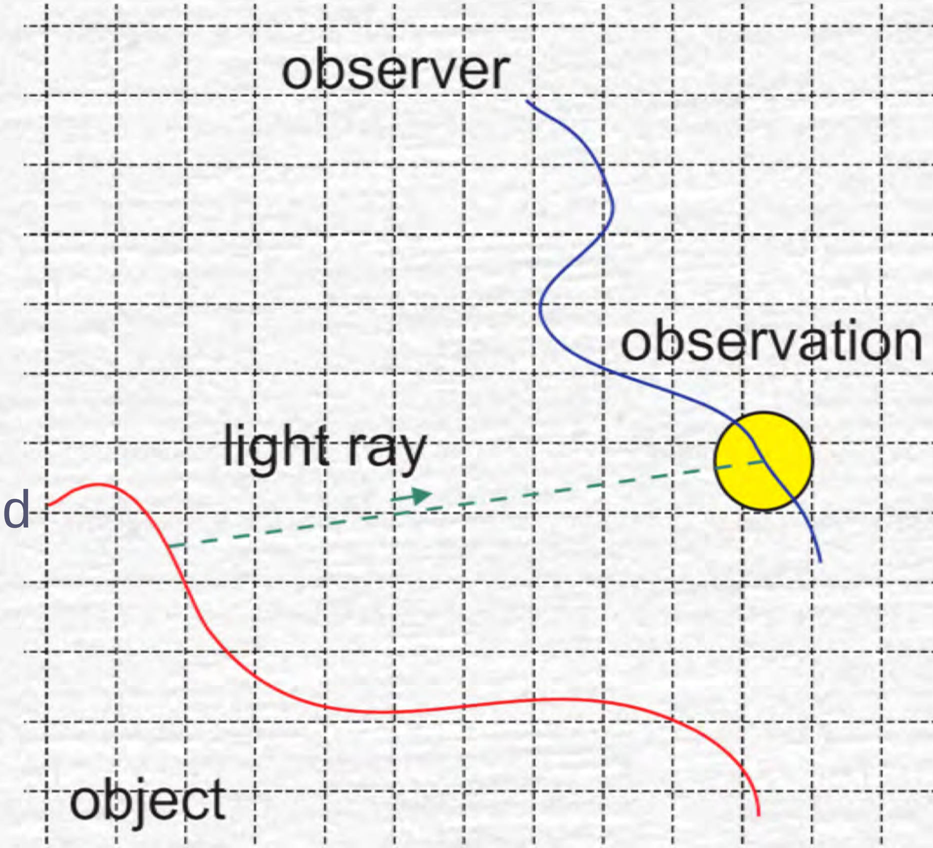
physically preferred global inertial coordinates



observables are directly related to the inertial coordinates

Modelling of positional observations in Newtonian physics

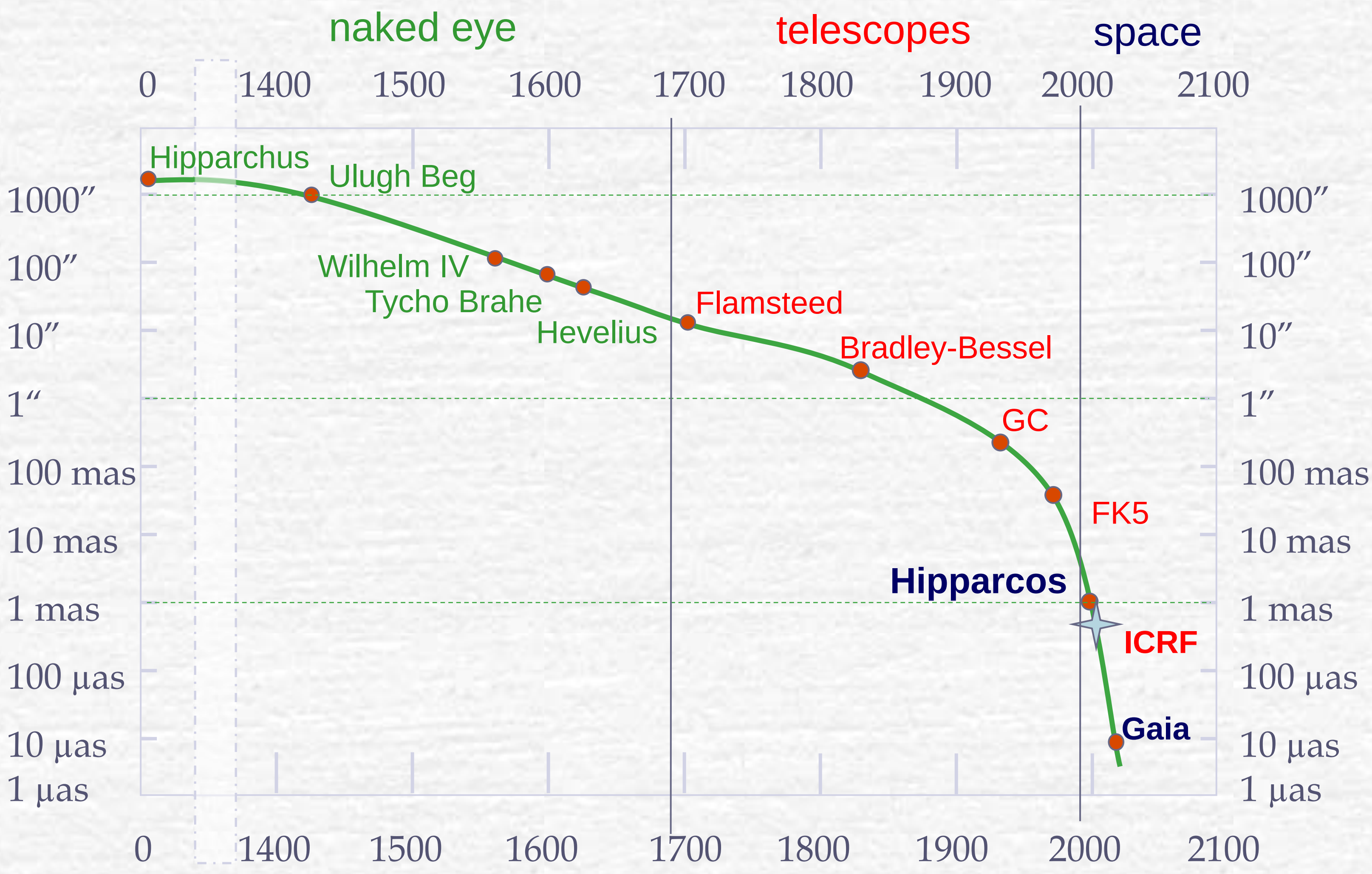
- Scheme:
    - aberration
    - parallax
    - proper motion



- All parameters of the model are defined in the preferred global coordinates:

