Testing a non-minimal coupling between matter and curvature — and beyond

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Outline

Motivation

1. Higher-order $f(R)$ curvature terms arise from string theory
2. Explore non-minimal couplings
   - Breaking the Equivalence Principle
3. Study analogy with multi-scalar-tensor theories
4. Study "dark gravity" $\neq$ dark matter/energy

Action functional

$$S = \int \left[\frac{1}{2} f_1(R) + [1 + f_2(R)] \mathcal{L}_m \right] \sqrt{-g} d^4 x \tag{1}$$

- GR: $f_1(R) = 2\kappa R$, $f_2(R) = 0$, $\kappa = c^4/16\pi G$
- Variation with respect to $g_{\mu\nu}$:

Modified Einstein field equations

$$\left(F_1 + 2F_2 \mathcal{L}_m\right) R_{\mu\nu} - \frac{1}{2} f_1 g_{\mu\nu} = \Delta_{\mu\nu} \left(F_1 + 2F_2 \mathcal{L}_m\right) + \left(1 + f_2\right) T_{\mu\nu} \tag{2}$$

O. Bertolami, C. Boehmer, T. Harko and F. Lobo (2007)

- $\Delta_{\mu\nu} = \nabla_\mu \nabla_\nu - g_{\mu\nu} \Box$, $F_i(R) \equiv f_i'(R)$
- Energy-momentum tensor: $T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta\left(\sqrt{-g}\mathcal{L}_m\right)}{\delta g^{\mu\nu}}$

Trace of Einstein field eqs.

$$\left(F_1 + 2F_2\mathcal{L}_m\right)R - 2f_1 = -3\Box\left(F_1 + 2F_2\mathcal{L}_m\right) + \left(1 + f_2\right)T \quad (3)$$

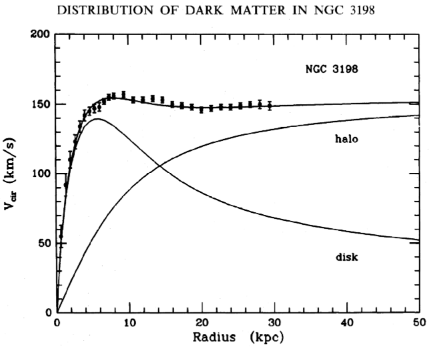
- Differential, not algebraic equation
- Bianchi identities, $\nabla^\mu G_{\mu\nu} = 0$ imply

Non-(covariant) conservation law

$$\nabla^\mu T_{\mu\nu} = \frac{F_2}{1+f_2}\left(g_{\mu\nu}\mathcal{L}_m - T_{\mu\nu}\right)\nabla^\mu R \quad (4)$$

  - Analogy with scalar fields for non-trivial $f_1(R), f_2(R)$
  - Energy exchange matter $\leftrightarrow$ scalar fields

Motivation



- Galactic rotation puzzle
  - Missing matter to account rotation velocity of stars
  - $\rightarrow$ Dark matter!
  - Or "Dark gravity"?
    - Maybe a non-minimal coupling of matter with geometry?

Procedure

- Typically
  - Solve modified Einstein eqs.
  - Obtain additional gravitational potential
  - Fit model parameters
  - Usually, no relation with “standard” DM models

S. Capozziello, V. F. Cardone, and A. Troisi (2007)

- Our approach
  - Write modified Einstein eqs.
  - Read “dark matter” density profile
  - Compare with models of DM $\rightarrow$ read parameter(s)
  - Fit remaining model parameters
  - Advantage: possible to mimic known DM models!

O. Bertolami and J. Páramos (2010)

Mechanism

- For simplicity, assume power-law

$$f_2(R) = \left(\frac{R}{R_n}\right)^n$$

- Also, trivial curvature term $f_1(R) = 2\kappa R$
- Flattening of galaxy rotation curves at large distances
  - low density $\rightarrow$ low $R \rightarrow n < 0$
- Visible matter content: Dust
  - Perfect fluid with $p = 0$
  - $T_{\mu\nu} = \rho U_{\mu\nu} U_\nu$
  - $\mathcal{L} = -\rho$

O. Bertolami, F. S. N. Lobo and J. Páramos (2008)

  - Assume known density profile (Hernquist)

$$\rho(r) = \frac{M}{2\pi}\frac{a}{r}\frac{1}{(r+a)^3} \quad (5)$$

Trace of modified Einstein eqs.

$$R = \frac{1}{2\kappa}\left[1 + (1-2n)\left(\frac{R}{R_n}\right)^n\right]\rho - \frac{3n}{\kappa}\Box\left[\left(\frac{R}{R_n}\right)^n \frac{\rho}{R}\right] \qquad (6)$$

Neglect linear ρ term → "static" solution

$$R = R_n\left[(1-2n)\frac{\rho}{\rho_0}\right]^{1/(1-n)} \qquad (7)$$

- Interpretation
  - At large distances, $R \propto \rho_{dm} \propto \rho^{1/(1-n)}$!
  - Tully-Fisher law: $M \sim M_{dm} \propto v^{2(1-n)}$!
- Substitute into modified Einstein eqs. → mimicked DM...
  - is dragged by visible matter (same four-velocity U^μ)
  - has non-vanishing pressure $p_{dm} = \frac{n}{1-4n} 2\kappa R$
  - Since $n < 0$, EOS parameter $\omega = n/(1-n) < 0$
    - Hint at unification with dark energy?
    - Dark matter matches cosmological background $\sim 100\ kpc$!

