Physics and coordinates in competition at highly accurate measurements

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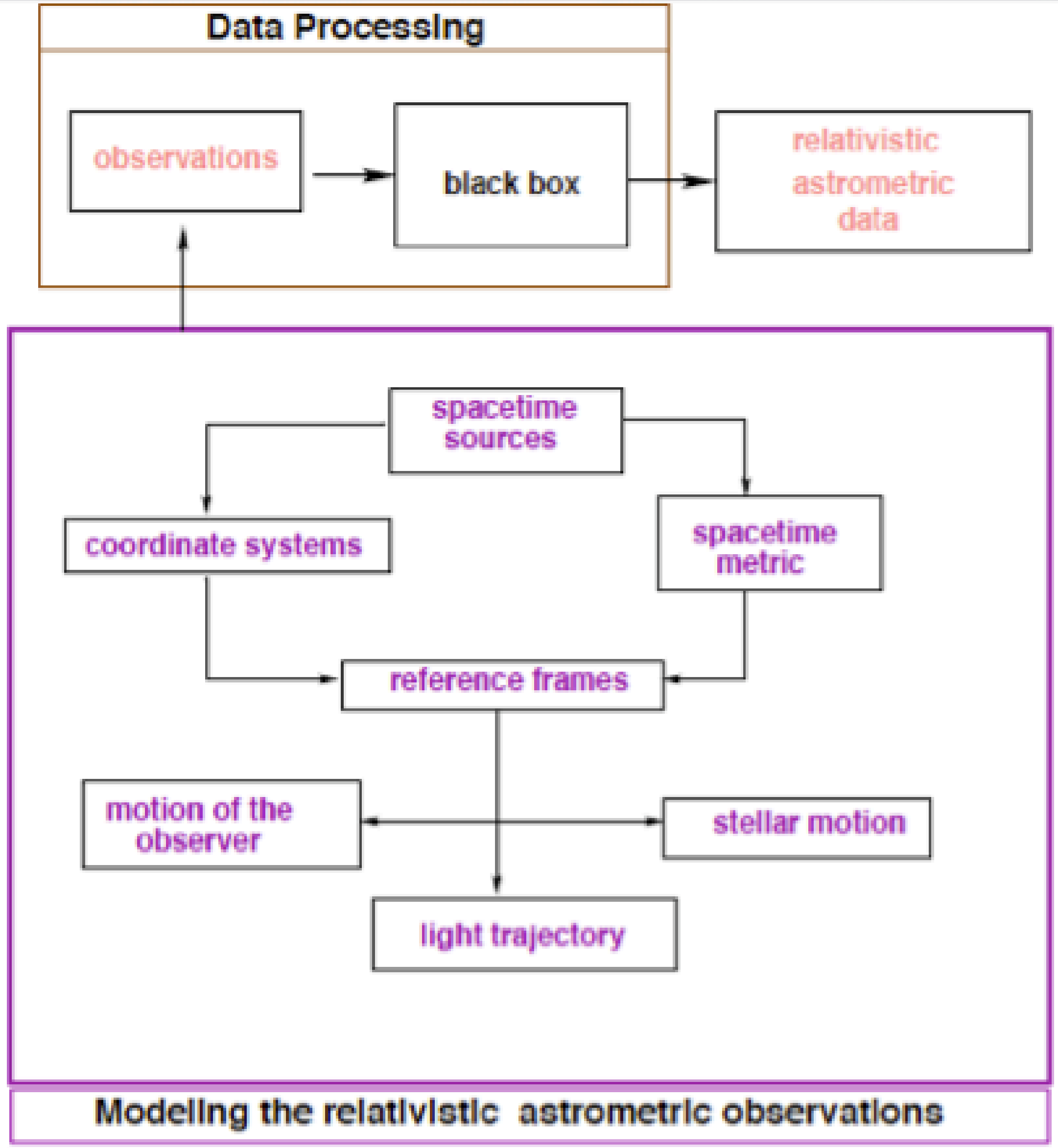
Outline