$$(\alpha, \delta), \quad (\mu_\alpha, \mu_\delta), \quad \pi, \quad \ldots$$

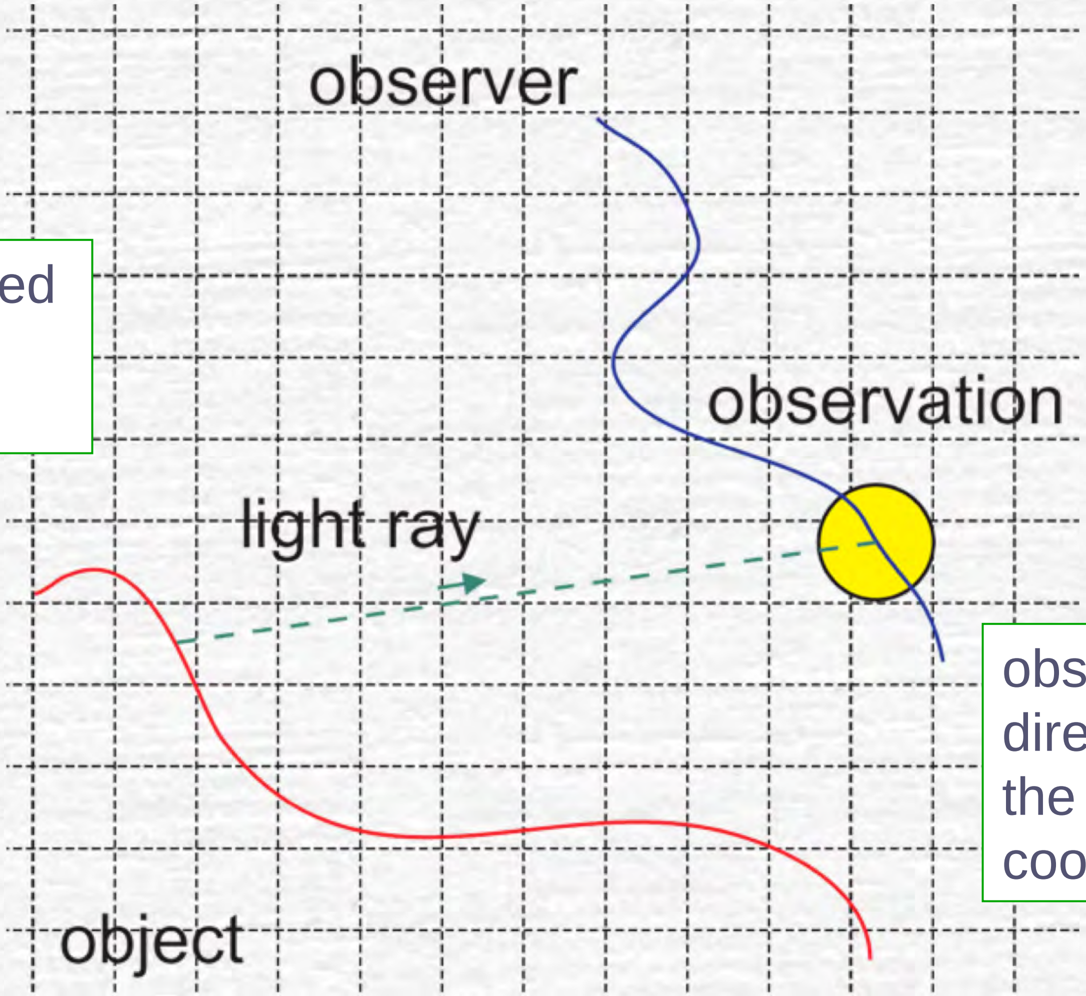
Accuracy of astrometric observations



1 µas is the thickness of a sheet of paper seen from the other side of the Earth

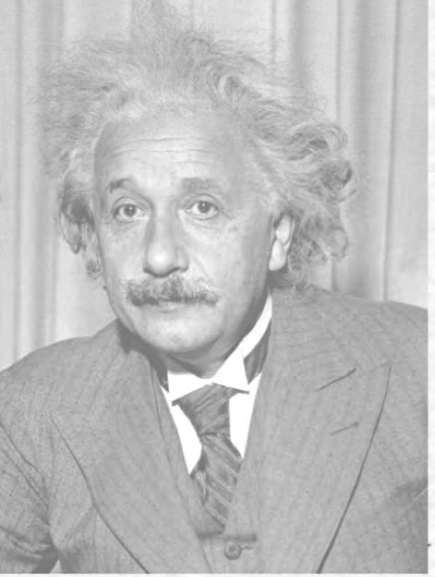
Astronomical observation

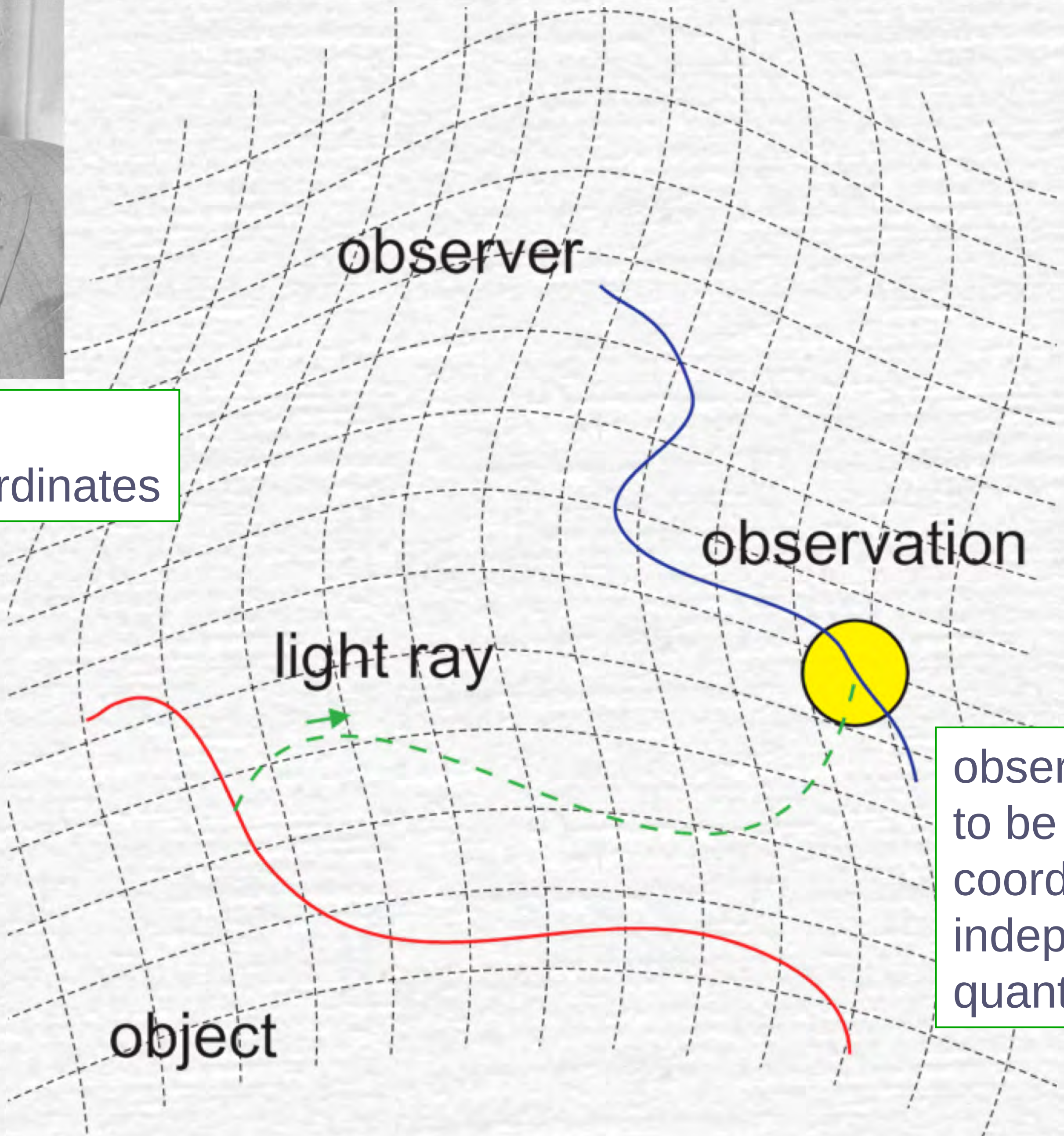
physically preferred global inertial coordinates



