# CLASSICAL MEASUREMENTS IN CURVED SPACE-TIMES

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# THE TASK OF THE BOOK

- A physical measurement is meaningful only if one identifies in a non ambiguous way who is the observer and what is being observed. The same observable can be the target of more than one observer so we need a suitable algorithm to compare their measurements. This is the task of the theory of measurement which we develop here in the framework of general relativity.

# THE MEASUREMENT PROTOCOL

(i) Identify the covariant equations which describe the phenomenon under investigation.

(ii) Identify the observer who makes the measurements.

(iii) Chose a frame adapted to that observer, allowing the space-time to be split into the observer's *space* and *time*.

(iv) Decide whether the intended measurement is local or non-local with respect to the background curvature.

(v) Identify the frame components of those quantities which are the observational targets.

(vi) Find a physical interpretation of the above components, following a suitable criterion such as a comparison with what is known from special relativity or from non-relativistic theories.

(vii) Verify the degree of residual ambiguity in the interpretation of the measurements and decide on a strategy to eliminate it.

$$\frac{D^2Y^\alpha}{d\tau_U^2} = -R^\alpha{}_{\beta\gamma\delta}U^\beta Y^\gamma U^\delta + Y^\sigma \nabla_\sigma a(U)^\alpha.$$

# WITH RESPECT TO A REFENCE FRAME

$$\ddot{Y}^{\hat{a}} + K_{(U,E)}{}^{\hat{a}}{}_{\hat{b}} Y^{\hat{b}} = 0,$$

# TIDAL MATRIX

$$\mathcal{K}_{(U,\,E)}{}^{\hat{a}}{}_{\hat{b}} = [T_{(\mathrm{fw},\,U,\,E)} - S(U) + \mathcal{E}(U)]^{\hat{a}}{}_{\hat{b}}.$$

# TWIST TENSOR

$$
\begin{aligned}
T_{(\mathrm{fw},\,U,\,E)}{}^{\hat{a}}{}_{\hat{b}} &= \dot{C}_{(\mathrm{fw})}{}^{\hat{a}}{}_{\hat{b}} - [C^2_{(\mathrm{fw})}]^{\hat{a}}{}_{\hat{b}} - 2C_{(\mathrm{fw})}{}^{\hat{a}}{}_{\hat{c}}\, k(U)^{\hat{c}}{}_{\hat{b}} \\
&= \delta^{\hat{a}}_{\hat{b}} \zeta^2_{(\mathrm{fw})} - \zeta^{\hat{a}}_{(\mathrm{fw})} \zeta_{(\mathrm{fw})\hat{b}} - \epsilon^{\hat{a}}{}_{bf}\, \dot{\zeta}^{\hat{f}}_{(\mathrm{fw})} \\
&\quad - 2\epsilon^{\hat{a}}{}_{\hat{f}\hat{c}}\, \zeta^{\hat{f}}_{(\mathrm{fw})} k(U)^{\hat{c}}{}_{\hat{b}},
\end{aligned}
$$

# FERMI-WALKER STRAIN TENSOR

$$S(U) = \nabla(U)a(U) + a(U) \otimes a(U),$$

# GRAVITOELECTRIC TENSOR

$$R(U,Y)U \equiv R^{\alpha}{}_{\beta\gamma\delta}U^{\beta}Y^{\gamma}U^{\delta} = \mathcal{E}(U)^{\alpha}{}_{\gamma}Y^{\gamma}$$

# THE RELATIVE ACCELERATION EQUATION IN TERMS OF TETRAD COMPONENTS

$$\frac{d^2Y^{\hat{r}}}{d\tau^2} = \left\{-\mathcal{E}(U)_{\hat{r}\hat{r}} + \partial_{\hat{r}}a(U)_{\hat{r}} + (a(U)_{\hat{r}})^2 + (\zeta_{(\mathrm{fw})_{\hat{\theta}}})^2\right\}Y^{\hat{r}},$$

$$\frac{d^2Y^{\hat{\theta}}}{d\tau^2} = \left\{-\mathcal{E}(U)_{\hat{\theta}\hat{\theta}} + \partial_{\hat{\theta}}a(U)_{\hat{\theta}} + \frac{1}{2}(E^{\theta}_{\hat{\theta}})^2(\partial_{\hat{r}}g_{\theta\theta})a(U)_{\hat{r}}\right\}Y^{\hat{\theta}},$$