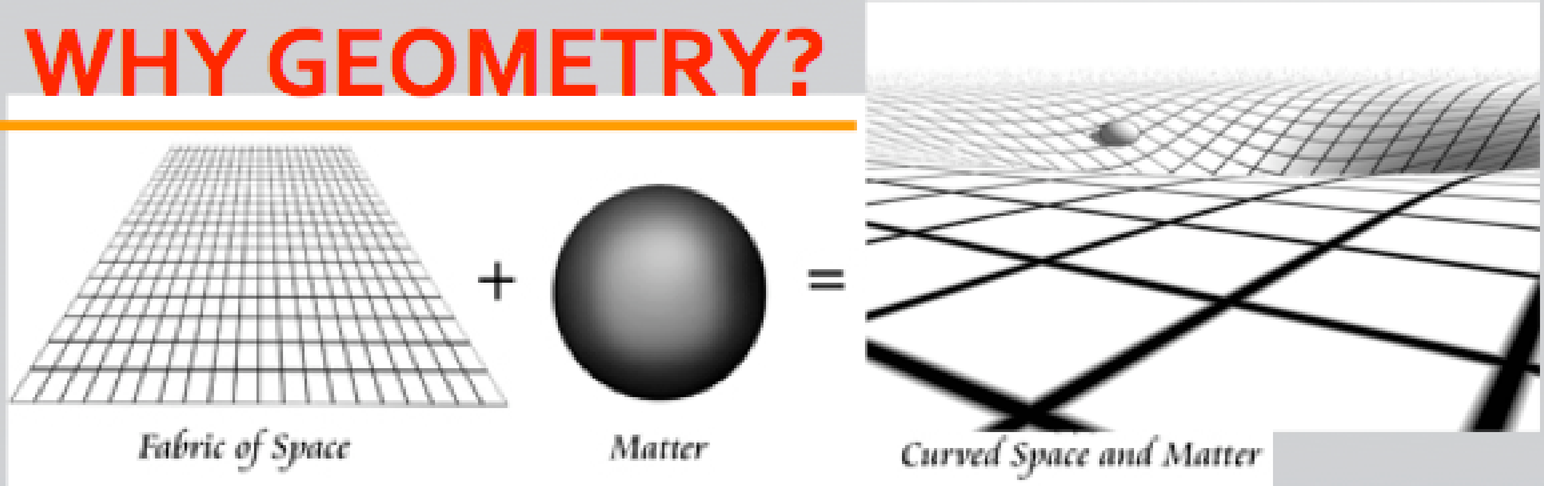
- The astrometric problem
- With or without vorticity
- Physics and coordinates: matching the interpretation at high accuracy
- Conclusion
- …and some confusion

The astrometric problem

star wanted



WHY GEOMETRY?



Brief summary of “obviousness”

- General Relativity is the theory in which geometry and physics are joined in order to explain how gravity works
- The trajectory of a photon is deduced by solving the null geodesic in a curved space-time
- The measurements of the light takes place in a geometrical background generated by a n-body distribution as, for example, the Solar System

Solar System background

weakly relativistic metric

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta} + O(h^2),$$

$$|h_{\alpha\beta}| \ll 1.$$

according to the virial theorem

$$|h_{\alpha\beta}| \leq U/c^2 \sim v^2/c^2,$$

ε ~ v^2/c^2 ~ GM/rc^2 ~ mas accuracy

which requires determination of

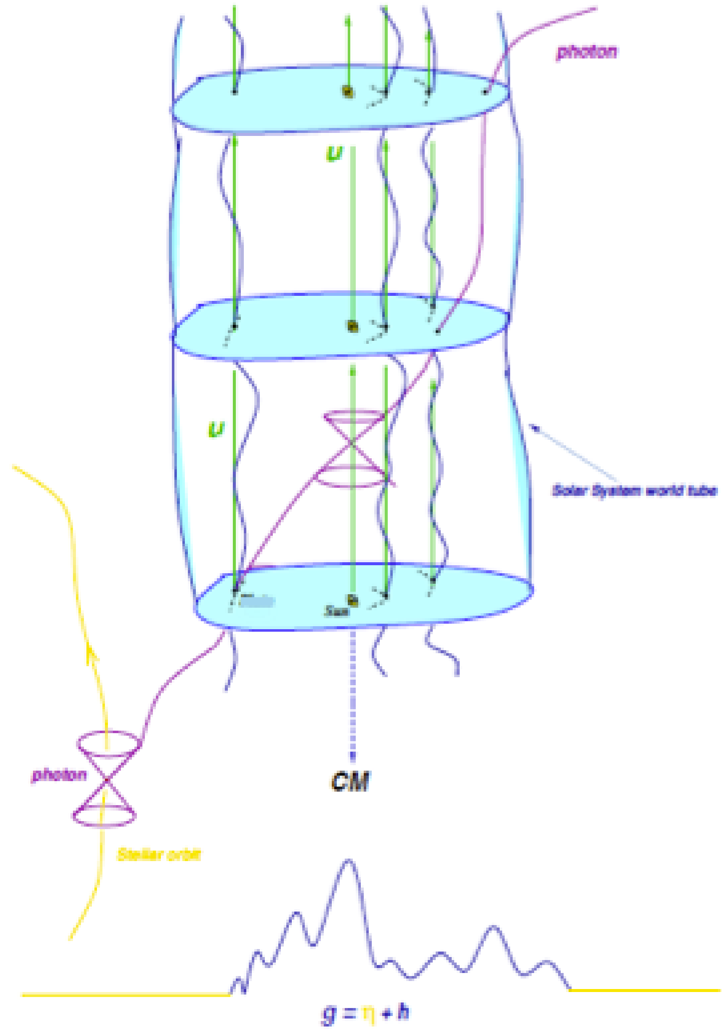
g_{oo} even terms in ε, lowest order $ε^2$~mas