observables are directly related to the inertial coordinates

Astronomical observation

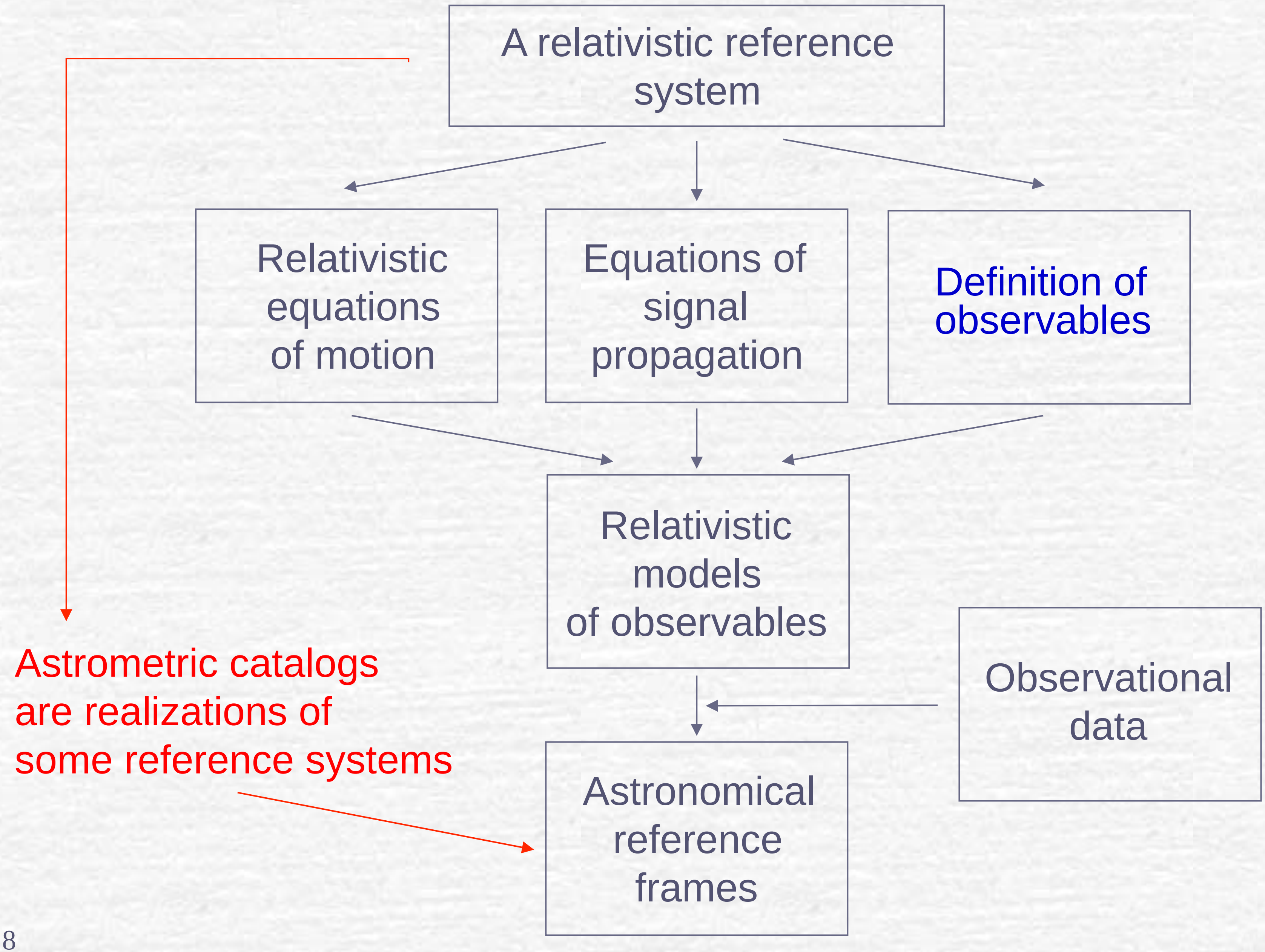




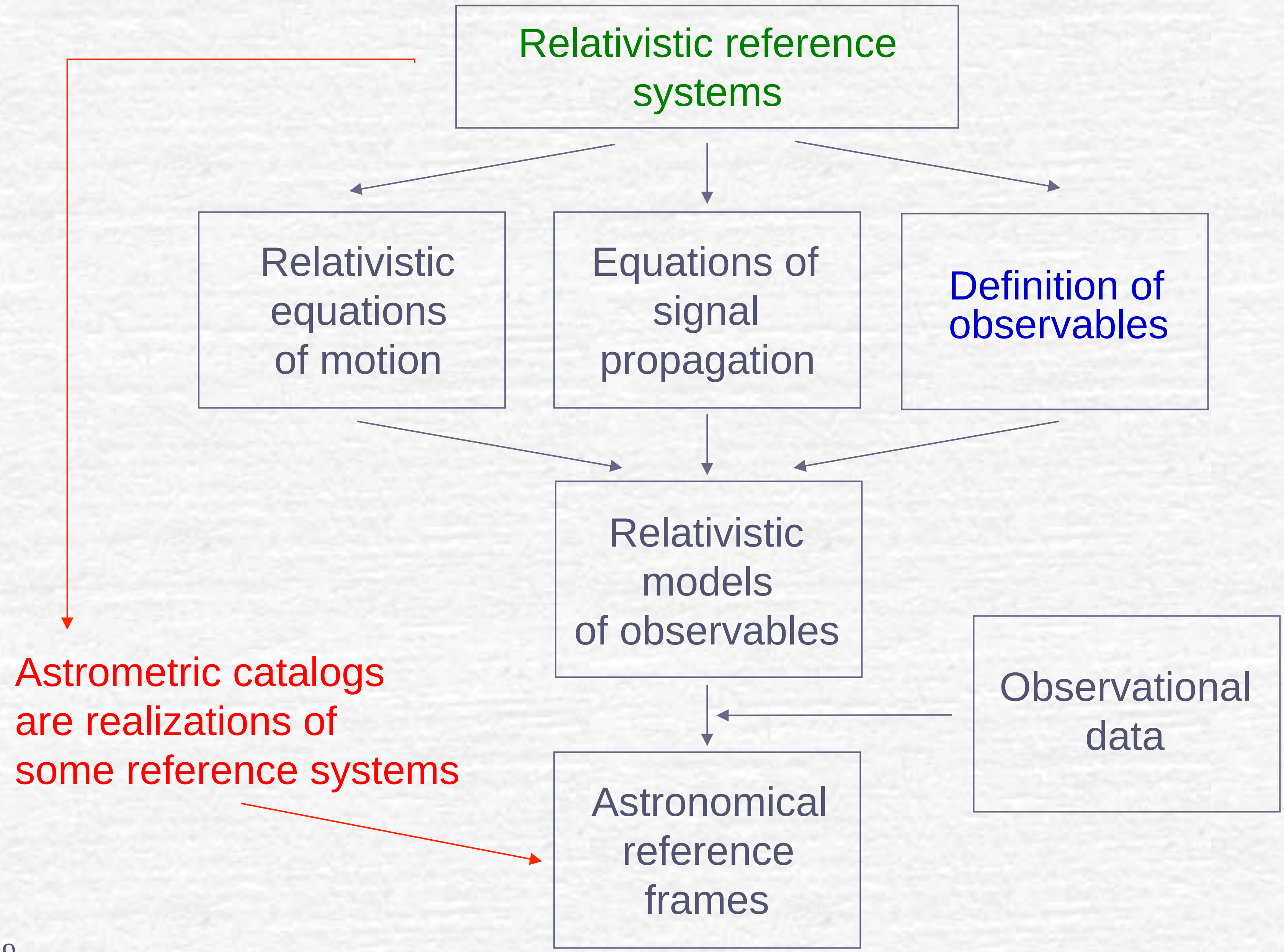
no physically preferred coordinates

observables have to be computed as coordinate independent quantities

General-relativistic modeling of astronomical data



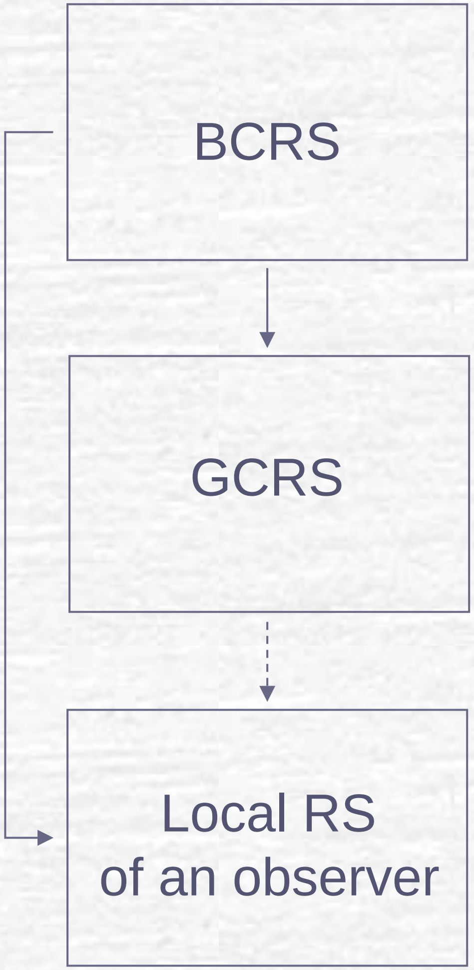
General-relativistic modeling of astronomical data



Astronomical relativistic reference systems

The IAU 2000 framework

- Three standard astronomical reference systems were defined
  - BCRS (Barycentric Celestial Reference System)
  - GCRS (Geocentric Celestial Reference System)
  - Local reference system of an observer



- All these reference systems are defined by

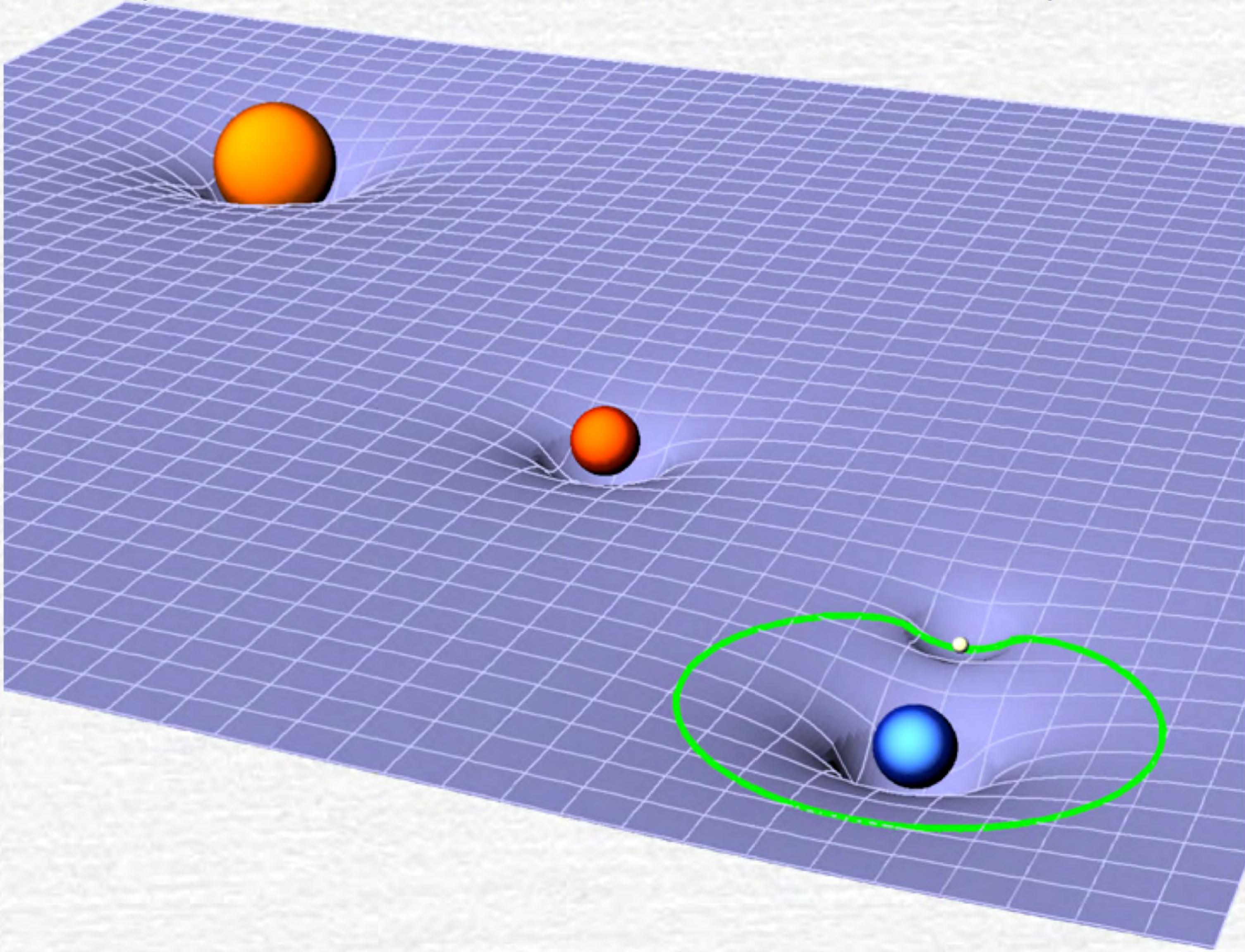
  the form of the corresponding metric tensors.

Technical details: Brumberg, Kopeikin, 1988-1992
Damour, Soffel, Xu, 1991-1994
Klioner, Voinov, 1993
Soffel, Klioner, Petit et al., 2003
Klioner,Soffel, 2000; Kopeikin, Vlasov, 2004

Relativistic Astronomical Reference Systems

particular reference systems in the curved space-time of the Solar system

- One can use any
- but one should fix one



Barycentric Celestial Reference System

The BCRS is suitable to model processes in the whole solar system

$$g_{00} = -1 + \frac{2}{c^2} w(t, \boldsymbol{x}) - \frac{2}{c^4} w^2(t, \boldsymbol{x}) \,,$$

$$g_{0i} = -\frac{4}{c^3} w^i(t, \boldsymbol{x}) \,,$$

$$g_{ij} = \delta_{ij} \left(1 + \frac{2}{c^2} w(t, \boldsymbol{x}) \right) .$$

w, w^i : relativistic gravitational potentials

Using the laws of GRT one can derive

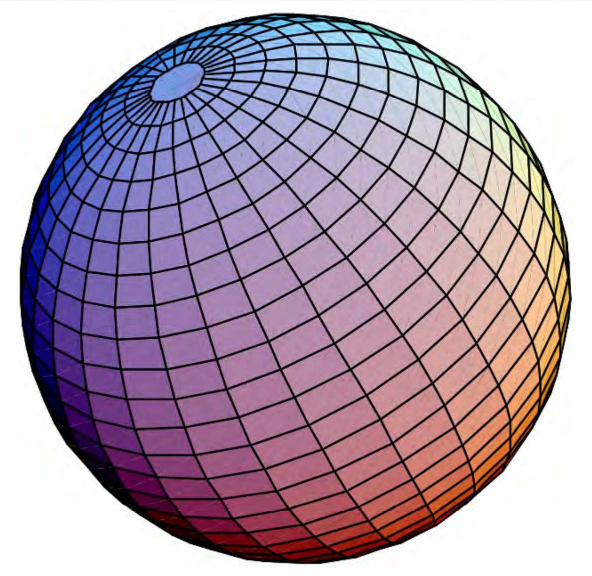
equations of motion and
definitions of observables

Why the BCRS is inconvenient for the Earth?

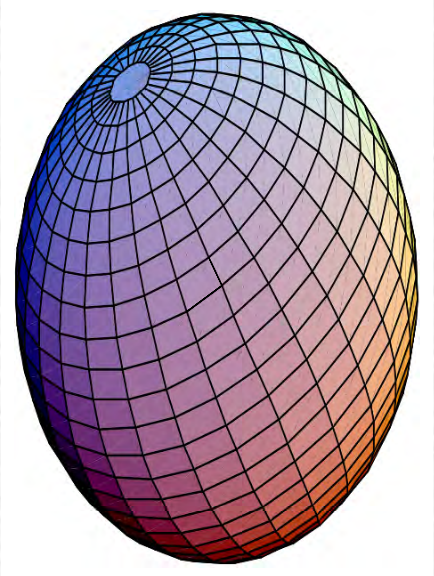
- Imagine a sphere (in inertial coordinates of special relativity), which is then forced to move in a circular orbit around some point…

- What will be the form of the sphere for an observer at rest relative to that point?

Lorentz contraction deforms the shape…



Direction of the velocity



Additional effect due to acceleration (velocity is not constant)

gravity (general relativity, not special one)

Geocentric Celestial Reference System

The GCRS is suitable to model physical processes in the vicinity of the Earth:

A: The gravitational field of external bodies is represented only in the form of a relativistic tidal potential.