$$\frac{d^2Y^{\hat{\phi}}}{d\tau^2} = \left\{-\mathcal{E}(U)_{\hat{\phi}\hat{\phi}} - \Gamma_{\hat{r}\hat{\phi}\hat{\phi}}a(U)_{\hat{r}} + (\zeta_{(\mathrm{fw})_{\hat{\theta}}})^2\right\}Y^{\hat{\phi}},$$

# THE CHOICE OF THE METRIC

$$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right) dt^2 - \frac{4aMr\sin^2\theta}{\Sigma} dt d\phi + \frac{A}{\Sigma}\sin^2\theta d\phi^2$$
$$+\frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2,$$

# THE CHOICE OF THE OBSERVER

$$U = \Gamma(\partial_t + \zeta\partial_\phi) ,$$

$$\Gamma = \left[1 - \frac{2Mr}{\Sigma}(1 - a\zeta \sin^2\theta)^2 - (r^2 + a^2)\zeta^2 \sin^2\theta\right]^{-1/2} .$$

# THE CHOICE OF THE REFERENCE FRAME

$$E_{\hat{t}} = U = \Gamma(\partial_t + \zeta\partial_\phi),$$

$$E_{\hat{r}} = (\Delta/\Sigma)^{1/2}\partial_r,$$

$$E_{\hat{\theta}} = \Sigma^{-1/2}\partial_\theta,$$

$$E_{\hat{\phi}} = \bar{\Gamma}(\partial_t + \bar{\zeta}\partial_\phi),$$

# THE STRESS TENSOR

$$a(U)^{\hat{r}} = \frac{\Gamma^2\sqrt{\Delta}}{\sqrt{\Sigma}}\left[\frac{M(r^2 - a^2\cos^2\theta)}{\Sigma^2}(1 - a\zeta\sin^2\theta)^2 - r\zeta^2\sin^2\theta\right],$$

$$a(U)^{\hat{\theta}} = -\frac{\Gamma^2\sin\theta\cos\theta}{\sqrt{\Sigma}}\left[\frac{2Mr}{\Sigma^2}[(r^2 + a^2)\zeta - a]^2 + \Delta\zeta^2\right].$$