Stability

Well behaved model:

- No instabilities in the Newtonian regime
- Follows all the energy conditions
  - Weak (WEC): $\rho \geq 0 \quad , \quad \rho + p \geq 0$
  - Null (NEC): $\rho + p \geq 0$
  - Strong (SEC): $\rho + p \geq 0 \quad , \quad \rho + 3p \geq 0$
  - Dominant (DEC): $\rho \geq |p|$

O. Bertolami and M. Sequeira (2009)

- Conserves energy:

$$\nabla_\mu T^{\mu\nu} \approx 0$$

Fitting known dark matter profiles

- Isothermal sphere (IS) $\rho_{dm} \propto 1/r^2$
- Cusped density profiles:

$$\rho = \frac{\rho_{cp}}{\left(\frac{r}{a}\right)^{\gamma}\left(1+\frac{r}{a}\right)^{m-\gamma}} \tag{8}$$

  - Navarro-Frenk-White (NFW): $\gamma = 1$, $m = 3$
  - Hernquist: $\gamma = 1$, $m = 4$

J. F. Navarro, C. S. Frenk, and S. D. M. White (1995); L. Hernquist (1990)

- For large distances
  - DM (NFW): $\rho_{dm} \propto r^{-3}$, (IS) $\rho_{dm} \propto r^{-2}$
  - Visible (Hernquist): $\rho \propto r^{-4}$
- $\rho_{dm} \propto \rho^{1/(1-n)} \rightarrow \quad n_{NFW} = -1/3 \quad , \quad n_{IS} = -1$
- Tully-Fisher law: : $M \sim M_{dm} \propto v^{2(1-n)} = v^{8/3} \quad \wedge \quad v^4$

- Sample of seven galaxies
  - Type $E0$: approximately spherical

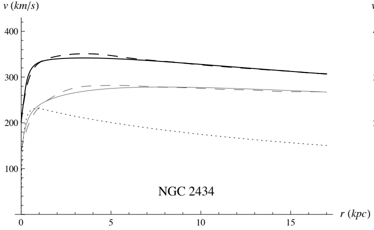
A. Kronawitter, R. P. Saglia, O. Gerhard, and R. Bender (2000, 2001)

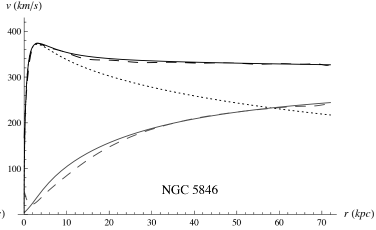
S. M. Faber, G. Wegner, D. Burstein, R. L. Davies (1989)

H. W. Rix *et al.* (1997)

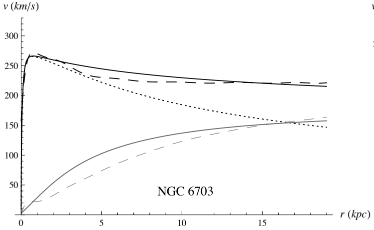
  - Composite model: both *NFW* and *IS* dark matter profiles
  - Objective: fit lengthscales $r_1 = R_1^{-1/2}$ and $r_3 = R_3^{-1/2}$
    - Variability $\rightarrow$ individual fits
    - Order of magnitude analysis

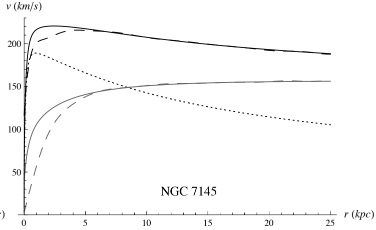
Results





Rotation curve (observed: dash, mimicked: full) = visible (dot) + DM (observed: dash grey, mimicked: full grey)





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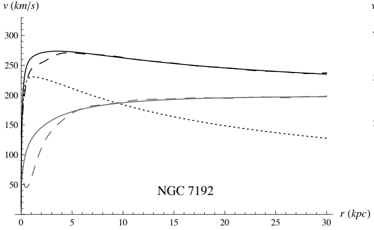