B: The internal gravitational field of the Earth coincides with the gravitational field of a corresponding isolated Earth.

$$G_{00} = -1 + \frac{2}{c^2} W(T, \boldsymbol{X}) - \frac{2}{c^4} W^2(T, \boldsymbol{X}),$$

$$G_{0a} = -\frac{4}{c^3} W^a(T, \boldsymbol{X}),$$

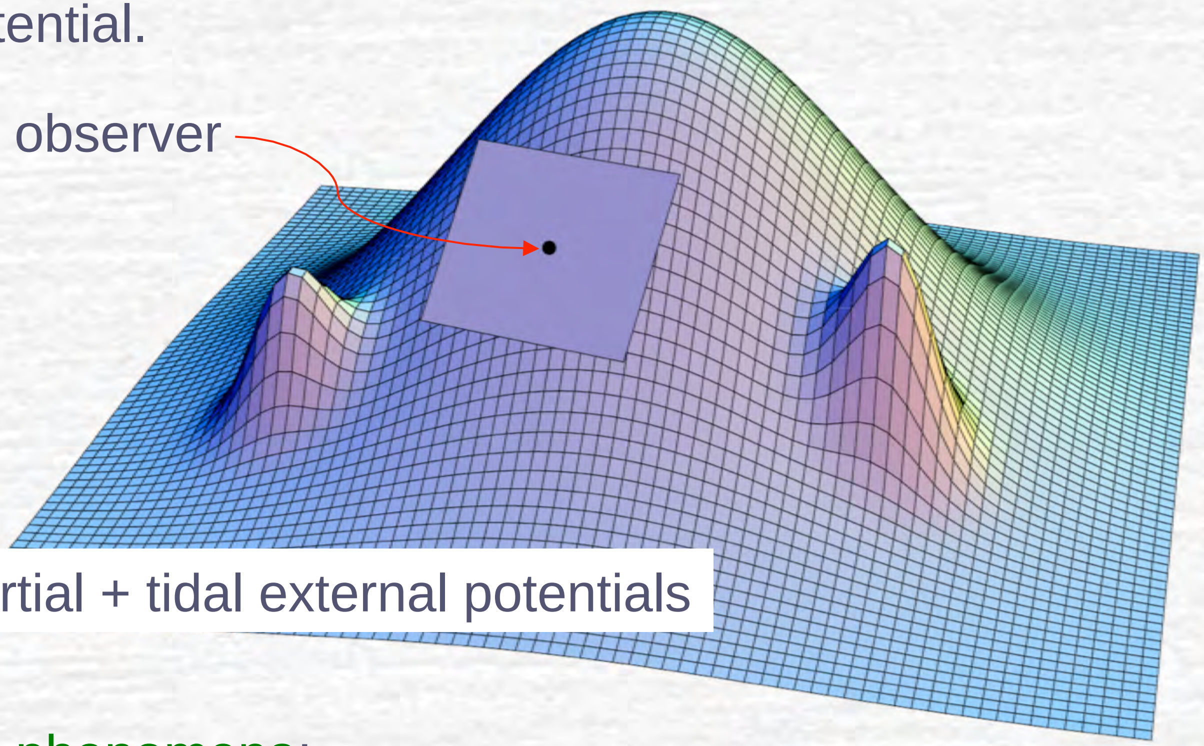
$$G_{ab} = \delta_{ab}\left(1 + \frac{2}{c^2} W(T, \boldsymbol{X})\right).$$

W, W^a : internal + inertial + tidal external potentials

Local reference system of an observer

The version of the GCRS for a massless observer:

A: The gravitational field of external bodies is represented only in the form of a relativistic tidal potential.



W, W^a : ~~internal~~ + inertial + tidal external potentials

• Modelling of any local phenomena:

observation,
attitude,
local physics (if necessary)

Local reference system of an observer

- The local coordinate basis at the origin of that reference system is is a BCRS-induced tetrad: ($\mathcal{X}^\alpha$ are the local coordinates)

$$\boxed{\mathrm{e}_\mu^{(\alpha)} = \left.\frac{\partial \mathcal{X}^\alpha}{\partial x^\mu}\right|_{\mathcal{X}^i=0}}$$

$$\boldsymbol{g}_{\mu\nu} = \eta_{\alpha\beta}\mathrm{e}_\mu^{(\alpha)}\mathrm{e}_\nu^{(\beta)}$$

- The computations of the observed direction are equivalent (Klioner, 2004):
  - from the tetrad formalism:

$$s^{(a)} = -\frac{dx^{(a)}}{dx^{(0)}} = -\frac{\mathrm{e}_\mu^{(a)}\,dx^\mu}{\mathrm{e}_\mu^{(0)}\,dx^\mu} = -\frac{\mathrm{e}_0^{(a)} + \mathrm{e}_i^{(a)}\,p^i}{\mathrm{e}_0^{(0)} + \mathrm{e}_j^{(0)}\,p^j}, \qquad p^i = \frac{1}{c}\frac{dx_p^i}{dt}$$

- projecting on the local axes:

$$\underline{s^a} = -\frac{d\mathcal{X}^a}{d\mathcal{X}^0} = -\frac{\frac{\partial \mathcal{X}^a}{\partial x^\mu}\,dx^\mu}{\frac{\partial \mathcal{X}^0}{\partial x^\mu}\,dx^\mu} = -\frac{\frac{\partial \mathcal{X}^a}{\partial x^0} + \frac{\partial \mathcal{X}^a}{\partial x^i}\,p^i}{\frac{\partial \mathcal{X}^0}{\partial x^0} + \frac{\partial \mathcal{X}^0}{\partial x^j}\,p^j} \underline{= s^{(a)}}$$

Further developments after 2000

- Local reference systems in the PPN formalism:

  Klioner, Soffel, 2000; Kopeikin, Vlasov, 2004

- BCRS for the non-isolated Solar system:

  Kopeikin et al., 2000-…; Klioner, Soffel, 2004; …

- BCRS and GCRS in the post-post-Newtonian approximation for the light propagation

  Xu et al, 2005; Minazzolli, Chauvineau, 2009; Klioner et al., 2011

The BCRS for non-isolated Solar system

- The standard BCRS considers the Solar system isolated

$$\boxed{\begin{aligned} g_{00} &= -1 + \frac{2}{c^2} w(t,\mathbf{x}) - \frac{2}{c^4} w^2(t,\mathbf{x}) \,, \\ g_{0i} &= -\frac{4}{c^3} w^i(t,\mathbf{x}) \,, \\ g_{ij} &= \delta_{ij} \left(1 + \frac{2}{c^2} w(t,\mathbf{x}) \right) . \end{aligned}}$$

$$w(t,\mathbf{x}) = G \int d^3x' \frac{\sigma(t,\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} + \frac{1}{2c^2} G \frac{\partial^2}{\partial t^2} \int d^3x' \sigma(t,\mathbf{x}') \, |\mathbf{x}-\mathbf{x}'| \,, \quad w^i(t,\mathbf{x}) = G \int d^3x' \frac{\sigma^i(t,\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|} \,,$$

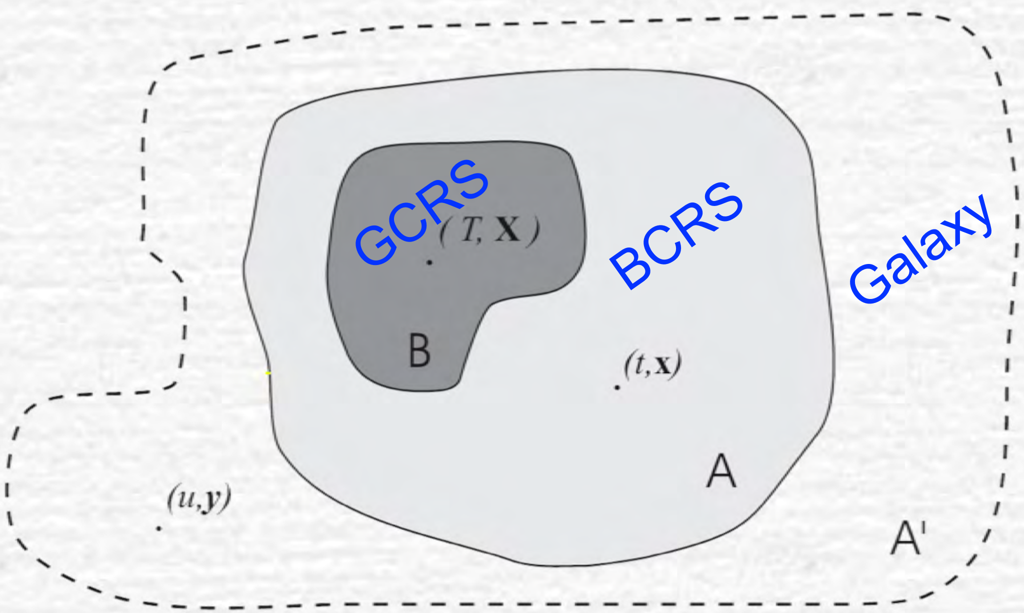
$\sigma = \left(T^{00} + T^{kk} \right) / c^2, \quad \sigma^i = T^{0i} / c, \quad T^{\mu\nu}$ is the BCRS energy-momentum tensor

$$\boxed{\lim_{\substack{|\mathbf{x}|\to\infty \\ t=\text{const}}} g_{\mu\nu} = \eta_{\mu\nu}} \quad \Rightarrow \quad \text{Isolated system!}$$

The BCRS for a non-isolated Solar system

I. Any “localized” sources: stars, galaxies, etc.

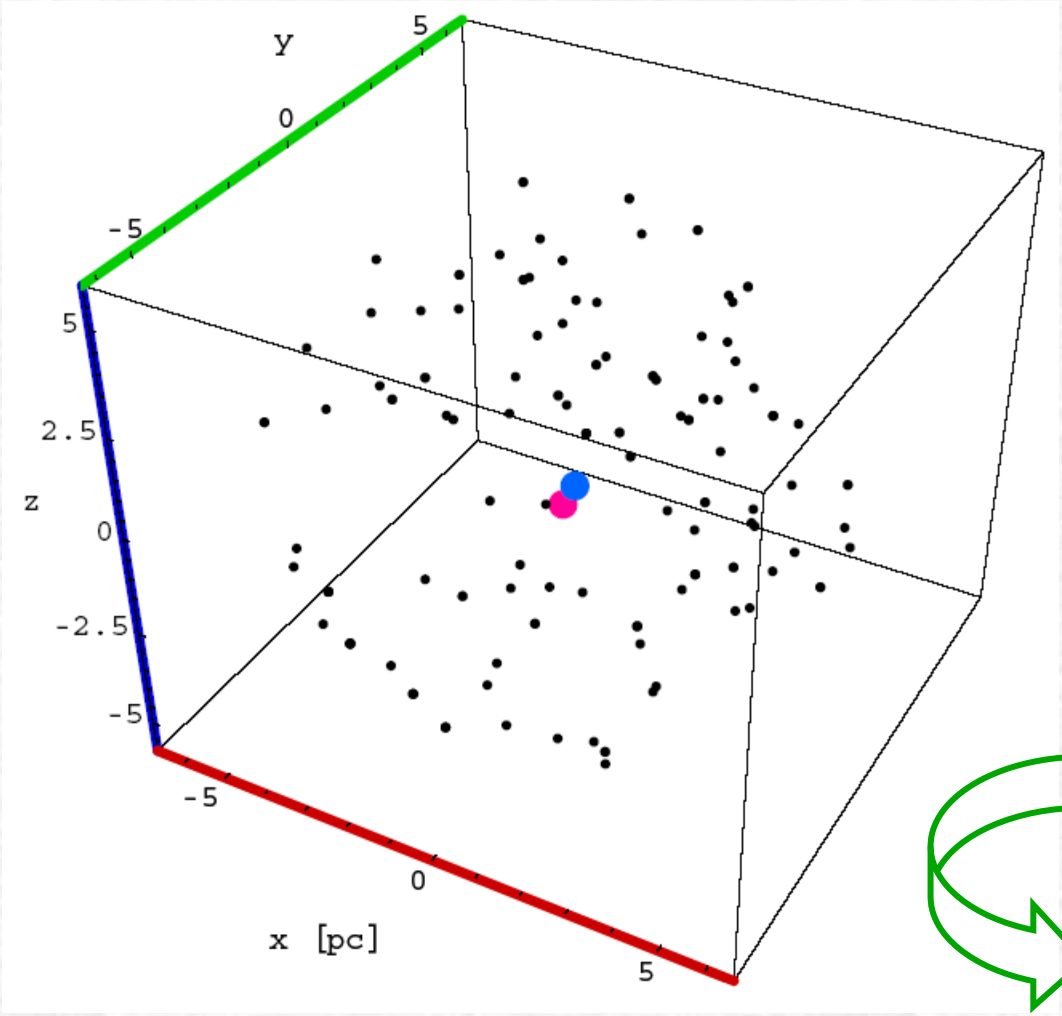
- solar system can be considered as “one body” among many
- a “GCRS”-like reference system can be constructed for that “one body”: gravitational influence of outer matter is effaced as much as possible
- the tidal gravitational potentials and the inertial forces have been estimated numerically and found to be negligible