# THE OUTCOME OF A MEASUREMENT

## THE DISCOVERY OF A NEW EFFECT:

## *THE RELATIVISTIC THRUST ANOMALY*

# A NEW GENERAL RELATIVISTIC EFFECT

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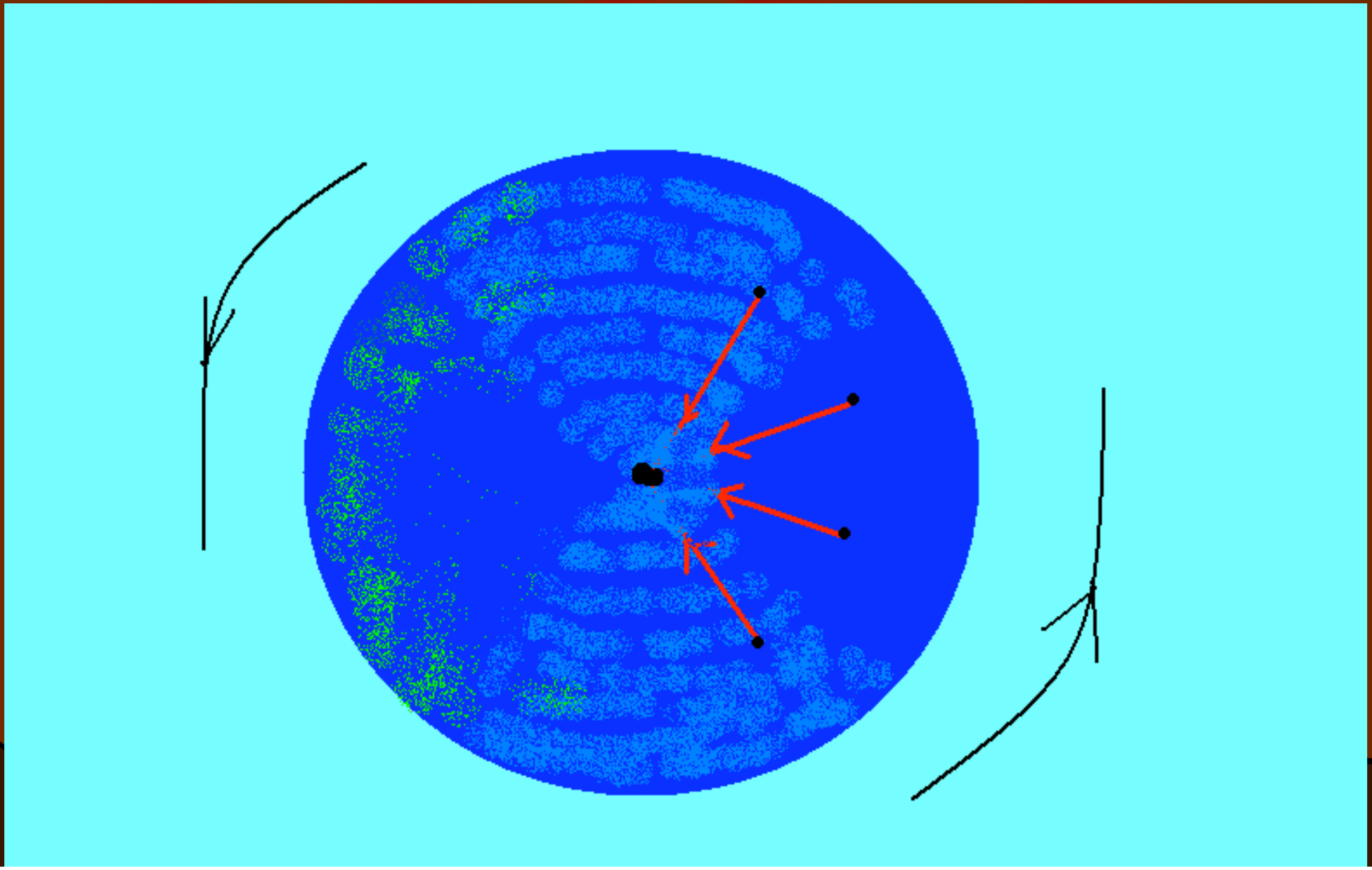
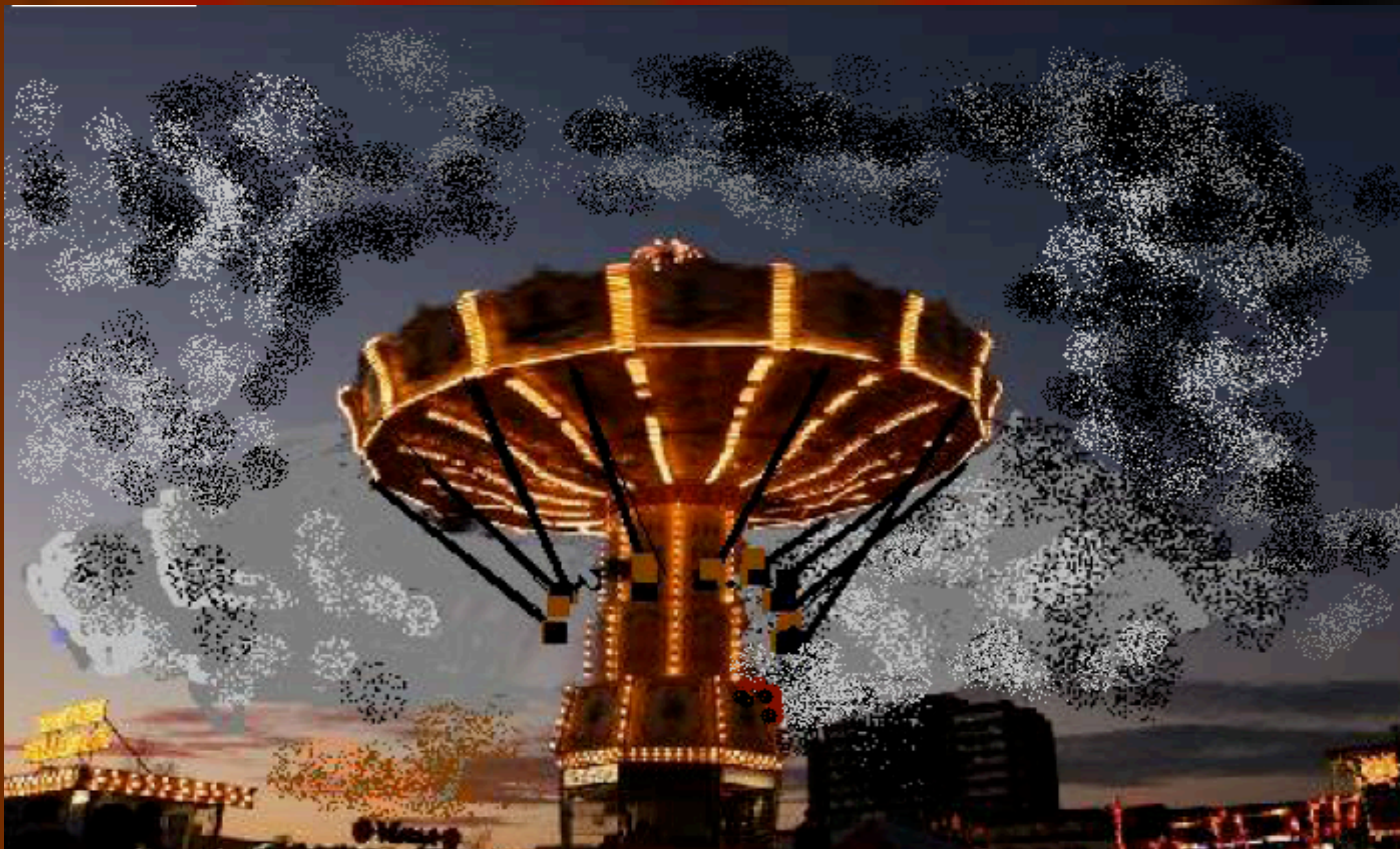
*Class. Quantum Grav. 11 (1994) 1283*