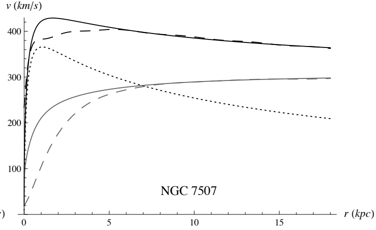

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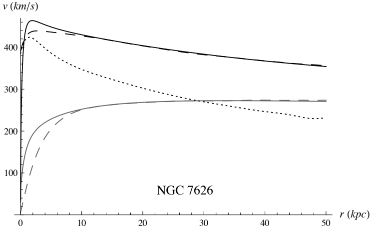


Results

Table

NGC	r_1	r_3	$r_{\infty\ 1}$	$r_{\infty\ 3}$
2434	∞	0.9	0	33.1
5846	37	∞	138	0
6703	22	∞	61.2	0
7145	22.3	47.3	60.9	14.2
7192	14.8	24	86.0	18.3
7507	4.9	2.9	178	31.1
7626	28	9.6	124	42.5

- Units
  - r_1: *Gpc*
  - r_3: 10^5 *Gpc*
  - Background matching distances $r_{\infty\ n}$: *kpc*

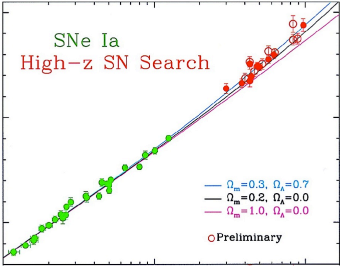
Universality

- Not very good...
  - Average $\bar{r}_1 = 21.5\ Gpc$, s.d. $\sigma_1 = 10.0\ Gpc$
  - Average $\bar{r}_3 = 1.69 \times 10^6\ Gpc$, s.d. $\sigma_3 = 1.72 \times 10^6\ Gpc$
- Reasons:
  - Deviation from sphericity
  - Relevance of $f_1(R)$ term
  - Too simplistic $f_2(R)$ power-law (Laurent series...)
  - Bad choices?
    - Visible matter density ρ
    - NFW or IS ρ_{dm} (different n)
    - Reconstructed density profiles
    - $\mathcal{L}$

Motivation

- Accelerated expansion of the Universe
  - Missing energy with negative pressure $\rightarrow$ Dark energy!
  - Or "Dark gravity"?
    - Multi-scalar-tensor analogy $\rightarrow$ two-field quintessence

M. C. Bento, O. Bertolami and N. M. C. Santos (2002)



- Similar to galactic rotation puzzle:
  - spherically symmetric $g_{\mu\nu}(r) \leftrightarrow$ FRW $g_{\mu\nu}(t)$
  - GR at small distances $\leftrightarrow$ earlier times
  - Dark Universe at large distances $\leftrightarrow$ late times $\rightarrow n < 0$

O. Bertolami, P. Frazão and J. Páramos (2010)

Mechanism

- Flat $k = 0$, isotropic and homogeneous Universe

FRW metric

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{\sqrt{1-kr^2}} + d\Omega^2\right) \tag{9}$$

- Dust filled Universe, $T^{\mu\nu} = \rho U^\mu U^\nu = (\rho, 0, 0, 0)$
- Constant deceleration parameter $\rightarrow a(t) = a_0(t/t_0)^\beta$
  - Expanding Universe $\rightarrow \beta > 0$, accelerating $\beta > 1$

Quantities

$$\begin{aligned} H &\equiv \frac{\dot{a}}{a} = \frac{\beta}{t} \\ R &\equiv 6\left[\left(\frac{\dot{a}}{a}\right)^2 + \frac{\ddot{a}}{a}\right] = \frac{6\beta}{t^2}(2\beta - 1) \\ q &\equiv -\frac{\ddot{a}a}{\dot{a}^2} = \frac{1}{\beta} - 1 \end{aligned} \tag{10}$$

Covariant energy conservation

- Energy is conserved!

Non-(covariant) conservation law, $\nu = 0$

$$\begin{aligned} \nabla^\mu T_{\mu 0} &= \frac{F_2}{1+f_2}\left(g_{\mu 0}\mathcal{L}_m - T_{\mu 0}\right)\nabla^\mu R = \\ & \frac{F_2}{1+f_2}\left(\mathcal{L}_m + T_{00}\right)\dot{R} = 0 \rightarrow \\ \dot{\rho} + 3H\rho &= 0 \end{aligned} \quad (11)$$

Matter density

$$\rho(t) = \rho_0 \left(\frac{a_0}{a(t)}\right)^3 \quad (12)$$

Modified dynamics

Modified Friedmann and Raychaudhuri Eqs.

$$H^2 + \frac{k}{a^2} = \frac{1}{6\kappa}(\rho_m + \rho_c) \tag{13}$$

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{1}{12\kappa}\left[\rho_m + \rho_c + 3(p_m + p_c)\right]$$