The BCRS for a non-isolated Soalr system

I. Any “localized” sources: stars, galaxies, etc.

Tidal quadrupole moment can be represented by two fictitious bodies:



	Body 1	Body 2
Mass	1.67 M_{sun}	0.19 M_{Sun}
Distance (chosen)	1 pc	1 pc
α	221.56°	285.11°
δ	-60.92°	13.91°

$$\left| X^a \right| = 40 \text{ AU}$$

$$\left| \delta \ddot{X}^a \right| \leq 4 \times 10^{-17} \text{ m / s}^{-2}$$

The BCRS for a non-isolated Solar system

II. Cosmological influences

Question: Does the cosmological expansion influence local physics?

Long history: McVittie(1933), …, Einstein, Straus (1945), …
a lot of newer papers using different approaches and…
giving different answers

A contribution of Klioner & Soffel (2004):

- The answer depends crucially on the model for the Universe
- But the “worst-case” answer can be given

The BCRS for a non-isolated Solar system

A general Robertson-Walker metric

$$g_{00} = -1,$$
$$g_{0i} = 0,$$
$$g_{ij} = a^2(t)\left(1 + k\,r^2/4\right)^{-2}$$

can be transformed into
an local reference system

$$G_{00} = -1 + \sum_{s=1}^{\infty} A_s \left(R/c\right)^{2s},$$
$$G_{0a} = 0,$$
$$G_{ab} = \delta_{ab}\left(1 + \sum_{s=1}^{\infty} B_s \left(R/c\right)^{2s}\right)$$

by a coordinate transformation

$$T = t + \sum_{s=1}^{\infty} C_s \,(a(t)\,r/c)^{2s},$$
$$X^a = \delta^{ai} x^i a(t)\left(1 + \sum_{s=1}^{\infty} D_s \,(a(t)\,r/c)^{2s}\right).$$

$$(\, r = \left|x^i\right|, \quad R = \left|X^a\right| \,)$$

The BCRS for a non-isolated system

Unknown functions can be uniquely defined from matching:

$$p_s = \frac{1}{a(t)}\,\frac{d^s}{dt^s}\,a(t),$$

$$q = \frac{k\,c^2}{a^2(t)}.$$

$$\begin{aligned}
A_1 &= p_2,\\
A_2 &= \frac{1}{4}\,(p_1^4 - 2\,p_1^2\,p_2 - p_2^2 + q\,(p_1^2 - p_2)),\\
B_1 &= -\frac{1}{2}\,(p_1^2 + q),\\
B_2 &= -\frac{1}{16}\,(p_1^4 - 4\,p_1^2\,p_2 - 2\,q\,p_1^2 - 3\,q^2),\\
C_1 &= \frac{1}{2}\,p_1\\
C_2 &= \frac{1}{8}\,p_1\,(2\,p_2 - q),\\
D_0 &= 1,\\
D_1 &= \frac{1}{4}\,p_1^2,\\
D_2 &= \frac{1}{16}\,p_1^2\,(2\,p_2 - q),\\
&\dots
\end{aligned}$$

The BCRS for a non-isolated Solar system

Equations of motion in the local coordinates $(\boldsymbol{T},\mathbf{X})$ get the correction

$$\delta \ddot{X}^a = \frac{\ddot{a}}{a} X^a = -q H^2 X^a \approx 3.2 \times 10^{-36} s^{-2} X^a$$

This gives for $\left| X^a \right| = 40 \text{ AU}$

$$\left| \delta \ddot{X}^a \right| = 2 \times 10^{-23} \text{ m / s}^{-2}$$

For comparison: the tidal force due to all matter of the Galaxy (same place)

$$\left| \delta \ddot{X}^a \right| \simeq 5 \times 10^{-17} \text{ m / s}^{-2}$$

The post-post-Newtonian metric for light rays

The metric tensors get additional terms:

BCRS: $$g_{ij} = \delta_{ij}\left(1 + \frac{2}{c^2} w(t,\boldsymbol{x}) + \frac{2}{c^4} w^2(t,\boldsymbol{x})\right) + \frac{4}{c^4} q_{ab}(t,\boldsymbol{x}).$$

GCRS: $$G_{ab} = \delta_{ab}\left(1 + \frac{2}{c^2} W(T,\mathbf{X}) + \frac{2}{c^4} W^2(T,\mathbf{X})\right) + \frac{4}{c^4} Q_{ab}(T,\boldsymbol{X}).$$

plus the coordinate transformation BCRS-GCRS and
a set of matching conditions, e.g.:

$$Q_{\mathrm{E}}^{ab} = R^a{}_i R^b{}_j \left(q_{\mathrm{E}}^{ij} + v_E^i v_E^j w_E - 2\, v_E^{(i} w_E^{j)} + \delta^{ij}\left(2 v_E^k w_E^k - v_E^2 w_E\right) + 2 a_E^{(i} \chi_{\mathrm{E}\,,j)}\right)$$

Applications:

post-post-Newtonian light propagation with multipoles and motion

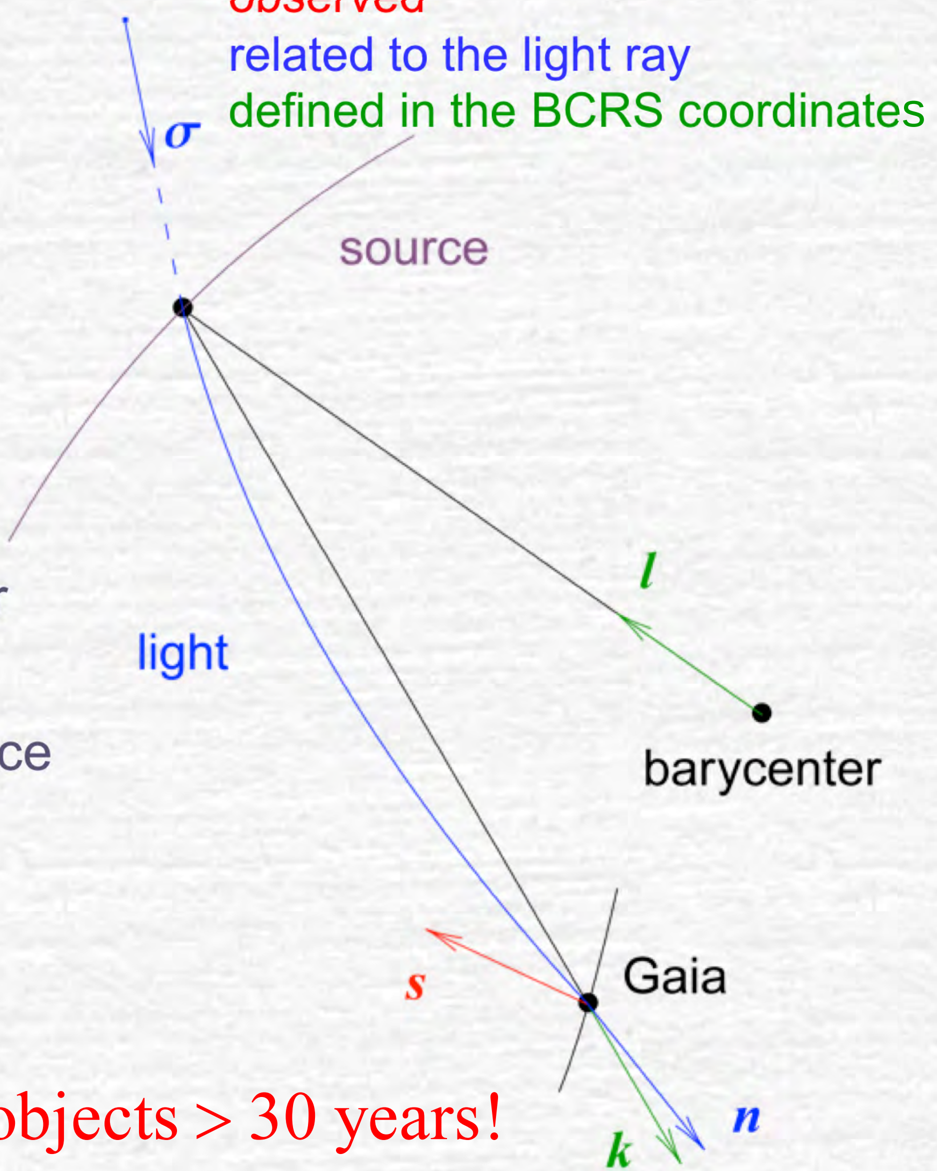
marginally interesting for Gaia,
important for next-generation projects

Application for high-accuracy positional observations

General structure of the model

Klioner, AJ, 2003; PhysRevD, 2004:

- s the observed direction
- n tangential to the light ray at the moment of observation
- σ tangential to the light ray at $t = -\infty$
- k the coordinate direction from the source to the observer
- l the coordinate direction from the barycentre to the source
- π the parallax of the source in the BCRS



The model must be optimal: $1\ s \cdot\ 10^9$ objects > 30 years!