g_{oj} odd terms in ε, lowest order $ε^3$~μ-as

g_{ij} even terms in ε, lowest order $ε^2$~mas

Time variation of the order of

$$\epsilon |h_{\alpha\beta}|.$$



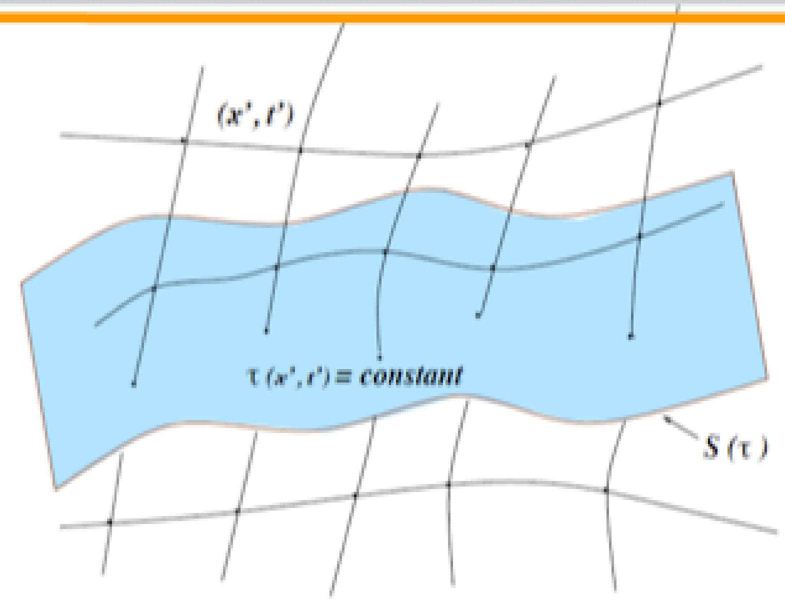
Observers&coordinates according to space-time evolution

✓Family of observes: a time-like congruences of curves ***u***
✓evolution: vorticity, which measures how a world-line of a congruence rotates around a neighboring one

$$\boxed{\omega_{ij} = \partial_{[j} h_{i]0} + O\left(1/c^4\right)}$$

h_{0i} and $\partial_j h_{0i}$ are $\sim O(1/c^3)$

i.e. ~ μ-arcsecond



if vorticity ~ 0, from the Frobenius theory there exists a space-time foliation->
✓constant hypersurfaces ***S*** with spatial coordinates x^i fixed on it
✓congruence of time-like lines ***u*** orthogonal to it with parameter σ which runs uniformely with the coordinate time τ

$$\begin{cases} x^0 = \tau(x', t') \\ x^i = x^i(x', t') \end{cases}$$

$$\begin{cases} u_\alpha(x^0, x^i) = -\delta^0_\alpha e^\psi \\ u^\alpha(x^0, x^i) = -g^{0\alpha} e^\psi = \dfrac{dx^\alpha}{d\sigma} \end{cases}$$

With or without vorticity?

The vorticity term cannot be neglected at the the order of microarcsecond, the possibility to ignore it locally can be applied only to a small neighborhood with respect to the scale of the vorticity itself

Within the scale of the Solar System and the accuracy of Gaia, then, there is no slice that extends from the observer up to the star that emits light

=><u>we have to scrutinize if a measurement can be considered local or not with respect to the curvature</u>.

“spatial coordinates” suffer the shift law from a slice to another

$$\Delta x^i = \int_0^{\Delta\tau} N^i(\tau')d\tau'.$$

$N^i=-u^i/u^0$

With or without

light direction is <u>always</u> a physical measurement: the local-line-of-sight represents what locally the observer measures of the collected light

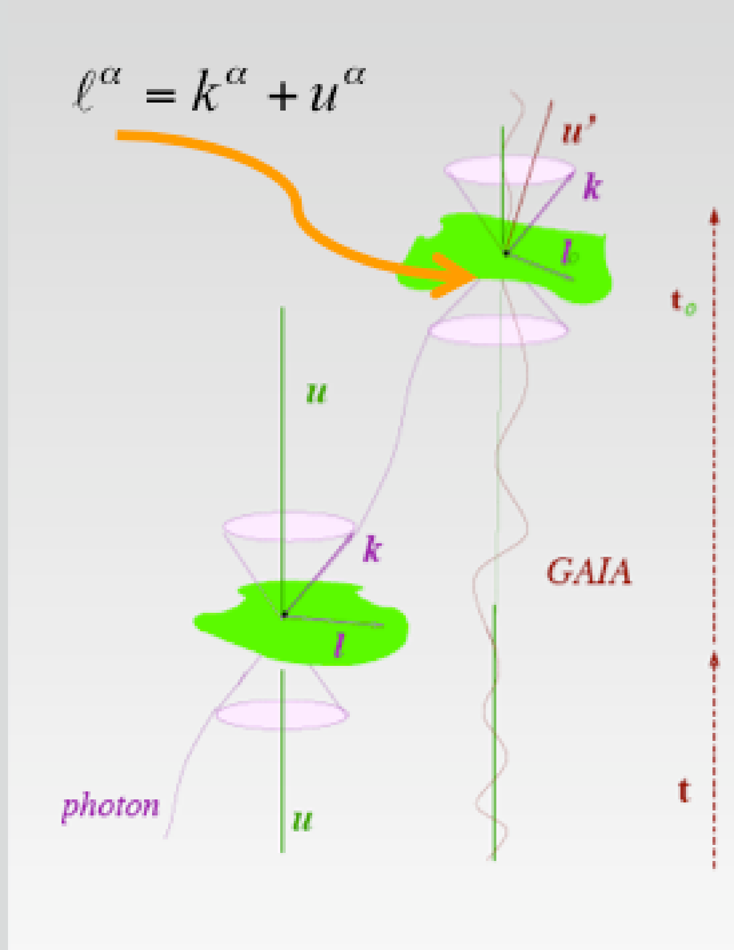
$$\ell^\alpha = P^\alpha_\beta(u) k^\beta(\tau)$$

