Putting Einstein to Test

Astrometric Experiments in Fundamental Physics

A. Vecchiato[1]

[1]INAF - Astronomical Observatory of Torino

QSO Astrophysics, Fundamental physics, and Astrometric Cosmology in the Gaia era
Porto, June 8, 2011

Outline

Why Astrometry

First experimental evidences of General Relativity came from Astrometry.

- *One can thus hope that the discussion of the observations of Mercury simply confirm previous research. Now this is not negligible: we see here that the approximately 3-fold secular movement of the eccentricity, added to the secular movement of the perihelion, gives a sum in which the observations are greater by 39″ than those which result from calculation.* (Le Verrier, 1859)
- *Thus the results of the expeditions to Sobral and Principe can leave little doubt that a deflection of light takes place in the neighbourhood of the Sun and that it is of the amount demanded by Einstein's generalised theory of relativity, as attributable to the sun's gravitational field.* (Dyson, Eddington and Davidson, 1920)

The PPN Framework

The Parametrized Post-Newtonian formalism is a framework which has been designed to describe the Post-Newtonian limit of all the metric theories of gravity by means of a set of 10 parameters.

These parameters are associated to specific physical properties.

Parameter	Meaning	Parameter	Meaning
γ	How much space-curvature produced by unit rest mass?	α_3	Violation of conservation of total momentum?
β	How much "nonlinearity" in the superposition law for gravity?	ζ_1	
ξ	Prefered-location effects?	ζ_2	
α_1	Prefered-frame effects?	ζ_3	
α_2		ζ_4	
α_3			

The PPN Framework

The Parametrized Post-Newtonian formalism is a framework which has been designed to describe the Post-Newtonian limit of all the metric theories of gravity by means of a set of 10 parameters. These parameters are associated to specific physical properties.

Parameter	Meaning	Parameter	Meaning
γ	How much space-curvature produced by unit rest mass?	α_3	Violation of conservation of total momentum?
β	How much "nonlinearity" in the superposition law for gravity?	ζ_1	
ξ	Prefered-location effects?	ζ_2	
α_1	Prefered-frame effects?	ζ_3	
α_2		ζ_4	
α_3			

Theoretical Implications

A precise estimation of the PPN γ and β parameters, and in general precise tests of fundamental physics, have theoretical and observational implications.

Fulfilling theoretical needs

They are the phenomenological "trace" of a scalar field coupled with gravity which is related to:

- theories aiming to provide a formulation of a quantum theory of gravity;
- theories fully compatible with the Mach principle;
- cosmological scenarios with inflationary stage.

Explaining observational evidences

They can provide constraints on theories which claim of being able to account for several astrophysical and cosmological problems without any need for DM or DE like, e.g.:

- acceleration of cosmological expansion (attributed to DE);
- galactic rotation curves, galaxy cluster masses and dynamics (attributed to DM);
- observational data from gravitational lensing;
- Tully-Fisher relation.

Theoretical Implications

A precise estimation of the PPN γ and β parameters, and in general precise tests of fundamental physics, have theoretical and observational implications.

Fulfilling theoretical needs

They are the phenomenological “trace” of a scalar field coupled with gravity which is related to:

- theories aiming to provide a formulation of a quantum theory of gravity;
- theories fully compatible with the Mach principle;
- cosmological scenarios with inflationary stage.

Explaining observational evidences

They can provide constraints on theories which claim of being able to account for several astrophysical and cosmological problems without any need for DM or DE like, e.g.:

- acceleration of cosmological expansion (attributed to DE);
- galactic rotation curves, galaxy cluster masses and dynamics (attributed to DM);
- observational data from gravitational lensing;
- Tully-Fisher relation.

Theoretical Implications

A precise estimation of the PPN γ and β parameters, and in general precise tests of fundamental physics, have theoretical and observational implications.

Fulfilling theoretical needs

They are the phenomenological "trace" of a scalar field coupled with gravity which is related to:

- theories aiming to provide a formulation of a quantum theory of gravity;
- theories fully compatible with the Mach principle;
- cosmological scenarios with inflationary stage.

Explaining observational evidences

They can provide constraints on theories which claim of being able to account for several astrophysical and cosmological problems without any need for DM or DE like, e.g.:

- acceleration of cosmological expansion (attributed to DE);
- galactic rotation curves, galaxy cluster masses and dynamics (attributed to DM);
- observational data from gravitational lensing;
- Tully-Fisher relation.

Present Experimental Limits

Present best results for γ

Experiment	Effect	Technique	$\|\gamma - 1\|$ lower bound
HIPPARCOS	Light deflection	Global Astrometry	$3 \cdot 10^{-3}$
VLBI	Light deflection	Radio Interferometry	$4.5 \cdot 10^{-4}$
Cassini	Shapiro time delay	Round-trip travel time of radar signals	$2.3 \cdot 10^{-5}$

References: Froeschlé et al. (1997), Shapiro et al. (2004), Bertotti et al. (2003).