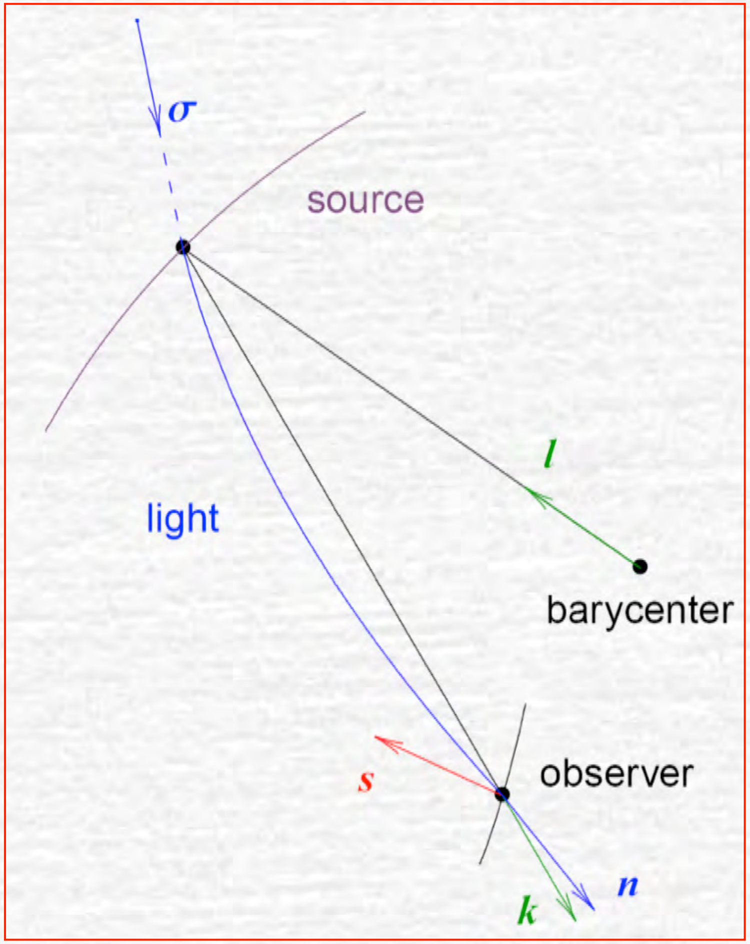
Sequences of transformations

- Stars:

$$s \overset{(1)}{\leftrightarrow} n \overset{(2)}{\leftrightarrow} \sigma \overset{(3)}{\leftrightarrow} k \overset{(4)}{\leftrightarrow} l(t), \pi(t) \overset{(5)}{\leftrightarrow} l_0, \pi_0, \mu_0, \dot{\pi}_0, \ldots$$

- Solar system objects:

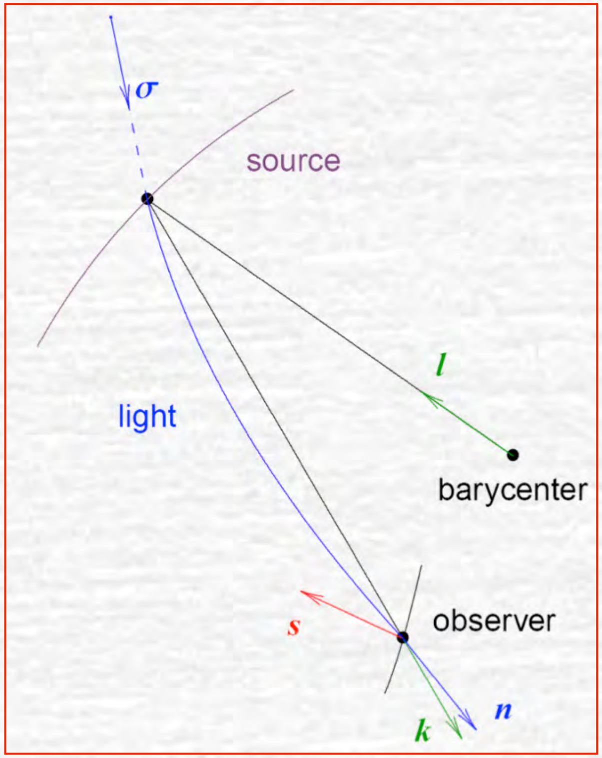
$$s \overset{(1)}{\leftrightarrow} n \overset{(2,3)}{\leftrightarrow} k \overset{(6)}{\leftrightarrow} \text{orbit}$$



(1) aberration
(2) gravitational deflection
(3) coupling to finite distance
(4) parallax
(5) proper motion, etc.
(6) orbit determination

Aberration: $s \leftrightarrow n$

- Lorentz transformation with the scaled velocity of the observer:



$$\boldsymbol{s} = \left(-\boldsymbol{n} + \left\{ \frac{\gamma}{c} - (\gamma - 1)\frac{\boldsymbol{v}\cdot\boldsymbol{n}}{v^2} \right\} \boldsymbol{v} \right) \frac{1}{\gamma\,(1 - \boldsymbol{v}\cdot\boldsymbol{n}/c)},$$

$$\gamma = \left(1 - v^2/c^2\right)^{-1/2},$$

$$\boldsymbol{v} = \dot{\boldsymbol{x}}_o \left(1 + \frac{2}{c^2} w(t, \boldsymbol{x}_o) \right)$$

- For an observer on the Earth or on a typical satellite:
  - Newtonian aberration ~ 20″
  - relativistic aberration ~ 4 mas
  - second-order relativistic aberration ~ 1 μas

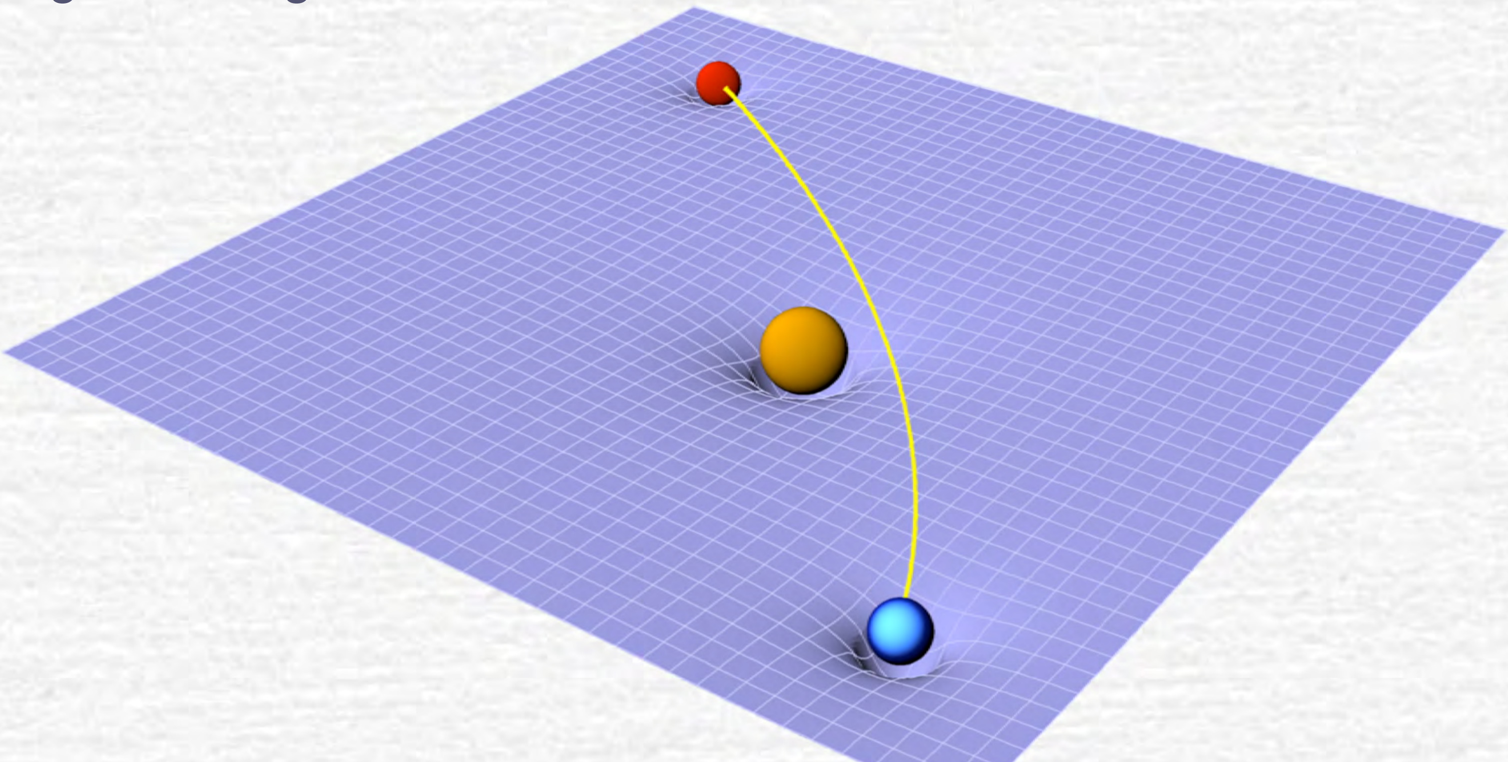
- Requirement for the accuracy of the orbit: $\left|\delta \boldsymbol{s}\right| \le 1\,\mu\mathrm{as} \quad \Rightarrow \quad \left|\delta \dot{\boldsymbol{x}}_o\right| \le 1\,\mathrm{mm/s}$

Gravitational light deflection: $n \leftrightarrow \sigma \leftrightarrow k$

- Several kinds of gravitational fields deflecting light

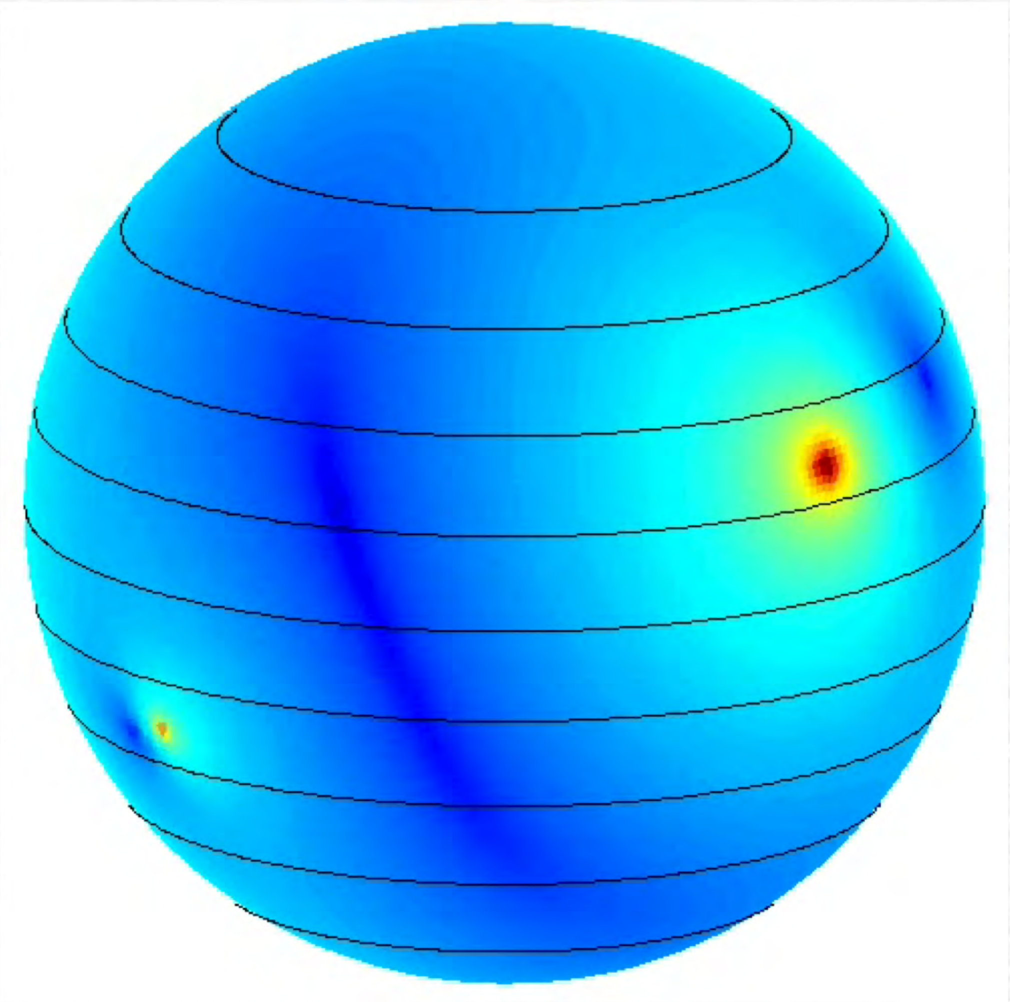
For the accuracy of 1 µas for an observer close to the Earth:

- monopole field (post-Newtonian + enhanced ppN terms)
- quadrupole field
- gravitomagnetic field due to translational motion



Gravitational light deflection: $n \leftrightarrow \sigma \leftrightarrow k$

- The monopole effects due to the major bodies of the solar system in μas
- The maximal angular distance to the bodies where the effect is still >1 μas



body	Monopole	ψ_{max}
Sun	1.75×10^6	180 °
(Mercury)	83	9 ′
Venus	493	4.5 °
Earth	574	125 °
Moon	26	5 °
Mars	116	25 ′
Jupiter	16270	90 °
Saturn	5780	17 °
Uranus	2080	71 ′
Neptune	2533	51 ′

Analytical theory of light propagation for numerical accuracy of 1 μas

When the theory meets the practice…

- only numerical magnitude is interesting for practical work
- analytical orders of magnitude are often used

Detailed investigation of numerical magnitudes of various terms
In the equations of light propagation

Klioner & Zschocke, 2010

Schwarzschild field: the big formula

$$\boxed{\boldsymbol{d}_\sigma = \boldsymbol{\sigma} \times (\boldsymbol{x} \times \boldsymbol{\sigma})}$$

N	$\boldsymbol{n} = \boldsymbol{\sigma}$
pN	$-(1+\gamma)\, m \dfrac{\boldsymbol{d}_\sigma}{d_\sigma^2} \left(1 + \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{x}}{x}\right)$
ΔpN	$+(1+\gamma)^2\, m^2 \dfrac{\boldsymbol{d}_\sigma}{d_\sigma^3}\, \dfrac{x}{d_\sigma} \left(1 + \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{x}}{x}\right)^2$
scaling	$-\dfrac{1}{2}\, m^2 (1+\gamma)^2\, \dfrac{\boldsymbol{\sigma}}{d_\sigma^2} \left(1 + \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{x}}{x}\right)^2$
ppN	$-\dfrac{1}{2}\, m^2 \alpha\, \epsilon\, \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{x}}{x^4}\, \boldsymbol{d}_\sigma$
ppN	$+(1+\gamma)^2\, m^2 \dfrac{\boldsymbol{d}_\sigma}{d_\sigma^2}\, \dfrac{1}{x} \left(1 + \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{x}}{x}\right)$
ppN	$-\dfrac{1}{4}\, \left(8\,(1+\gamma-\alpha\,\gamma)\,(1+\gamma) - 4\,\alpha\,\beta + 3\,\alpha\,\epsilon\right)\, m^2\, \dfrac{\boldsymbol{d}_\sigma}{d_\sigma^2}\, \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{x}}{x^2}$
ppN	$-\dfrac{1}{4}\, \left(8\,(1+\gamma-\alpha\,\gamma)\,(1+\gamma) - 4\,\alpha\,\beta + 3\,\alpha\,\epsilon\right)\, m^2\, \dfrac{\boldsymbol{d}_\sigma}{d_\sigma^3}\, \left(\pi - \delta(\boldsymbol{\sigma},\boldsymbol{x})\right)$
	$+\mathcal{O}(m^3),$

Schwarzschild field: estimates

$$|\psi_{\mathrm{pN}}| = 2\,m\,\frac{|\boldsymbol{\sigma}\times\boldsymbol{d}_\sigma|}{d_\sigma^2}\,\left(1+\frac{\boldsymbol{\sigma}\cdot\boldsymbol{x}}{x}\right) \leq 4\,\frac{m}{d_\sigma}$$

$$|\psi_{\Delta\mathrm{pN}}| = 4m^2\,\frac{|\boldsymbol{\sigma}\times\boldsymbol{d}_\sigma|}{d_\sigma^3}\,\frac{x}{d_\sigma}\left(1+\frac{\boldsymbol{\sigma}\cdot\boldsymbol{x}}{x}\right)^2 \leq 16\,\frac{m^2}{d_\sigma^2}\,\frac{x}{d_\sigma}$$

$$\boxed{|\psi_{\mathrm{ppN}}| \leq \frac{15}{4}\,\pi\,\frac{m^2}{d_\sigma^2}}$$

Schwarzschild field: simplified formula

Second-order effect at very low computational costs:

$$\boldsymbol{n} = \boldsymbol{\sigma} + \boldsymbol{d}_\sigma\, Q\,(1 + Q\,x) + \mathcal{O}\left(\frac{m^2}{d_\sigma^2}\right) + \mathcal{O}(m^3)\,,$$

$$Q = -(1+\gamma)\,\frac{m}{d_\sigma^2}\,\left(1 + \frac{\boldsymbol{\sigma}\cdot\boldsymbol{x}}{x}\right)\,.$$

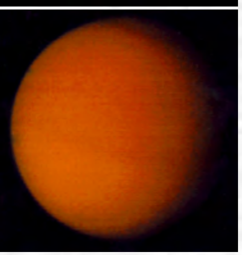
The enhanced ppN term in μas:

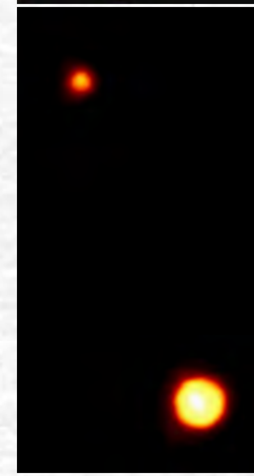
Sun	Sun at 45°	Jupiter	Saturn	Uranus	Neptune
3192.8	0.663×10^{-3}	16.11	4.42	2.58	5.83

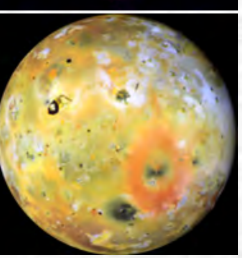
Gravitational light deflection: $n \leftrightarrow \sigma \leftrightarrow k$

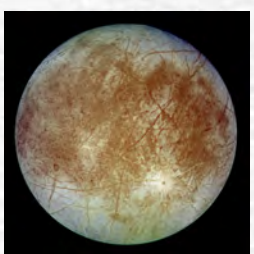
- A body of mean density ρ produces a light deflection not less than δ if its radius:

$$R \geq \left(\frac{\rho}{1\,\mathrm{g/cm^3}}\right)^{-1/2} \times \left(\frac{\delta}{1\,\mu\mathrm{as}}\right)^{1/2} \times 650\ \mathrm{km}$$









Ganymede	35
Titan	32
Io	30
Callisto	28
Triton	20
Europe	19

Pluto	7
Charon	4
Titania	3
Oberon	3
Iapetus	2
Rea	2
Dione	1
Ariel	1
Umbriel	1
Ceres	1

Gravitational light deflection: $n \leftrightarrow \sigma \leftrightarrow k$

- Quadrupole light deflection:

body	Quadrupole	$\psi_{\max}$
Jupiter	240	152 ″
Saturn	95	46 ″
Uranus	8	4 ″
Neptune	10	3 ″

- Efficient evaluation (Zschocke, Klioner, 2011):

radius of the body

$$|\,\delta\boldsymbol{\sigma}_{\mathrm{Q}}^{A}\,| \leqslant \frac{9}{8}\,|J_2^A|\,\frac{P_A^2}{d_A^2}\,|\,\delta\boldsymbol{\sigma}_{\mathrm{pN}}^{A}|$$

quadrupole deflection

monopole deflection

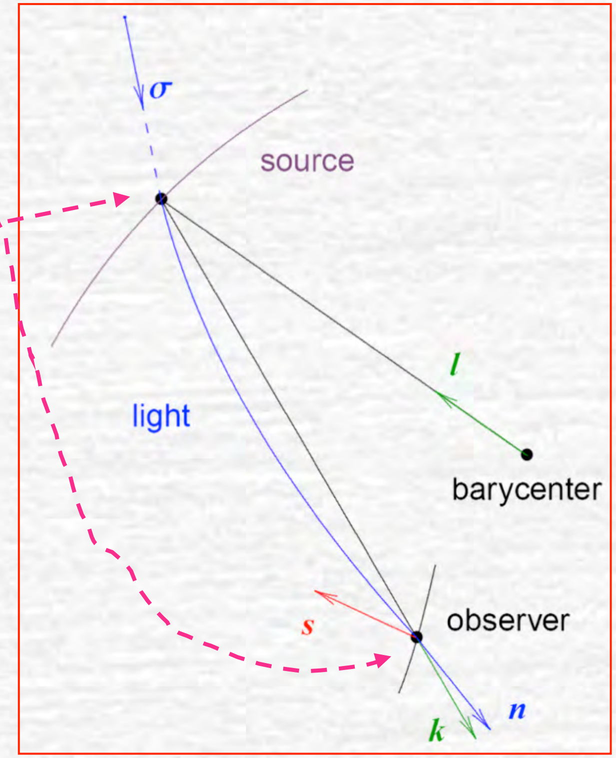
impact parameter of the light ray

Only 3 multiplications are needed!
The mean value of the estimate is 0.494… of the real quadrupole deflection.