- Curvature pressure p_c, density ρ_c depend on $f_1(R), f_2(R)$

Modified dynamics

- Use trivial $f_1(R) = 2\kappa R$ and power-law $f_2(R) = (R/R_n)^n$

Curvature density and pressure

$$\rho_c \approx -6\rho_0\beta \frac{1-2\beta+n(5\beta+2n-3)}{\left(\frac{t}{t_0}\right)^{3\beta}\left(\frac{t}{t_n}\right)^{2n}[6\beta(2\beta-1)]^{1-n}} \quad (14)$$

$$p_c \approx -2\rho_0 n \frac{2+4n^2-\beta(2+3\beta)+n(8\beta-6)}{\left(\frac{t}{t_0}\right)^{3\beta}\left(\frac{t}{t_n}\right)^{2n}[6\beta(2\beta-1)]^{1-n}}$$

- $t_n \equiv R_n^{-1/2}$ marks onset of accelerated phase
- Solve Friedmann Eq. $\rightarrow \beta(n) = 2(1-n)/3$

- Deceleration parameter

$$n = 1 - \frac{3}{2(1+q)} \quad , \qquad q = -1 + \frac{3}{2(1-n)} \tag{15}$$

- EOS parameter $p_c = \omega \rho_c$

$$\omega = \frac{n}{1-n} \tag{16}$$

  - Same as in DM scenario
  - $n \rightarrow \infty$, $\omega \rightarrow -1$ and $q \rightarrow -1$
    - Cosmological constant Λ
    - Unobtainable with $f_2(R)$: matter term is not constant!
- Previous R_1 and R_3 for $n = -1$ (IS) and $n = -1/3$ (NFW)
  - $r_1, r_3 \ll r_H$ Hubble radius
  - No cosmological role

Fitting $q(z)$ profiles

- Numerically solve Friedmann Eq. for fixed n
- Fit t_n to available $q(z)$ curve

Y. G. Gong and A. Wang (2007)

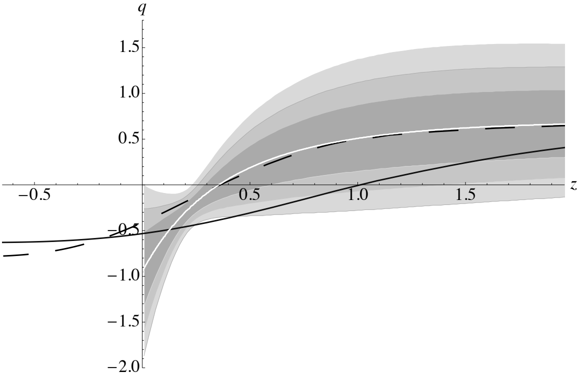


Figure: $q(z)$ for $n = -4$, $t_2 = t_0/4$ (full) and $n = -10$, $t_2 = t_0/2$ (dashed); 1σ, 2σ and 3σ regions shaded, best fit (white)

Motivation

Inflation: early phase of fast expansion of the Universe

- Driven by scalar field slow-rolling down suitable potential
  - Or non-trivial curvature term $f_1(R)$
  - Formal equivalence with scalar tensor theory
  - Starobinsky inflation: $f_1(R) = 2\kappa R + R^2/6M^2$

A. A. Starobinsky (1980)

- Problem: at the end of inflation, Universe is too cold!
- "Old reheating": scalar field oscillates around minimum
  - Decays into particles and reheats the Universe
  - Problem: fine tuning of parameters, overproduction
- Solution: preheating

Dolgov and Kirilova (1990) , Traschen and Brandenberger (1990)

Kofman *et al.* (1994) ; Shtanov *et al.* (1995)

Preheating

Quantum field χ with mass m coupled to scalar curvature:

Lagrangean density

$$\mathcal{L}_\chi = -\frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} m^2 \chi^2 - \frac{1}{2} \xi R \chi^2 \tag{17}$$

- Spacetime dependent effective mass $m_{eff}^2 = m^2 + \xi R$

Fourier decomposition

$$\chi(t, \mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left[a_k \chi_k(t) e^{-i\mathbf{k}\cdot\mathbf{x}} + a_k^\dagger \chi_k^*(t) e^{i\mathbf{k}\cdot\mathbf{x}} \right] \tag{18}$$

- Particle creation/annihilation with mass m, momentum $\mathbf{k}$

Parametric resonance

During oscillatory phase

$$\ddot{\chi}_k + \left(\frac{k^2}{a^2} + m^2 + \xi R - \frac{9}{4} H^2 - \frac{3}{2} \dot{H} \right) \chi_k = 0 \rightarrow \tag{19}$$

$$\ddot{\chi}_k + \left(\frac{k^2}{a^2} + m^2 - \frac{4 M \xi}{t - t_o} \sin \left[M (t - t_o) \right] \right) \chi_k \simeq 0$$

- Varying frequency $\rightarrow$ parametric resonance $\rightarrow$ explosive particle production