*Class. Quantum Grav. 12 (1995) 1119*

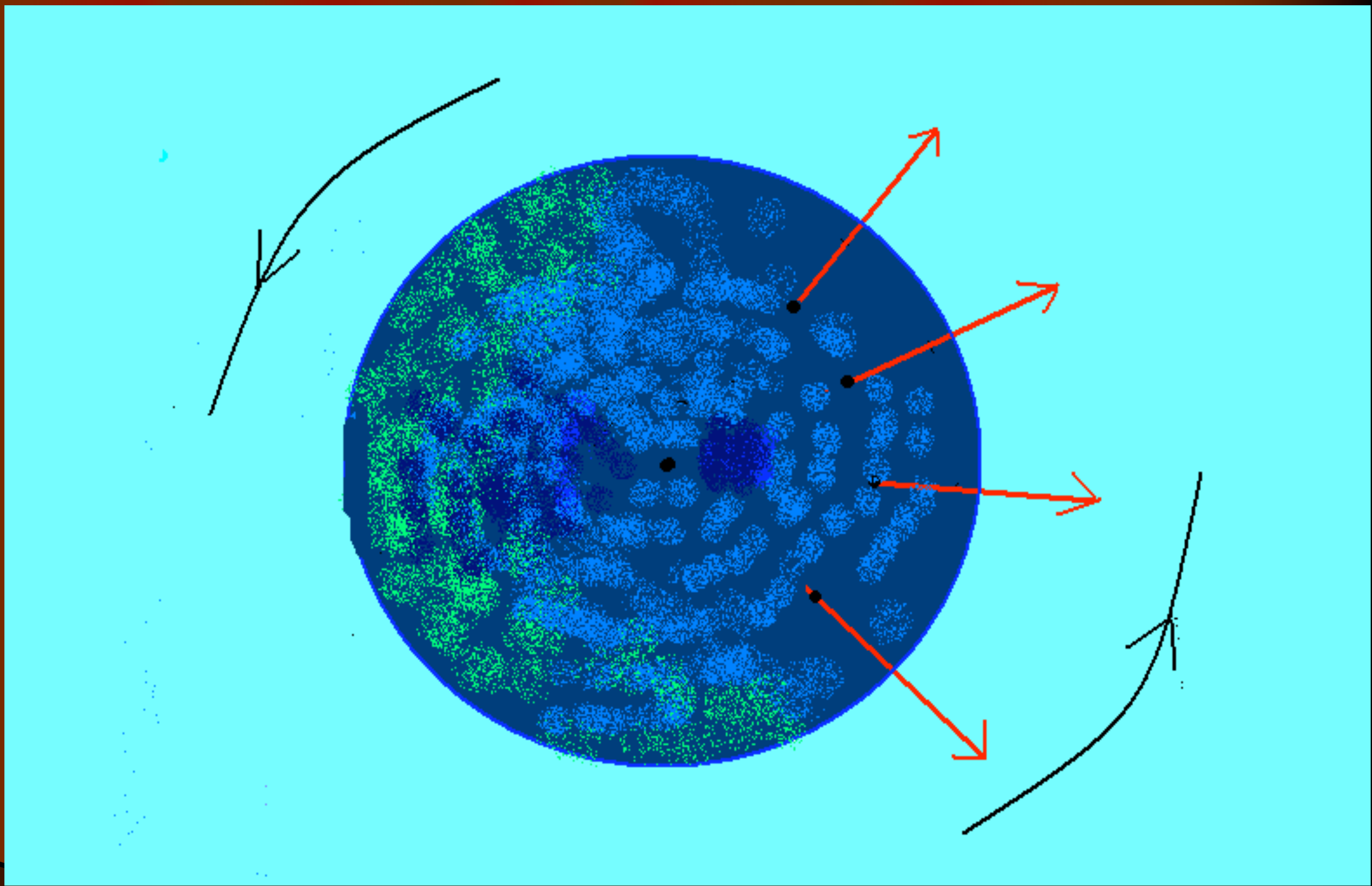
# CENTRIFUGAL FORCE

- Normal behaviour: the thrust acts inwardly





# The thrust acts outwardly
# *The thrust anomaly*



# IT MANIFESTS IN TWO SITUATIONS

| |
|---|
| In Schwarzschild |
| at $2m < r < 3 M$ |
| Abramowicz |
| Acta Phys. Pol. B5 (1974) 327 |

| |
|---|
| In Kerr |
| at $r_+ < r < 00$ |
| de Felice |
| 1994 |

# Circular orbits in the space-time of a rotating source

$$y = \frac{\zeta}{1 - a\zeta}$$

$$\Delta = r^2 + a^2 - 2\mathcal{M}r$$

$$y_{K\pm} \equiv \pm\sqrt{\frac{\mathcal{M}}{r^3}}$$

$$y_{c_\pm} = (a \pm \sqrt{\Delta})/r^2$$

# Specific thrust acting on a general non-geodesic equatorial spatially circular orbit in the space-time of a rotating source

$$a(U) = \frac{\Delta^{1/2}}{r^2} \frac{(y - y_{k_+})(y - y_{k_-})}{(y - y_{c+})(y - y_{c-})},$$

$$\theta = \pi/2 ,$$

# Gradient of the specific thrust with respect to the angular velocity *y*

$$\left.\frac{\partial a(U)}{\partial y}\right|_r = -\frac{2a\Delta^{1/2}}{r^4}\frac{(y-y_{0_+})(y-y_{0_-})}{(y-y_{c_+})^2(y-y_{c_-})^2}$$