Parallax and proper motion: $k \leftrightarrow l \leftrightarrow l_0, \mu_0, \pi_0$

- All formulas here are formally Euclidean:

$$\boldsymbol{k} = \frac{\boldsymbol{x}_o(t_o) - \boldsymbol{X}_s(t_e)}{|\boldsymbol{x}_o(t_o) - \boldsymbol{X}_s(t_e)|}, \qquad \boldsymbol{l} = \frac{\boldsymbol{X}_s(t_e)}{|\boldsymbol{X}_s(t_e)|},$$



$$\boldsymbol{X}_s(t_e) = \boldsymbol{X}_s(t_{e0}) + \boldsymbol{V}_s(t_{e0})\,(t_e - t_{e0}) + \dots$$

- Expansion in powers of several small parameters:

$$\pi = \frac{1\ \mathrm{AU}}{|\boldsymbol{X}_s(t_e)|}, \qquad \mu = \frac{|\boldsymbol{V}_s(t_e)|}{|\boldsymbol{X}_s(t_e)|}$$

$$\boldsymbol{k} = -\boldsymbol{l} + \dots, \qquad \boldsymbol{l} = \boldsymbol{l}_0 + \dots$$

Parallax and proper motion: $k \leftrightarrow l \leftrightarrow l_0, \mu_0, \pi_0$

- All formulas here are formally Euclidean, but the finite light velocity should be taken into account:
  - "superluminal effect": light emission by a moving source

$$V_{\mathrm{rad}}^{\mathrm{ap}} = V_{\mathrm{rad}} \left(1 + c^{-1}\, V_{\mathrm{rad}}\right)^{-1}$$

$$\boldsymbol{V}_{\mathrm{tan}}^{\mathrm{ap}} = \boldsymbol{V}_{\mathrm{tan}} \left(1 + c^{-1}\, V_{\mathrm{rad}}\right)^{-1}$$

  - "Roemer effect" due to the barycentric motion of the observer: up to 150 μas for high-proper-motion stars

$$\boldsymbol{l}(t\ ) = \boldsymbol{l}_0 + \boldsymbol{\mu}_{\mathrm{ap}} \Delta t_o + \boldsymbol{\mu}_{\mathrm{ap}} c^{-1} \{[\boldsymbol{x}_o(t) - \boldsymbol{x}_o(t_0)] \cdot \boldsymbol{l}_0\} + \cdots$$

$$\boldsymbol{\mu}_{\mathrm{ap}} = \pi_0 \frac{\boldsymbol{V}_{\mathrm{tan}}^{\mathrm{ap}}}{1\ \mathrm{AU}}$$

The BCRS coordinates of the observer at the moment of observation and at some reference epoch

Celestial Reference Frame

- All astrometrical parameters of sources obtained from astrometric observations are defined in BCRS coordinates:
    - positions
    - proper motions
    - parallaxes
    - radial velocities
    - orbits of minor planets, etc.
    - orbits of binaries, etc.
- These parameters represent a realization (materialization) of the BCRS
- This materialization is „the goal of astrometry“ and is called

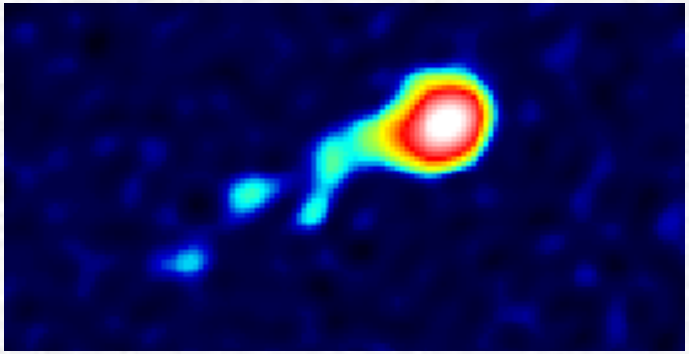
Celestial Reference Frame

The concept of ICRS

- It is *assumed* that on average quasars do not rotate with respect to our global reference system, BCRS

- This is a *cosmological* assumption to be verified by dynamical observations

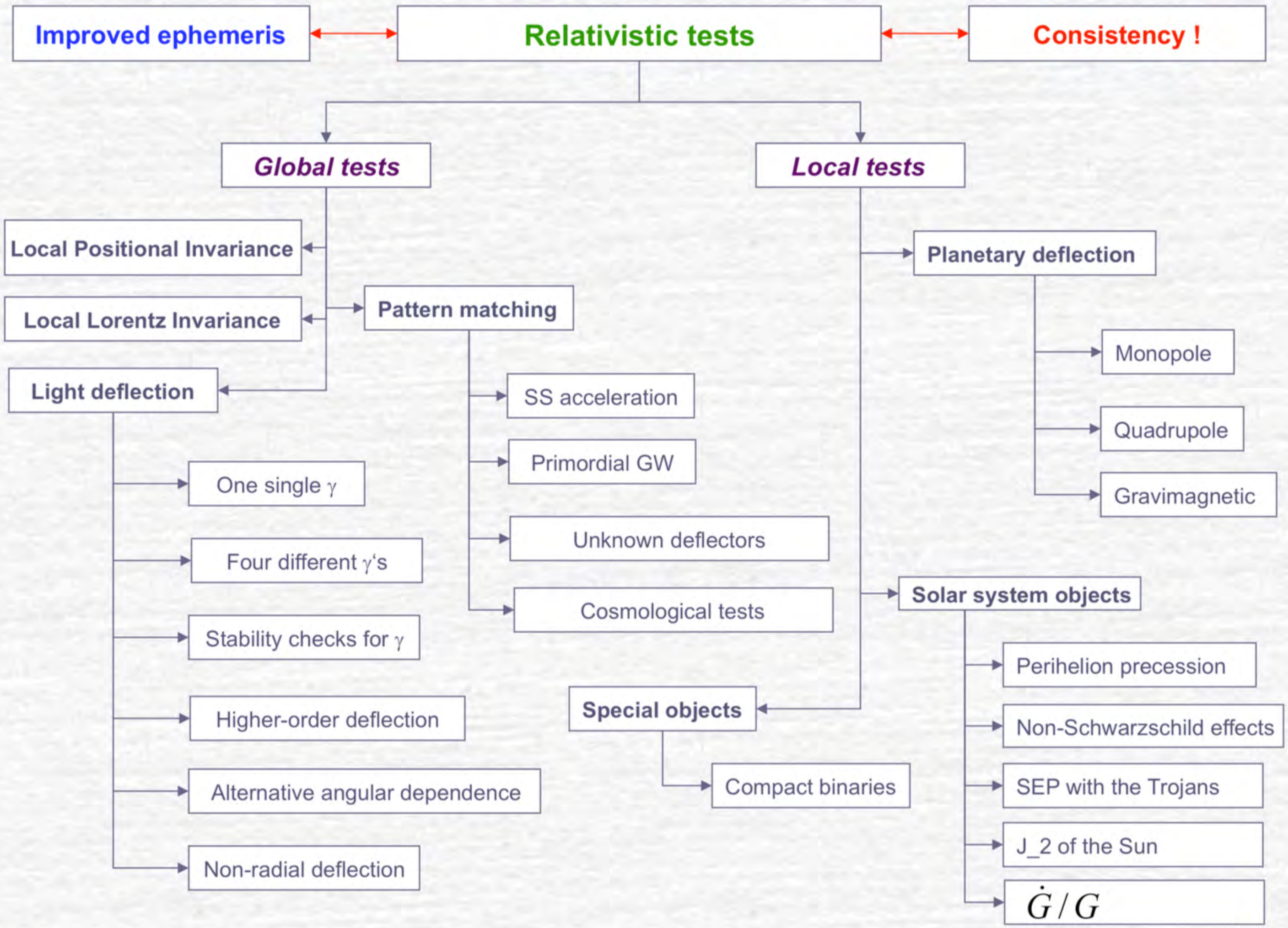
- This concept will by tested by millisecond pulsars and Gaia with unprecedented accuracy



Testing Relativity with Gaia

- Each effect included in the model can be used to test the theory (of relativity)

- Many tests are planned:



Is nanoarcsecond astrometry possible?

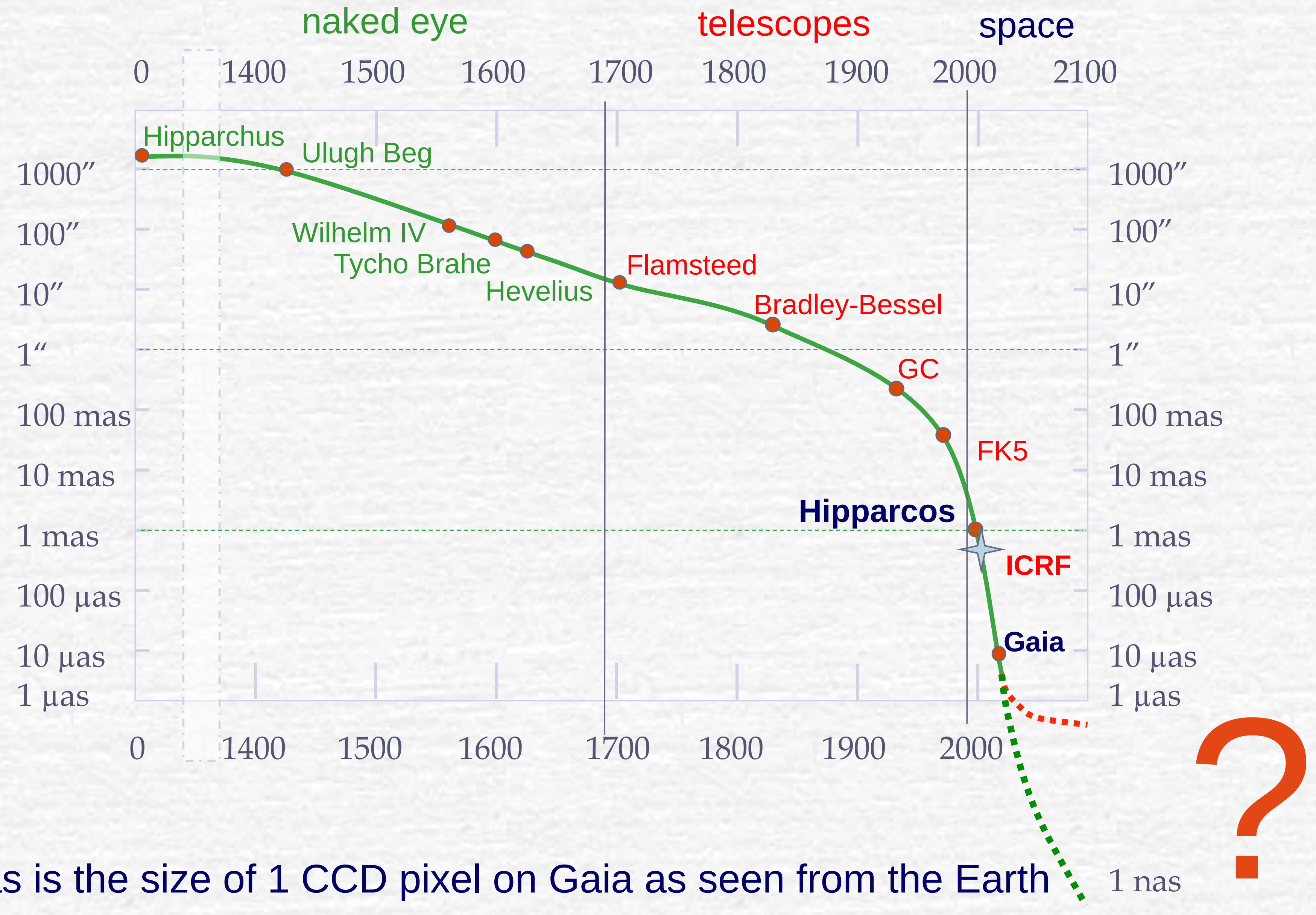
- Leaving aside all the source structure of quasars and stars…
- Leaving aside the stability of the instruments
- At which level of accuracy does the relativistic model becomes chaotic or too complicated to apply in practice?

BCRS velocity at the level of $10^{-6}\ \mathrm{m/s}$? (position to 0.1 m?)
Is it possible to do astrometry without knowing velocity?
(Butkevich, Klioner, 2006-)

Microlensing as noise (too many unknown objects in the Galaxy)

Deflection on small asteroids (from R=500 m) – chaotic?

Is nanoarcsecond astrometry possible?



1 nas is the size of 1 CCD pixel on Gaia as seen from the Earth