$$P(u)_{\alpha\beta} = g_{\alpha\beta} + u_\alpha u_\beta$$

With respect to the perturbation term of the metric

$$\bar{\ell}^i = -\frac{k^i}{u_\alpha k^\alpha} \approx -\frac{k^i}{u^0 k^0 \left[-1 + h_{00} + h_{0i}(k^i/k^0)\right]}$$

$$\begin{aligned} k^\alpha k_\alpha &= 0 \\ \frac{dk^\alpha}{d\lambda} + \Gamma^\alpha_{\rho\sigma} k^\rho k^\sigma &= 0; \end{aligned} \longrightarrow$$



Master Equations

$$\boxed{\frac{d\ell^\alpha}{d\sigma} = F^\alpha(\partial_\beta h(x, y, z, t), \ell^i(\sigma(x)))}$$

With: the dynamical case

$$\frac{d\bar{\ell}^0}{d\sigma} - \bar{\ell}^i\bar{\ell}^j h_{0j,i} - \frac{1}{2}h_{00,0} = 0$$

One more equation to be integrated: time component

$$\frac{d\bar{\ell}^k}{d\sigma} - \frac{1}{2}\bar{\ell}^k\bar{\ell}^i\bar{\ell}^j h_{ij,0} + \bar{\ell}^i\bar{\ell}^j\left(h_{kj,i} - \frac{1}{2}h_{ij,k}\right) + \frac{1}{2}\bar{\ell}^k\bar{\ell}^i h_{00,i}$$
$$+ \bar{\ell}^i\left(h_{k0,i} + h_{ki,0} - h_{0i,k}\right) - \frac{1}{2}h_{00,k} - \bar{\ell}^k\bar{\ell}^i h_{0i,0} + h_{k0,0} = 0.$$

$$\bar{\ell}^i = \dot{x}^i\left(1 + \frac{1}{2}h_{00} + h_{0i}\dot{x}^i\right) + \mathcal{O}\left(h^2\right)$$

DYNAMICAL CASE: RAMOD4

$$\frac{d\bar{\ell}^k}{d\sigma} \approx \ddot{x}^k + \dot{x}^k\left(\frac{1}{2}h_{00,i}\dot{x}^i + \frac{1}{2}h_{00,0} + h_{0i,j}\dot{x}^i\dot{x}^j + h_{0i,0}\dot{x}^i\right) + \mathcal{O}(h^2)$$

"Tracing light propagation to the intrinsic accuracy of space-time geometry" M. Crosta, **arXiv:1012.5226, submitted (and references tehrein)**

$$\ddot{x}^k \approx \frac{1}{2}h_{00,k} - h_{0k,0} - \frac{1}{2}h_{00,0}\dot{x}^k - h_{ki,0}\dot{x}^i - \left(h_{0k,i} - h_{0i,k}\right)\dot{x}^i -$$
$$h_{00,i}\dot{x}^k\dot{x}^i - \left(h_{ki,j} - \frac{1}{2}h_{ij,k}\right)\dot{x}^i\dot{x}^j + \left(\frac{1}{2}h_{ij,0} - h_{0i,j}\right)\dot{x}^i\dot{x}^j\dot{x}^k.$$

i.e. the first pM approximation of the null geodesic, consistently to the weak field assumption

solution in the pM/pN approximation

Relativistic effects or perturbation Ξ in the solar systems depending on the method adopted in order to integrate the null geodesic

