Using Ring Laser Systems to Measure Gravitomagnetic Effects on Earth

Matteo Luca Ruggiero[1]

[1]RELGRAV @ Politecnico di Torino, INFN Sezione di Torino

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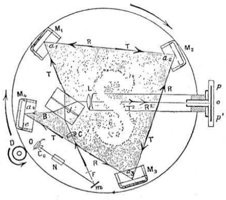
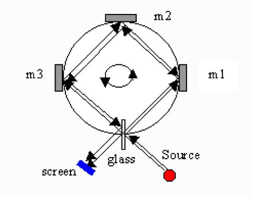
Reference Frames in Newtonian Physics

Focault's Pendulum

There are many mechanical devices for detecting the state of acceleration of a system and, in particular, the state of rotation of a frame. A Foucalut's pendulum can be used to detect and measure the Earth's rotation rate.

Electromagnetic Effects in Rotating Frames: Sagnac Experiments



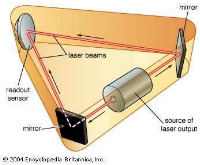


Sagnac (1913)

Electromagnetic detection of the rotation of a reference frame: he measured the fringe shift Δz for monochromatic light waves in vacuum, counter-propagating along a closed path (delimiting an oriented area $\boldsymbol{A} = A\boldsymbol{u}$) in an interferometer rotating with angular velocity $\boldsymbol{\Omega}$:

$$\Delta z = \frac{4\boldsymbol{\Omega} \cdot \boldsymbol{A}}{\lambda c}$$

Applications of the Sagnac Effect: the Ring Laser



- A ring laser gyroscope is a ring cavity around which two laser beams propagate in opposite directions around a closed circuit or ring, which is usually rectangular or triangular
- In a rotating frame (or in a non time-orthogonal metric) the two propagation directions are not equivalent, so that two oscillation frequencies are not the same
- What is measured is a frequency shift between two opposite directed traveling waves: the output intensity is modulated at the beat frequency
- The frequency shift is proportional to $\mathbf{\Omega} \cdot \boldsymbol{A}$, so that three laser rings are able to detect the rotation rate $\mathbf{\Omega}$ with respect to an inertial frame

Ring laser gyros are today very accurate rotation sensors: we can use them to test General Relativity!

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Ring Laser Measurements in General Relativity

What a ring laser would measure in General Relativity?

- Define the reference frame of the laboratory
- Define the space-time metric in this reference frame

Reference Frames and Coordinates in General Relativity

Democracy of Reference Frames and Coordinates

General Covariance requires that physics laws are expressed by means of tensorial equations in a pseudo-Riemannan manifold, which is the (mathematical model of the) four-dimensional space-time.

- there are no privileged reference frames
- within a frame, there are no privileged coordinates sets

Measurements in Space-Time

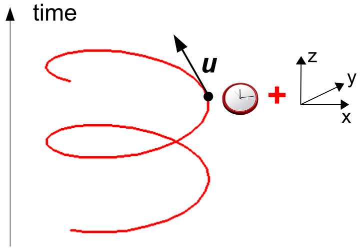
In Democracy Elections take place

In order to define the results of a measurement in the four-dimensional space-time, it is then necessary to focus on the (class of) observers that are performing such measurements:

- observers posses their own space-time, in the neighborhood of their world-lines
- covariant physics laws are then projected onto local space and time, by means of splitting techniques
- predictions for the outcome of measurements in the locally Minkowskian neighborhood of the observer are then obtained

$\longrightarrow$ Talks by F. de Felice e D. Bini

Measurements in Space-Time



Space-Time Splitting along the observer's world-line *u*

Gravitoelectromagnetic (GEM) fields can be introduced whenever one applies splitting techniques: the field equations of general relativity and geodesics equation can be recast in a 3+1 space+time form, in which they are analogous to Maxwell's equations and Lorentz force law

Physics is simple only when analyzed locally: the laboratory frame

Up to linear displacements from the observer's world-line

The space-time metric in the laboratory is

$$ds^2 = (1 + 2\boldsymbol{\mathcal{A}} \cdot \boldsymbol{x})\, dt^2 - d\boldsymbol{x} \cdot d\boldsymbol{x} - 2\,(\boldsymbol{\Omega} \wedge \boldsymbol{x}) \cdot d\boldsymbol{x} dt + O(|\boldsymbol{x}|^2)$$

- $\boldsymbol{\mathcal{A}}$ is the spatial projection of the observer's four-acceleration $\rightarrow$ failure of free fall
- $\boldsymbol{\Omega}$ is the precession rate of the local tetrad with respect to a Fermi-Walker transported tetrad $\rightarrow$ rotation of the gyroscopes with respect to the observer's tetrad
- the observer's frame is non rotating when its axes are Fermi-Walker transported, so $\boldsymbol{\Omega}$ measures the rotation rate of the frame