Equivalent to Mathieu equation

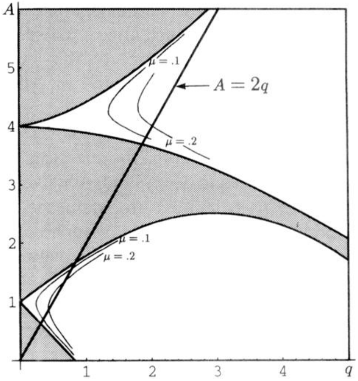
$$\frac{d^2 \chi_k}{dz^2} + \left[A_k - 2q \cos(2z) \right] \chi_k \simeq 0 \tag{20}$$

Parametric resonance

Mathieu equation

$$\frac{d^2\chi_k}{dz^2} + [A_k - 2q\cos(2z)]\,\chi_k \simeq 0 \qquad (21)$$

- Flouquet chart shows resonance bands



Linear non-minimal coupling

Generalize coupling with curvature

$$\mathcal{L}_\chi = -\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}m^2\chi^2 - \frac{1}{2}\xi R\chi^2 \rightarrow \tag{22}$$

$$\mathcal{L}_\chi = -\left(1 + 2\xi\frac{R}{M^2}\right)\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + \frac{1}{2}m^2\chi^2\right)$$

- $f_2(R)$ couples with all matter contributions
  - radiation, ultra-relativistic...
- Subdominant during slow-roll inflation: $1 < \xi < 10^4$

Linear non-minimal coupling

During oscillatory phase

$$\ddot{\chi}_k + \left(\frac{k^2}{a^2} + m^2 + \xi R - \frac{9}{4} H^2 - \frac{3}{2} \dot{H} \right) \chi_k = 0 \rightarrow (23)$$

$$\ddot{\chi}_k + \left(3H + 2\xi \frac{\dot{R}}{M^2} \right) \dot{\chi}_k + \left(\frac{k^2}{a^2} + m^2 \right) \chi_k = 0$$

- $X_k \equiv a^{3/2} f_2^{1/2} \chi_k$: friction term transforms into mass term
- Also leads to parametric resonance!

O. Bertolami, P. Frazão and J. Páramos (2011)

Linear coupling

- Probe $f_2(R) = R/R_1$
  - Where is curvature is high, but not too much? The Sun!
  - Perturbative treatment
  - Observable: central temperature

O. Bertolami and J. Páramos (2008)

- Birkhoff theorem: spherically symmetric, static $g_{\mu\nu}$

Tolman-Oppenheimer-Volkoff equation

$$p' + G(\rho + p)\frac{m_e + 4\pi p r^3}{r^2 - 2Gm_e r} = a\left[\left(\left[\frac{5}{8}p'' - 4\pi G p\rho\right] r - \frac{p'}{4}\right)\rho + p\rho'\right] . \tag{24}$$

- $a \equiv 16\pi G/R_1$, $[a] = M^{-4}$
- Suitably defined effective mass m_e
- 2^{nd}, not 1^{st} order ODE

- Newtonian limit

Modified hydrostatic equilibrium

$$p' + \frac{G m_e \rho}{r^2} = a \left[\left(\left[\frac{5}{8} p'' - 4\pi G p \rho \right] r - \frac{p'}{4} \right) \rho + p \rho' \right] . \quad (25)$$

Polytropic equation of state

$$p = K \rho_B^{(n+1)/n} \quad (26)$$

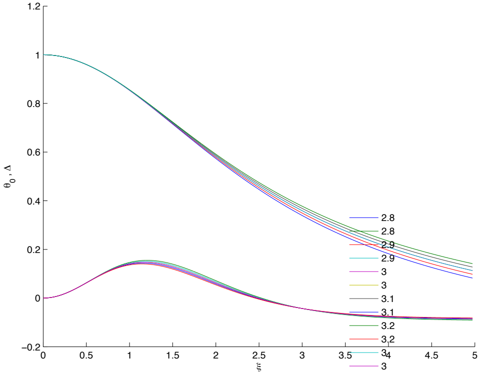
- n polytropic index
  - $n = -1$: isobaric
  - $n = 0$: isometric
  - $n \rightarrow +\infty$: isothermal
  - $n = 1/(\gamma - 1)$: adiabatic ($\gamma \equiv c_p / c_V$)
  - $n = 1.5$: giant planets, white/brown dwarfs, red giants
  - $n = 5$: boundless system
  - $n = 3$: first solar model (A. Eddington)
- K polytropic constant

- $\rho = \rho_c \theta^n(\xi) \quad , \quad p = p_c \theta^{n+1}(\xi)$
- $\xi = r/R_n \quad , \quad R_n^2 \equiv (n+1)p_c/4\pi G \rho_c^2$
- $\rho_c = 1.622 \times 10^5 \; kg/m^3 \quad , \quad p_c = 2.48 \times 10^{16} \; Pa$