$$x^i = x^i_o + k^i \Delta t + \Xi^i$$

pN/pM

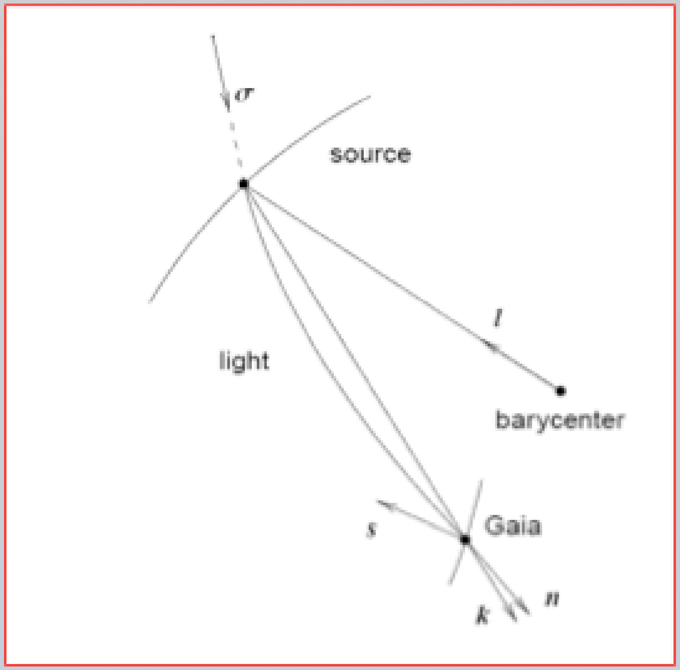
$$k^\alpha \equiv \left(1, k^i\right)$$

$$\frac{d^2\Xi^i}{dt^2} = F^i$$

parametrization of the light trajectory

(see Kopeikin et al., Phys. Rev. D, 1999)

$$\frac{1}{2}k^\alpha k^\beta (\check{h}_{\alpha\beta})_{,k} - \left(\frac{1}{2}\check{h}_{00}k^k - k^k k^p \check{h}_{p0} + \frac{1}{2}k^k k^p k^q \check{h}_{pq}\right)_{,\tau}$$



s -> the observed direction

- An application in pM approx. exists using the Time Transfer Function (no integration of the differential equation of the null geodesic, talk of Tessandyer and Bertone)

Without: the static case and mapped trajectories

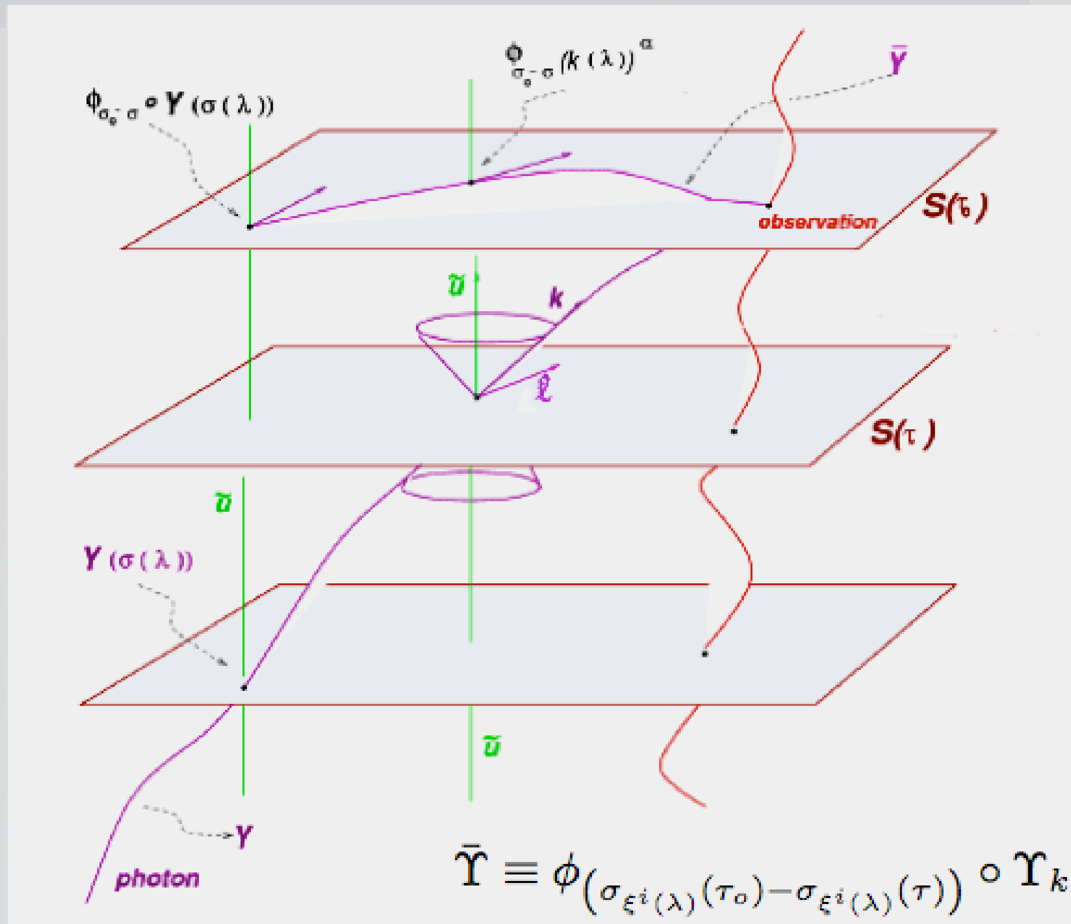
STATIC CASE: RAMOD3

$$\frac{d\bar{\ell}^k}{d\sigma} + \bar{\ell}^k\left(\frac{1}{2}\bar{\ell}^i h_{00,i}\right) + \delta^{ks}\left(h_{sj,i} - \frac{1}{2}h_{ij,s}\right)\bar{\ell}^i\bar{\ell}^j - \frac{1}{2}\delta^{ks}h_{00,s} = 0.$$