Present Experimental Limits

Experimental bounds on β depends on the estimation of γ.

Present best results for β

Data	Effect/Technique	Combination with γ	$\lvert\beta-1\rvert$ lower bound
Radar obs. of Mercury (1966-1990)	Shift excess for Mercury	$2\gamma-\beta$	$3\cdot10^{-3}$
Lunar Laser Ranging	Nordtvedt effect	$4\beta-\gamma$	$\sim10^{-4}$
Radar obs. of inner bodies (1963-2003)	"Grand fits" of Solar System eqs. of motion	$2\gamma-\beta$	$1\div2\cdot10^{-4}$

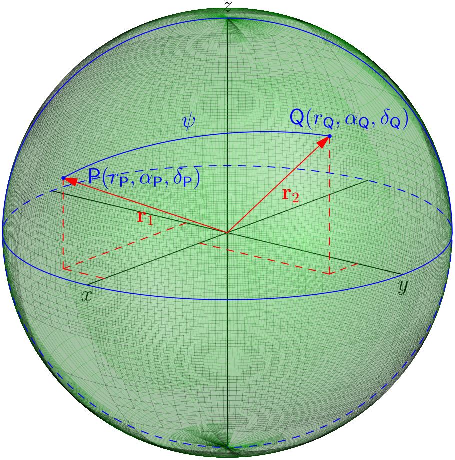
References: Shapiro (1990), Pitjeva (2005, 2010), Williams et al. (2004), Will (2006).

The Observable in the Euclidean World

- The basic astrometric observable is an angle between two viewing directions

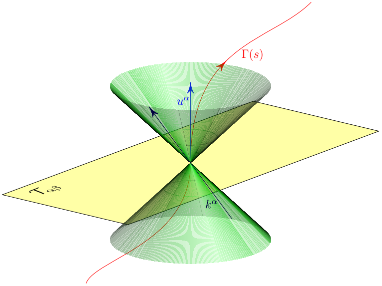
$$\cos\psi = \frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{|\mathbf{r}_1|\,|\mathbf{r}_2|} \qquad (1)$$

- We need to find a relativistic equivalent of this formula.



Theory of Measurements for the Impatient

- A physical observer is identified by his/her worldline Γ, whose four-velocity is $u^\alpha \equiv \dot{\Gamma}$ ($\Rightarrow$the parameter s of Γ represents the proper time of the observer).
- $\forall$ point $\in \Gamma$, u^α splits the space-time into a 1D subspace $|| \, u^\alpha$ and a 3D subspace $\perp u^\alpha$, which are the *time direction* and the *space* relative to the observer u^α respectively.



$$\mathscr{P}_{\alpha\beta} = -u_\alpha u_\beta \qquad \text{Parallel projector}$$

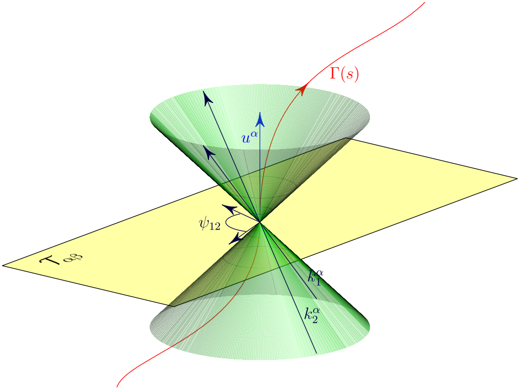
$$\mathscr{T}_{\alpha\beta} = g_{\alpha\beta} + u_\alpha u_\beta \qquad \text{Transverse projector}$$

$$\begin{aligned} \mathrm{d}s^2 &= g_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta \\ &= -c^2 \underbrace{\left(-c^{-2}\mathscr{P}_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta\right)}_{\mathrm{d}t_u^2} + \underbrace{\left(\mathscr{T}_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta\right)}_{\mathrm{d}l_u^2} \end{aligned}$$

A Relativistic formula for the Astrometric Observable

The equivalent of formula (1) can then be found considering that the relativistic viewing directions k_1^μ and k_2^ν (null four-vectors) have to be projected in the spatial hypersurface relative to u^α.

$$\cos\psi = \frac{\mathscr{T}_{\mu\nu} k_1^\mu k_2^\nu}{\sqrt{\mathscr{T}_{\mu\nu} k_1^\mu k_1^\nu}\sqrt{\mathscr{T}_{\mu\nu} k_2^\mu k_2^\nu}}$$