Physics is simple only when analyzed locally: the laboratory frame

Up to linear displacements from the observer's world-line

The space-time metric in the laboratory is

$$ds^2 = (1 + 2\boldsymbol{\mathcal{A}} \cdot \boldsymbol{x})\, dt^2 - d\boldsymbol{x} \cdot d\boldsymbol{x} - 2\,(\boldsymbol{\Omega} \wedge \boldsymbol{x}) \cdot d\boldsymbol{x} dt + O(|\boldsymbol{x}|^2)$$

- The Output of the Ring Laser is

$$\delta f = \frac{4A}{\lambda P} \boldsymbol{\Omega} \cdot \boldsymbol{u}$$

- $\boldsymbol{\Omega}$ is related to the g_{0i} terms of the observer's metric, so it measures the gravitomagnetic field in the laboratory frame

Laboratory on the Earth

In order to define $\mathbf{\Omega}$, we have to consider that

- the laboratory is fixed on the Earth surface
- the space-time of the rotating Earth can be described by the post-Newtonian metric

$$ds^2 = (1-2U(R))dT^2 - (1+2\gamma U(R))\,\delta_{ij}dX^i dX^j + 2\left[\frac{(1+\gamma+\alpha_1/4)}{R^3}(\boldsymbol{J}_\oplus \times \boldsymbol{R})_i - \alpha_1 U(R) W_i\right] dX^i dT,$$

where $\gamma = 1, \alpha_1 = 0$ in GR; $U(R)$ is the gravitational potential of the Earth, $\boldsymbol{J}_\oplus$ is its angular momentum, W_i measures preferred frames effect.

Laboratory on the Earth

The Rotation Rate measured by a Ring Laser in a terrestrial laboratory would be

$$\boldsymbol{\Omega} = \boldsymbol{\Omega}_0 + \boldsymbol{\Omega}_{REL}$$

where $\boldsymbol{\Omega}_0$ is the terrestrial rotation rate (the laboratory axes rotate) and

$$\boldsymbol{\Omega}_{REL} = \boldsymbol{\Omega}_G + \boldsymbol{\Omega}_B + \boldsymbol{\Omega}_W + \boldsymbol{\Omega}_T$$

$$\begin{aligned}
\boldsymbol{\Omega}_G &= -(1+\gamma)\frac{GM}{c^2R}\sin\vartheta\Omega_0\boldsymbol{u}_\vartheta \rightarrow \text{Geodetic Precession} \\
\boldsymbol{\Omega}_B &= -\frac{1+\gamma+\alpha_1/4}{2}\frac{G}{c^2R^3}\left[\boldsymbol{J}_\oplus - 3\left(\boldsymbol{J}_\oplus\cdot\boldsymbol{u}_r\right)\boldsymbol{u}_r\right] \rightarrow \text{Lense} - \text{Thirring} \\
\boldsymbol{\Omega}_W &= -\frac{\alpha_1}{4}\frac{GM}{c^2R^2}\boldsymbol{u}_r\wedge\boldsymbol{W} \rightarrow \text{Preferred Frame Effect} \\
\boldsymbol{\Omega}_T &= -\frac{1}{2c^2}\Omega_0^2R^2\sin^2\vartheta\boldsymbol{\Omega}_0 \rightarrow \text{Thomas Precession}
\end{aligned}$$

Orders of magnitude of the leading contributions to Ω

Leading GR contributions

- Geodetic

$$\Omega_G \simeq \frac{M_\oplus}{R_\oplus}\Omega_0 \simeq 6 \cdot 10^{-10}\Omega_0,$$

- Lense-Thirring

$$\Omega_B \simeq \zeta \frac{M_\oplus}{R_\oplus}\Omega_0 \simeq 6 \cdot 10^{-10}\,\zeta\,\Omega_0.$$

Measured by GP-B $\rightarrow$ Talk by N. Bartel

Ring laser gyros could make it possible to attain a precision ranging from $10^{-9}\,\Omega_0$ to $10^{-11}\,\Omega_0$: the detection of local gravitomagnetic field is within the range of current precision.

Detection of the gravitational contributions

Modeling the Leading Kinematical Contribution

- The above estimates suggest that the kinematical effect due to the Earth rotation rate overwhelms the other contributions due to the gravitational field by 10 orders of magnitude.
- In order to the detect the gravitational effects, it is necessary to correctly model and subtract from Ω the leading contribution Ω_0, due to the rotation of the Earth: this can be done by using the value determined by the IERS.