congruence of Killing vector everywhere orthogonal to the slice

time derivative of the metric are null

$g_{0i}=0$

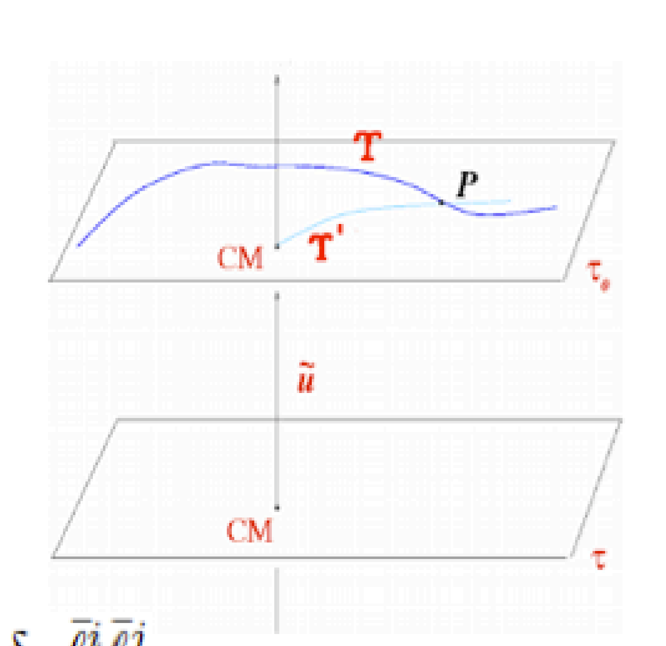


Without: parametrizated trajectories

The mapped trajectory allows to introduce two independent parameters :

impact parameter $\hat{\xi}^i$ $\hat{\tau} \equiv \sigma - \hat{\sigma}$

$$\xi^i(\sigma) = \hat{\xi}^i + \int_0^{\hat{\tau}} \bar{\ell}^i d\hat{\tau}'$$



by considering an appropriate coordinates transformation and the Euclidean scalar product

$$\hat{\xi}^k = P(\bar{\ell})^k_i \xi^i - \bar{\ell}^k_0 \hat{\tau} + \Xi^k,$$
$$\hat{\xi}^0 = \hat{\tau},$$

$$\bar{\ell}^i \bar{\ell}_i = \delta_{ij} \bar{\ell}^i \bar{\ell}^j$$

RAMOD3 master equations recovers the parametrized geodesic equation of Kopeikin et al. (Phys. Rev. D, 1999:

$$\frac{d^2\Xi^k}{d\hat{\tau}^2} \approx \frac{1}{2}\left(\hat{h}_{00} - 2\ell_0^p \hat{h}_{p0} + \ell_0^p \ell_0^q \hat{h}_{pq}\right)_{,\hat{k}} - \left(\frac{1}{2}\hat{h}_{00}\ell_0^k - \hat{h}_{0k} + \ell_0^q \hat{h}_{kq} - \frac{1}{2}\ell_0^k \ell_0^p \ell_0^q \hat{h}_{pq}\right)_{,\hat{\tau}}$$

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Retarded distances contributions

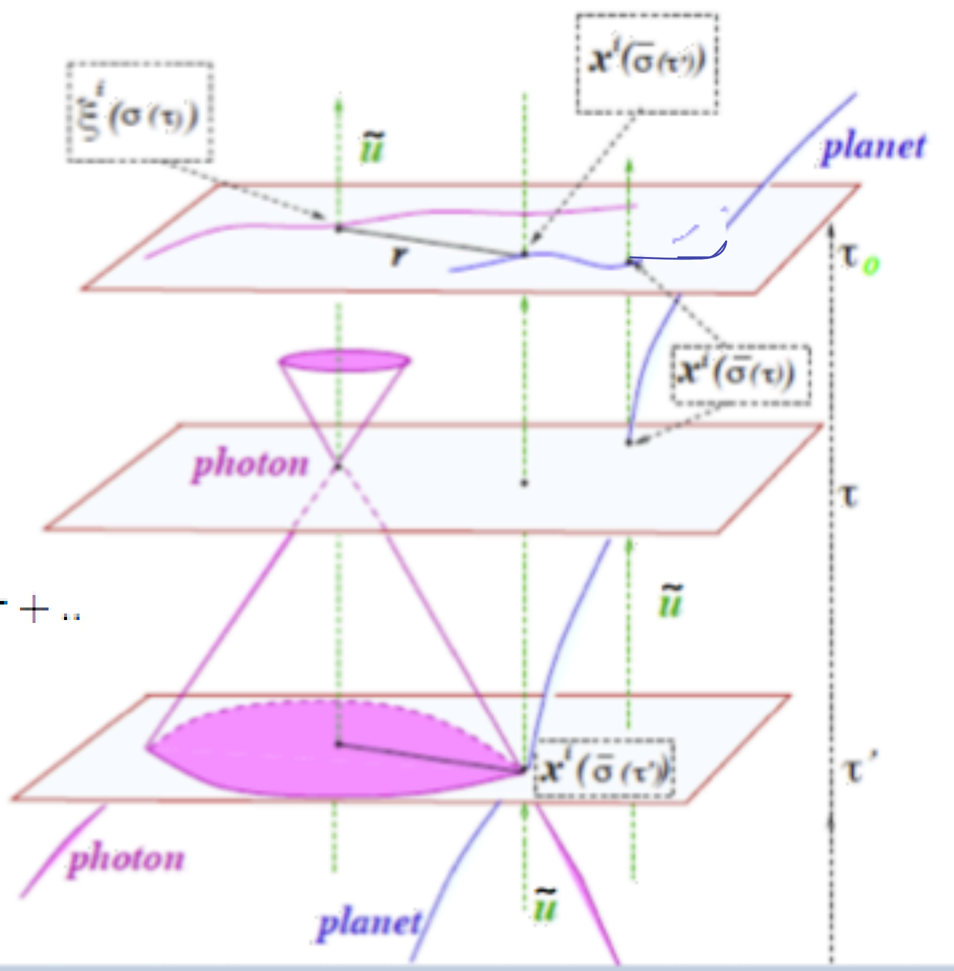
$$h_{00}^{(a)} = \left(\frac{2G\mathcal{M}^{(a)}}{c^2 r_{\mathcal{R}}^{(a)}}\right) + O\left(\frac{1}{c^4}\right)$$