Modified Lane-Emden equation

$$\frac{1}{\xi^2}\left[\xi^2\theta'\left(1 + \frac{3n-1}{4(n+1)} + A_c\theta^n\left[\left\{\frac{5}{8}\left(\theta'' + n\frac{\theta'^2}{\theta}\right\} - N_c\theta^{n+1}\right]\frac{\xi}{\theta'}\right]\right)\right]' = -\theta^n\left[1 + A_c\left(\frac{3}{8}\left[\theta'' + n\frac{\theta'^2}{\theta}\right] + \frac{\theta'}{4\xi} - \frac{\theta^n}{2}\right)\right] \quad (27)$$

- 3^{rd} order ODE! Linearize...

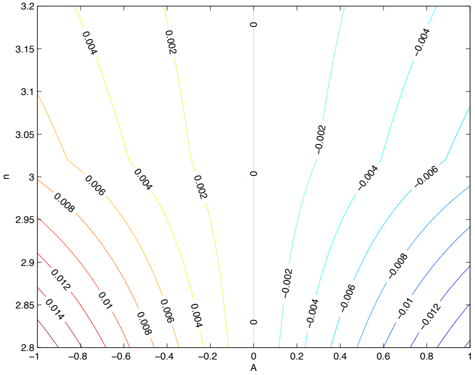


Unperturbed solution $\theta_0(\xi)$ and perturbation

Central temperature bound

$$\left| \frac{T_c}{T_{c0}} - 1 \right| < 6\% \rightarrow |R_1| > \left(1.53 \times 10^{-17}\ eV\right)^2 \sim 10^{-90} M_P^2$$

- Not very interesting...



Relative central temperature T_c deviation

Central temperature bound

$$\left| \frac{T_c}{T_{c0}} - 1 \right| < 6\% \rightarrow |R_1| > \left(1.53 \times 10^{-17}\, eV\right)^2 \sim 10^{-90} M_P^2$$

- Not very interesting...

Parameterized Post-Newtonian formalism

- Expand metric as sum of all potentials down to $O(c^{-4})$
- 10 PPN parameters related to fundamental properties
  - β: nonlinearity in the superposition law for gravity
  - γ: space-curvature produced by unit rest mass
  - General Relativity: $\beta = \gamma = 1$, other parameters vanish
- Momentum conservation, no preferred-frame/location $\rightarrow$

PPN metric

$$g_{00} = -1 + 2U - 2\beta U^2 \quad , \quad g_{ij} = (1 + 2\gamma U)\delta_{ij}$$

C. Will (2006)

Equivalence with a multiscalar-tensor theory

- $f(R)$ theories $\leftrightarrow$ Jordan-Brans-Dicke theory with $\omega = 0$

P. Teyssandier and P. Tourranc (1983), H. Schmidt (1990), D. Wands (1994)

$f(R)$ action

$$S = \int (f(R) + \mathcal{L}) \sqrt{-g} \, d^4x \tag{28}$$

JBD with $\omega = 0$ action

$$S = \int (F(\phi)R - V(\phi) + \mathcal{L}) \sqrt{-g} \, d^4x \tag{29}$$

- $F(\phi) = f'(\phi) \quad , \quad V(\phi) = \phi F(\phi) - f(\phi)$
- Varying action (29) w.r.t. ϕ yields $\phi = R$

Non-minimal coupling: two scalar fields required

$$\varphi^1 \propto \log[F_1(R) + F_2(R)\mathcal{L}] \quad , \quad \varphi^2 = R \qquad (30)$$

- Conformal transformation from Jordan frame ($F(\phi)R$ term) to Einstein frame (R uncoupled from ϕ):

$$g_{\mu\nu} \rightarrow g^*_{\mu\nu} = A^{-2}(\varphi_1) g_{\mu\nu} \quad , \quad A(\varphi_1) = \exp\left(-\frac{\varphi_1}{\sqrt{3}}\right)$$

T. Damour and G. Esposito-Farese (1992)

Multi-scalar-tensor model

$$S = \int \left[R^* - 2g^{*\mu\nu}\sigma_{ij}\varphi^i_{,\mu}\varphi^j_{,\nu} - 4U + f_2(\varphi^2)\mathcal{L}^*\right]\sqrt{-g}d^4x$$

$$\mathcal{L}^* = A^4(\varphi_1)\mathcal{L} \quad , \quad U = \frac{1}{4}A^2(\varphi_1)\left[\varphi^2 - A^2(\varphi_1)f_1(\varphi^2)\right]$$

- Kinetic term $g^{*\mu\nu}\sigma_{ij}\varphi^i_{,\mu}\varphi^j_{,\nu}$

Field metric

$$\sigma_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad (31)$$

- Only φ^1 is a dynamical field
- Solar System: perturbative coupling $\rightarrow F_2/f_2 \sim 0$