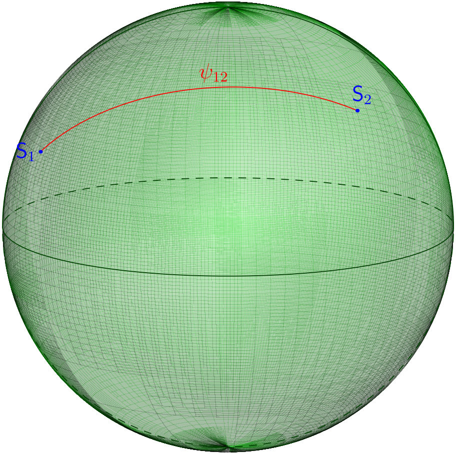
The Concept of the Global Sphere Reconstruction

The ideal picture

$$N_* = 2$$

$$N_{\text{unk}} = 4$$

$$N_{\text{arcs}} = 1$$



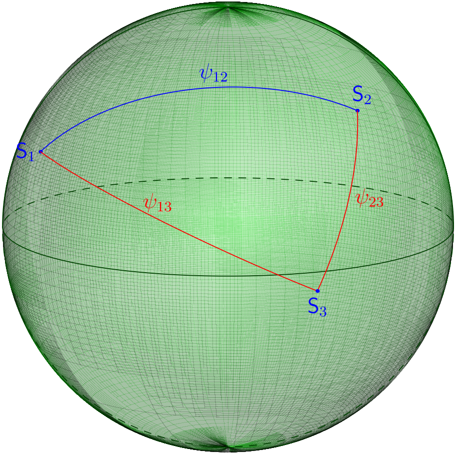
The Concept of the Global Sphere Reconstruction

The ideal picture

$$N_* = 3$$

$$N_{\mathrm{unk}} = 6$$

$$N_{\mathrm{arcs}} = 3$$



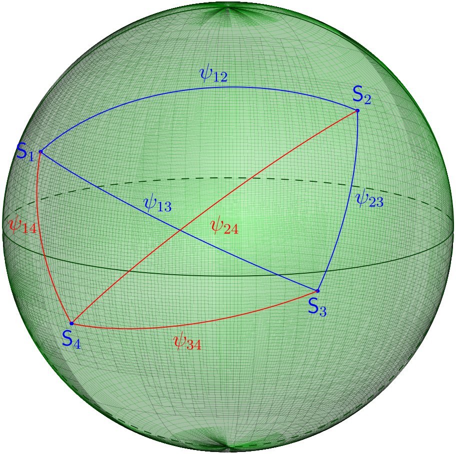
The Concept of the Global Sphere Reconstruction

The ideal picture

$$N_* = 4$$

$$N_{\mathrm{unk}} = 8$$

$$N_{\mathrm{arcs}} = 6$$



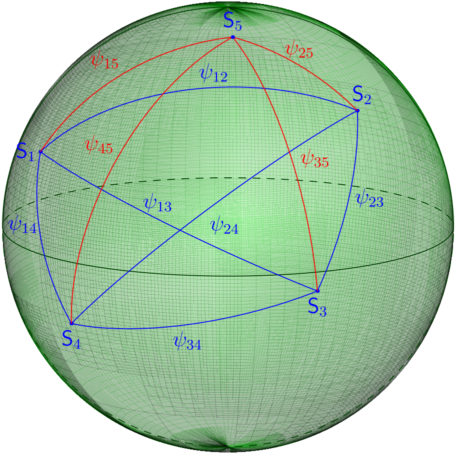
The Concept of the Global Sphere Reconstruction

The ideal picture

$$N_* = 5$$

$$N_{\text{unk}} = 10$$

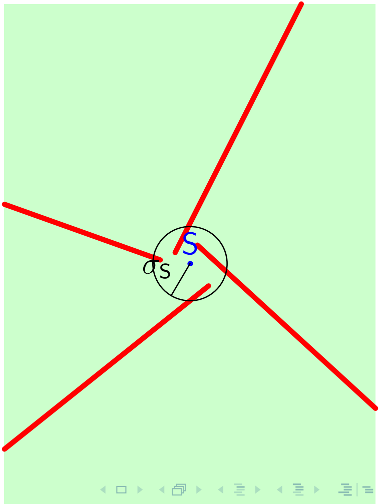
$$N_{\text{arcs}} = 10$$



The Concept of the Global Sphere Reconstruction

The (almost) real picture

- To take into account of observational errors:
  1. overdetermined system of equations;
  2. solution in the least-squares sense.