$$h_{jk}^{(a)} = \left(\frac{2G\mathcal{M}^{(a)}}{c^2 r_{\mathcal{R}}^{(a)}}\right)\delta_{jk} + O\left(\frac{1}{c^4}\right)$$

$$h_{0j}^{(a)} = -\frac{2w_j^{(a)}}{c^3} + O\left(\frac{1}{c^4}\right),$$

$$r = |\xi^i(\sigma(\tau)) - x^i(\bar{\sigma}(\tau'))|$$

$$= \xi^i(\sigma(\tau)) - x^i(\bar{\sigma}(\tau)) + \int_{\tau'}^{\tau} (u_\beta \bar{u}^\beta)^2 \mathrm{e}^{-\psi} \frac{\bar{v}^i}{c} d\tau + ..$$



Brushing up on the vorticity

- RAMOD3 can be parameterized since the geometry allows to define a rest-space of an observer everywhere, from the observer to the star, i.e. simultaneous “positions” with respect the to barycenter and a unique impact parameter.
- Only a dynamical space-time allows to consider the “active” contents of gravity

Modelling light propagation is intrinsically connected to the geometry where photons naturally move

Keeping the physical definitions of the quantities entering the process of observation guarantees the consistency of the measurements with the “intrinsic” accuracy of the space-time

Up to what accuracy do we have to use coordinates in order to interpret the “physics” ?

1. The spatial local-line-of-sight is not exactly equal to the light direction used in the pM or pN approximation

$$\cos\psi_{(E_{\hat{a}},\ell_{obs})} \equiv \mathbf{e}_{\hat{a}} = \frac{P(u')_{\alpha\beta}k^{\alpha}\boldsymbol{E}^{\beta}_{\hat{a}}}{(P(u')_{\alpha\beta}k^{\alpha}k^{\beta})^{1/2}}$$

$$\bar{l}^i = n^i\left(1-\frac{U}{c^2}\right)+\mathcal{O}\left(\frac{v^4}{c^4}\right)$$

n^i “aberration free” direction

U potential

It is possible to ``extract'' the aberration effect in the global observable, but keeping the physical definition of the local-light-of-sight turns out to have more terms at the milliarcsecond level of accuracy and different ones at the microarcsecond.,

Within the appropriate approximations and according to the BCRS definition adopted by IAU, RAMOD recovers the same expression adopted for the Gaia observable of GREM (up to the $(v/c)^3$ expansion).

Crosta M. and Vecchiato, A. 2010a, A&A, 509, A37

$\mathbf{e}_a$ results equivalent to $\mathbf{s}^a$ in pN/pM approach, under specific assumptions

2. Fully general-relativistic Doppler shift formula

- ω_* is the frequency of a photon as emitted by the star
- ω_{sat} is the corresponding reference frequency relative to the satellite rest-observer

$$\frac{\omega_*}{\omega_{\rm sat}} = \frac{1 - v_{*\rm rad}}{\sqrt{1 - v_*^2}}$$

$$\frac{\omega_*}{\omega_{\rm sat}} = \frac{1}{A(M_*, R_*)} \cdot \frac{\sqrt{(-g_{00})_{\rm sat}}}{\gamma_{\rm sat}(1 - v^{\alpha}_{\rm sat}\bar{\ell}_{(0)\alpha})} \times \exp\left[-\frac{1}{2}\int_{\sigma_*}^{\sigma_0}(\bar{\ell}^i\bar{\ell}^j\partial_0 h_{ij} + \partial_0 h_{00})d\sigma\right] \cdot \frac{1 - v_{*\rm rad}}{\sqrt{1 - v_*^2}}$$