Non-conservation law

$$\nabla^\mu T_{\mu\nu} = -\frac{\sqrt{3}}{3} T\varphi^1_{,\nu} + \frac{F_2}{f_2}\left(g_{\mu\nu}\mathcal{L} - T_{\mu\nu}\right)\nabla^\mu\varphi^2 \simeq \alpha_i T\varphi^i_{,\nu}$$

$$\text{with} \quad \alpha_i \equiv \frac{\partial \log A}{\partial \varphi^i} \quad \rightarrow \quad \alpha_1 = -\frac{1}{\sqrt{3}} \quad , \quad \alpha_2 = 0$$

Parameterized Post-Newtonian formalism

- Use field metric to raise/lower latin indices: $\alpha^i \equiv \sigma^{ij}\alpha_j$

$$\alpha^2 \equiv \alpha_i \alpha^i = \sigma^{ij}\alpha^i\alpha_j = 0 \quad , \quad \alpha_{i,j} \equiv \frac{\partial \alpha_j}{\partial \varphi^i} = 0$$

Since $\alpha_2 = 0$

$$\beta = 1 + \frac{1}{2}\left[\frac{\alpha^i\alpha^j\alpha_{j,i}}{(1+\alpha^2)^2}\right]_\infty = 1$$

$$\gamma = 1 - 2\left[\frac{\alpha^2}{1+\alpha^2}\right]_\infty = 1$$

- Same as in GR!
- If perturbative effects are considered, $\beta \sim 1$ and $\gamma \sim 1$
  - Choose $f_2(R)$, solve Einstein field eqs., expand metric

Novel ways to break the Equivalence Principle

Non-conservation law

$$\nabla^\mu T_{\mu\nu} = \frac{F_2}{f_2}\left(g_{\mu\nu}\mathcal{L} - T_{\mu\nu}\right)\nabla^\mu R$$

- Where to look?
  - Very high curvature and density (magnitude and gradient)
  - Possible couplings with other sectors, *e.g.* electromagnetic
- Quasars! Accretion disk, jet emission
  - Toy models
  - Simulation

MOdified Newtonian Dynamics

Modification of Poisson equation

$$\nabla \cdot \left[\mu \left(\frac{\nabla \phi}{a_0} \right) \nabla \phi \right] = 4\pi G \rho \qquad (32)$$

$$a_0 \approx 10^{-10}\, ms^{-2} \qquad , \qquad \mu(x) \approx \begin{cases} x & , \quad x \ll 1 \\ 1 & , \quad x \gg 1 \end{cases}$$

- Alternative to dark matter
- Solves puzzle of the flattening of galaxy rotation curves
- Yields Tully-Fisher law $L \propto v_\infty^4$
- Classical → Relativistic underlying theory: TeVeS

M. Milgrom (1983)

Tensor-Vector-Scalar theory

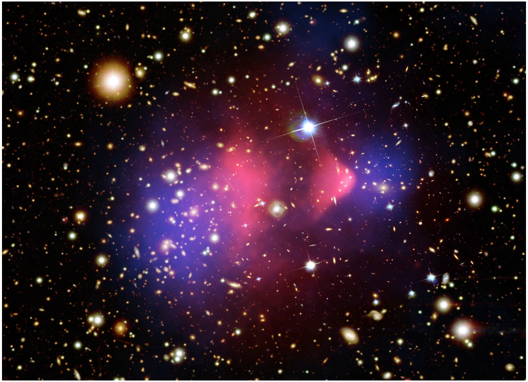
Action $S = S_G + S_V + S_S + S_M$

$$
\begin{aligned}
S_G &= \int R\sqrt{-g}\, d^4x \qquad (33)\\
S_V &= -\frac{\kappa}{2}\int \left[K U^{[\alpha,\mu]} U_{[\alpha,\mu]} - 2\lambda(U^\mu U_\nu + 1)\right] \sqrt{-g}\, d^4x \\
S_S &= -\frac{1}{2}\int \left[\sigma^2 h^{\alpha\beta} \phi_{,\alpha}\phi_{,\beta} + \frac{G}{2l^2}\sigma^4 F(kG\sigma^2)\right] \sqrt{-g} d^4x \\
S_M &= \int \mathcal{L}(\varphi_i, \tilde{g}_{\mu\nu})\sqrt{-\tilde{g}}\, d^4x
\end{aligned}
$$

- Three additional fields (one vector U^μ, two scalars σ, ϕ)
  - K, k and l are constants specific of the theory
  - Lagrange multiplier $\lambda \to U^\mu$ timelike
  - F is a free function
  - σ has no kinetic term
  - $h^{\alpha\beta} = g^{\alpha\beta} - U^\alpha U^\beta$
  - "physical metric" $\tilde{g} = g_{\alpha\beta} - 2U_\alpha U_\beta\, sinh(2\phi)$