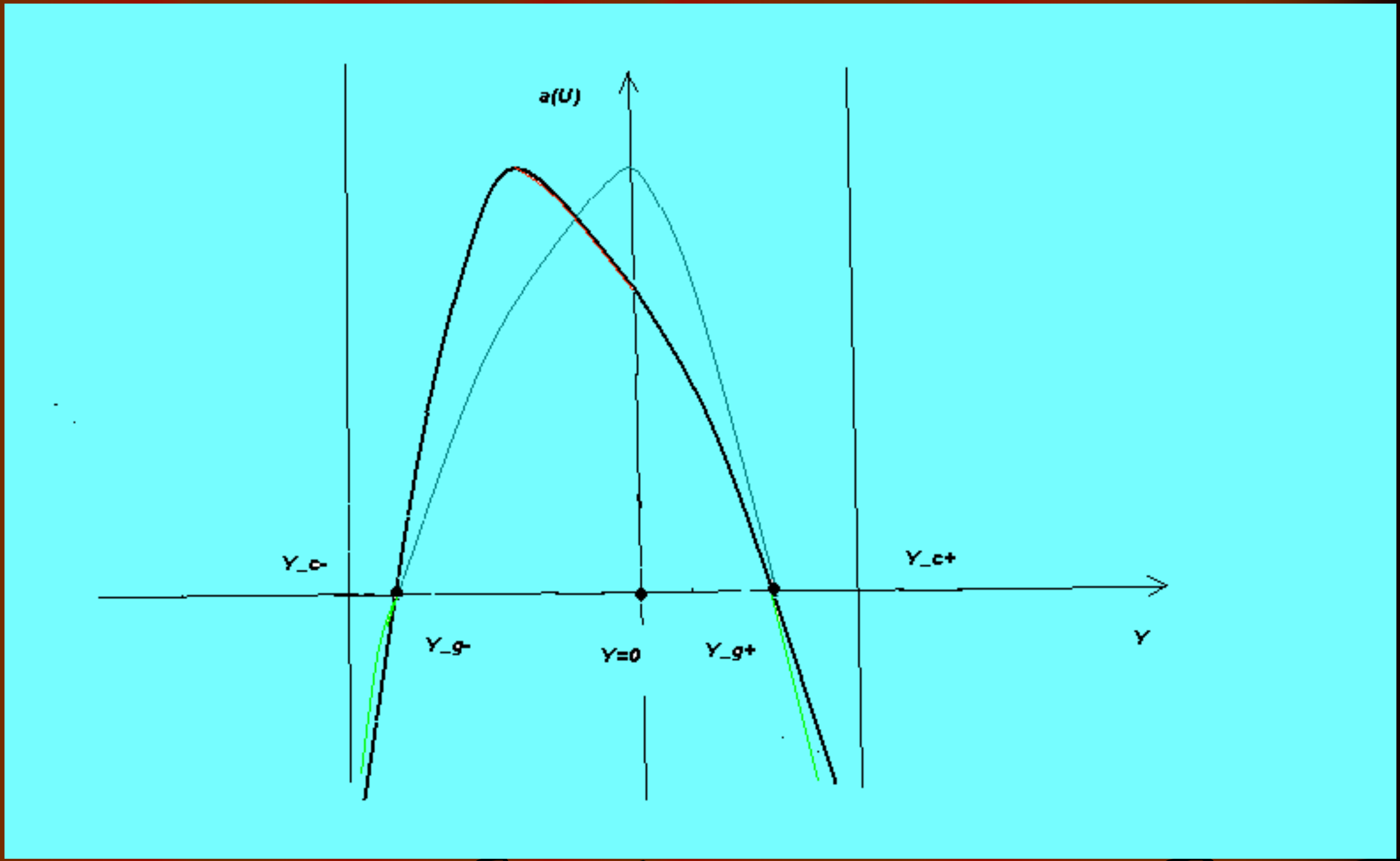
# THE MEASUREMENT OF THE THRUST

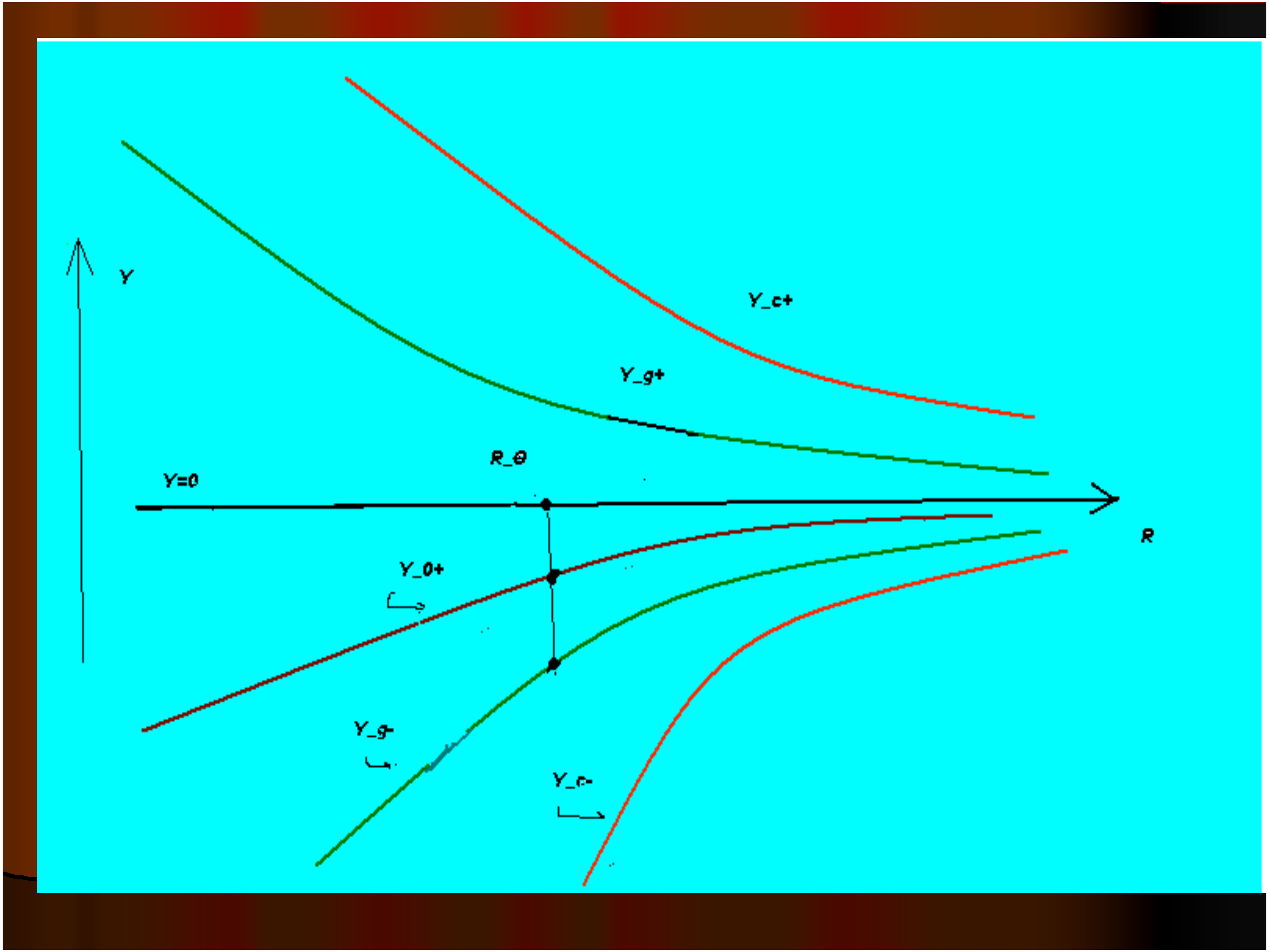
$$a\left(U_{(\mathrm{crit})-}\right) = \frac{\Delta^{1/2}}{r^2}\frac{\sqrt{1-\frac{4Ma^2}{r^3}\left(1-\frac{3M}{r}\right)^{-2}}-1}{\sqrt{1-\frac{4Ma^2}{r^3}\left(1-\frac{3M}{r}\right)^{-2}-\frac{2a^2}{r^2}\left(1-\frac{3M}{r}\right)^{-1}}-1},$$

# Values of y where the specific thrust has a maximum

$$y_{0\pm} = -\frac{1}{2a}\left[1 - \frac{3\mathcal{M}}{r} \mp \sqrt{\left(1 - \frac{3\mathcal{M}}{r}\right)^2 - \frac{4\mathcal{M}a^2}{r^3}}\right]$$

# Behaviour of the specific thrust as function of *y* at a fixed radius




Y
Y_c+
Y_g+
R_θ
Y=0
R
Y_0+
Y_g-
Y_c-