special-relativistic formula → general-relativistic Doppler shift formula

Determination of the stellar velocity by taking advantage both of the spectroscopic and of the astrometric data supplied by the Gaia observations

de Felice F Preti G Crosta M and Vecchiato A 2011 Astron. Astrophys. 528 A23+

3. maybe…just after Gaia

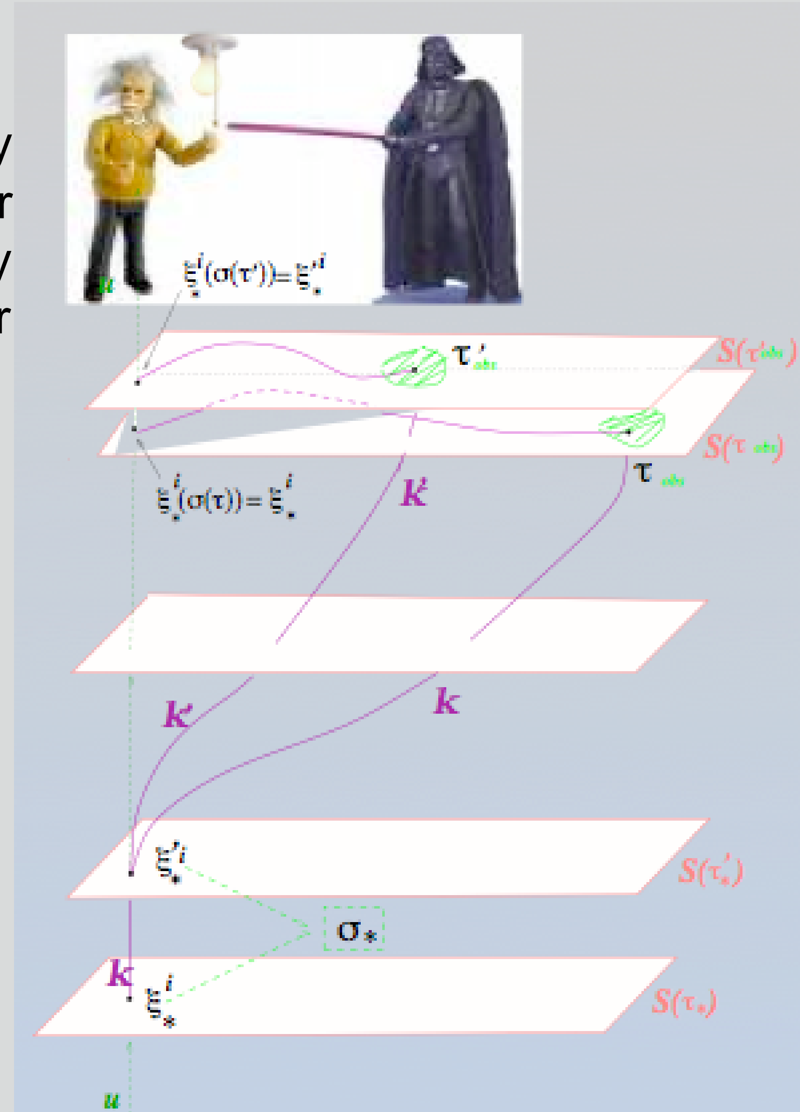
the local line-of-sight as a physical entity can be utilized in the “inverse parameter problem” approach in order to statistically determine the metric (maybe outside the Solar System?)

$$\boxed{\delta g^{\alpha\beta} = -\frac{1/2}{g_{\mu\nu}\, u^{\mu}\, \ell^{\nu}}\, \ell^{\alpha}\, \ell^{\beta}\, \delta\hat{\sigma} \quad .}$$

$$g_{\alpha\beta} \text{ to } g_{\alpha\beta} + \delta g_{\alpha\beta}$$

“The measurement act is not simply an ‘observed value’ (of a set of ‘observed values’) but an ‘state of information’ acquired on some observable parameter”

Inverse Problem Theory and Methods for Model Parameter Estimation by Albert Tarantola, SIAM 2005



Conclusion

✓In tracing back light ray we need to keep consistency, at any level of approximations, with General Relativity

✓The comparison between different light modeling approaches is EXTREMELY important since Gaia will “change” our scientific vision and we are implementing NEW methods using REAL data

✓By comparing different formulations of a null geodesic we have the opportunity the exploit the advantages of the different methods and improve on OUR understanding of light propagation

…and confusion

"Measurement with respect to the curvature makes no sense.
I even can not imagine what it can be"[it is General Relativity!]

"I do not understand AT ALL how the boundary conditions can be fixed by the measurements" [astrometric observable and Cauchy problem]

"It looks like more mathematical, specialized in differential geometry, than physical" [GR is based on differential geometry!]

"This technique [RAMOD] is quite controversial in my opinion as it does not match with the classic astrometric approach" [RAMOD matches the known approaches, but, most of all, we are doing RELATIVISTIC astrometry in the Gaia era!!]

==> ..“concordance general covariant model“:

suitable framework where any desired advancement in the light tracing problem and its subsequent detection as physical measurement can be contemplated

It is better to be flat....

Thank you
for your attention!