Issues with MOND

- Bullet cluster

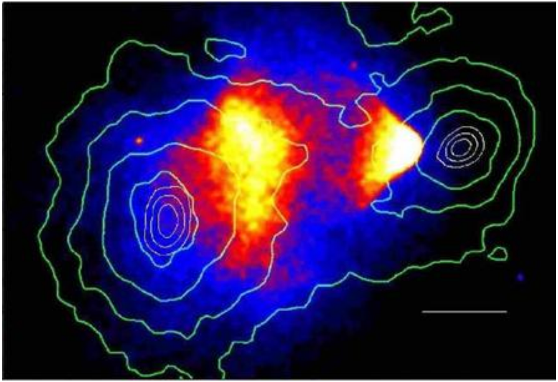


Bullet Cluster (false colors)

Angus *et al.* (2006)

Issues with MOND

- Bullet cluster



Bullet Cluster (with mass density contours)

Angus *et al.* (2006)

Issues with MOND

- Bullet cluster
  - Compatible only with heavy neutrinos $m_\nu \sim 2\ eV$ $(0.07\ eV < m_\nu < 2.2\ eV)$
  - Linear superposition of *ad-hoc* MOND potentials
- Early Universe: fluctuations of $\phi \to$ structure formation
  - Inconsistent with numerical findings

Pointecouteau (2006)

- PPN parameters $\beta = \gamma = 1$, as in General Relativity
  - Assumes $U^\mu = (U^0, 0, 0, 0)$ (allowed, but...)
  - If instead one assumes that U^μ is radial (more natural)

$$\beta = 1 + \frac{k}{8\pi} + \frac{K}{4} + \phi_c \left(3 + \frac{k}{\pi K} \pm \sqrt{\frac{2k}{\pi K} + 5} \right) \quad , \quad \gamma = 1$$

Giannios (2005)

Too complex and problematic!

Summary

Non-minimal coupling between matter and curvature

- Wide phenomenology, distinctive features
- Description of Dark Matter and Dark Energy!
- Elegant generalization of preheating
- Specific n for different regimes hints at Laurent expansion

$$f_2(R) = \sum_n \left(\frac{R}{R_n} \right)^n \tag{34}$$

- WIP: DM in clusters, Cosmological Constant...
- Clear signature of Equivalence Principle breaking $\rightarrow$ search in violent phenomena

Thank you!

Strong coupling between curvature and kitten

Choice of Lagrangian density: case of a perfect fluid

Non-(covariant) conservation law

$$\nabla^\mu T_{\mu\nu} = \frac{F_2}{1+f_2} \left(g_{\mu\nu} \mathcal{L}_m - T_{\mu\nu} \right) \nabla^\mu R \qquad (35)$$

- In GR, $\mathcal{L}_m$ serves to obtain $T_{\mu\nu}$ only
- If $f_2(R) \neq 0$, $\mathcal{L}_m$ appears in eqs. motion!

Perfect fluid

$$T_{\mu\nu} = (\rho + p) U_{\mu\nu} U_\nu + p g_{\mu\nu} \qquad (36)$$

$$p \equiv n \frac{\partial \rho}{n} - \rho$$

- U^μ: four-velocity
- n: particle number density
- $J^\mu = \sqrt{-g} n U^\mu$: flux vector of particle number density n

Action in GR

$$S_m = \int d^4x \left[-\sqrt{-g}\, \rho(n,s) + J^\mu \phi_\mu \right] \tag{37}$$

J. D. Brown (1993)

- ϕ_μ: contains thermodynamical potentials
  - particle number conservation
  - entropy exchange
  - definition of temperature
  - chemical free energy
- Equivalent Lagrangean densities:
  - Begin with $\mathcal{L}_0 = -\rho$
  - Substitute eqs. motion back into action Eq. (37)
  - Read "on-shell" $\mathcal{L}_i$:
    - $\mathcal{L}_1 = p$
    - $\mathcal{L}_2 = -na \quad , \quad a(n,T) = \rho(n)/n - sT$

- How to couple $f_2(R)$ to a perfect fluid?

Modified action

$$S_m = \int d^4x \left[-\sqrt{-g}\,[1+f_2(R)]\,\rho(n,s) + J^\mu \phi_\mu \right] \qquad (38)$$

Equivalent to on-shell Lagrangian?

$$S_m = \int d^4x \sqrt{-g}\,[1+f_2(R)]\,p \qquad (39)$$

- Yes, but...

Redefined thermodynamical quantities, *e.g.*

$$T = \frac{1}{n}\frac{\partial \rho}{\partial s}\bigg|_n = \frac{1}{1+f_2(R)}\theta_{,\mu}U^\mu \qquad (40)$$

O. Bertolami, F. S. N. Lobo and J. Páramos (2008)