Contents

G-GranSasso People

F. Bosi, G. Cella, A. Di Virgilio, *INFN, Pisa*

M. Allegrini, J. Belfi, N. Beverini, G. Carelli, I. Ferrante, A. Fioretti, E. Maccioni, F. Stefani, *Univ. Pisa and CNISM*

F. Sorrentino, *Univ. Firenze*

A. Porzio, S. Solimeno, *Univ. Napoli and CNISM*

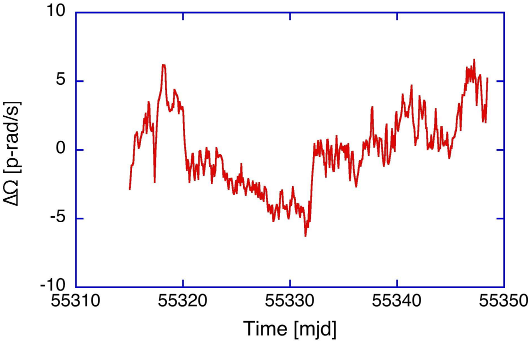
M. Cerdonio, A. Ortolan and J.P Zendri, *Univ. Padova and INFN-LNL*

MLR, A. Tartaglia, M. Sereno, *Politecnico di Torino and INFN*

U. Schreiber and team, *Technische Universitaet Muenchen - Fundamentalstation Wettzell and Forschungseinrichtung Satellitengeodaesie, Germany*

Jon-Paul Wells and team, *University of Christchurch, New Zealand*

The starting point: G-Wetzell



The large ring laser "G" at the Geodetic Observatory in Wettzell has a square contour with an area of 16 m^2 and a corresponding perimeter of 16 m and is placed on a very stable granite monument in a laboratory approximately 6 m below the Earth surface. A performance level of better than 1.26×10^{-11}rad/s is now routinely obtained.

The Reference Frame

$$\mathbf{\Omega} \cdot \boldsymbol{u}$$

Vectors in the same reference frame!

- The ring laser measures the rotation rate projected on to the normal to the ring area
- The gravitational contribution to the ring laser signal is $(\mathbf{\Omega}_G + \mathbf{\Omega}_B) \cdot \boldsymbol{u} = (\mathbf{\Omega} - \mathbf{\Omega}_0) \cdot \boldsymbol{u}$
- The orientation of the ring laser (local frame) should be known with an accuracy of 1 to 10^{10} w.r.t. the IERS frame (inertial frame)

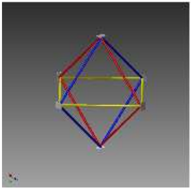
Is it possible to have some information about the vector $\mathbf{\Omega}$ without knowing *a priori* the relative orientation of the two reference frames?

A three axial detector

Focusing on G-Gran Sasso Proposal

- $\mathbf{\Omega}$ can be completely measured by means of its projections on at least 3 independent directions: we can use $M \geq 3$ ring lasers oriented along directions $\boldsymbol{u}^\alpha$ ($\alpha = 1 \dots M$), to obtain a three-axial detector

A three axial detector



Geometry of the Detector

- It is possible to exploit the properties of regular polyhedra with $M = 4$ (tetrahedron), 6 (cube), 8 (octahedron), 12 (dodecahedron) and 20 (icosahedron) to demonstrate that several constraints hold such as

$$\sum_{\alpha=1}^{M} \boldsymbol{u}_\alpha = \mathbf{0}, \quad \sum_{\alpha=1}^{M} (\boldsymbol{\Omega} \cdot \boldsymbol{u}_\alpha)^2 = \frac{M}{3} \left|\boldsymbol{\Omega}\right|^2 .$$

- These constraints can be used to reduce the impact of noise fluctuations or variations of the geometry of the configuration

Conclusion and Prospects

- Our ring laser system realizes a comparison between the local laboratory frame and the astrophysical inertial frame
- We do expect that a 10% accuracy in the measurements of the gravitomagnetic field can be achieved in three months by comparing the squared modulus of rotation vectors $\mathbf{\Omega}$ and $\mathbf{\Omega}_0$
- With highly accurate ring laser (shot noise limited) we can achieve the 1% accuracy (exploiting the polar motion to orient the local frame with the inertial frame)

Some Publications

- A. Di Virgilio, K. U. Schreiber and A. Gebauery, J-P. R. Wells, A. Tartaglia, J. Belfi and N. Beverini, A.Ortolan, A laser gyroscope system to detect the Gravito-Magnetic effect on Earth, MH GRF 2010, arXiv:1007.1861v1
- A. Di Virgilio, M. Allegrini, J. Belfi, N. Beverini, F. Bosi, G. Carelli, E. Maccioni, M. Pizzocaro, A. Porzio,U. Schreiber , S. Solimeno e F. Sorrentino, Performances of G-Pisa: a middle size gyrolaser, CQG 2010, SIF Award 2009
- MLR, A. Tartaglia, Gravitomagnetic Effects, NCB, 2002, arXiv:gr-qc/0207065v2
- G. Rizzi, MLR, The relativistic Sagnac Effect: two derivations in Relativity in Rotating Frames, 2003, arXiv:gr-qc/0305084v4