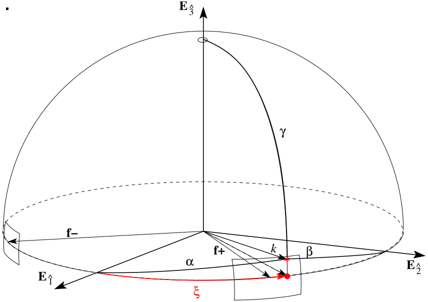


The Relativistic Observable for Gaia

The relativistic observable takes into account that Gaia measures abscissae along the scanning direction.

$$\cos\psi_{(\hat{a},k)} = \frac{\mathcal{T}_{\mu\nu} E^{\mu}_{\hat{a}} k^{\nu}}{\sqrt{\mathcal{T}_{\mu\nu} k^{\mu} k^{\nu}}}$$

$$\cos\xi = \frac{\cos\psi_{(\hat{1},k)}}{\sqrt{1-\cos^2\psi_{(\hat{3},k)}}}$$



Estimated Accuracy on γ for Gaia

- In principle, each observation is a function of Astrometric, Attitude, Instrument, and Global parameters.

$$k^{\nu} = k^{\nu}(\alpha_*, \delta_*, \pi_*, \mu_{\alpha*}, \mu_{\delta*}, \gamma)$$

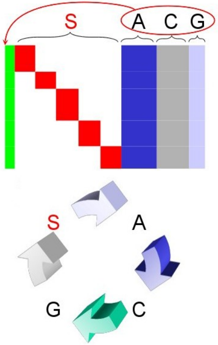
$$E^{\nu}_{\hat{a}} = E^{\nu}_{\hat{a}}\left(q_1^{(a)}, q_2^{(a)}, q_3^{(a)}, q_3^{(a)}, \gamma\right)$$

$$\cos\psi_{(\hat{a},k)} \equiv F_{\hat{a}}\left(\underbrace{\alpha_*, \delta_*, \pi_*, \mu_{\alpha*}, \mu_{\delta*}}_{\text{Astrometric parameters}}, \underbrace{q_1^{(a)}, q_2^{(a)}, q_3^{(a)}, q_4^{(a)}}_{\text{Attitude parameters}}, \underbrace{\gamma, \ldots}_{\text{Global}}\right)$$

- The dependence on γ gives the estimation of this parameter as a by-product of the sphere reconstruction.
- Current simulations suggest a final accuracy of $\sigma_\gamma \sim 10^{-6}$.

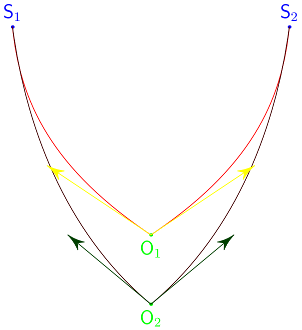
Problems and Pitfalls

- Dimension and complexity of the problem ($\sim 10^{10} \times 10^{8}$)
  - Is convergence ensured?
  - Is the right convergence ensured?
- Correlation with parallaxes
- Correlation with Basic Angle variations



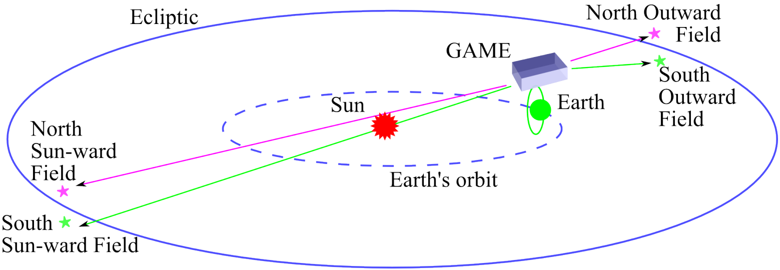
Problems and Pitfalls

- Dimension and complexity of the problem ($\sim 10^{10} \times 10^{8}$)
  - Is convergence ensured?
  - Is the right convergence ensured?
- Correlation with parallaxes
- Correlation with Basic Angle variations



The GAME Concept

GAME: a concept for a dedicated mission of relative astrometry

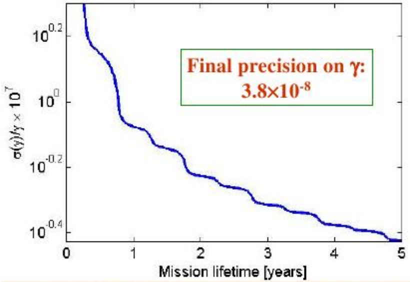


Estimating γ with GAME

Magnitude	σ_{100s} (mas)	σ_{500s} (mas)
10	0.058	0.026
11	0.093	0.041
12	0.150	0.065
13	0.250	0.103
14	0.423	0.164
15	0.782	0.263
16	1.598	0.425
17	3.581	0.704
18	8.523	1.225

$$\Delta\psi = \frac{(1+\gamma)\,GM_\odot}{c^2 r_o}\sqrt{\frac{1+\cos\psi}{1-\cos\psi}} \Rightarrow \frac{\sigma_{\Delta\psi}}{\Delta\psi} \sim \frac{\sigma_\gamma}{\gamma}$$

$$\Delta\psi \sim 0''2 \text{ at } \psi = 2^\circ \Rightarrow \frac{\sigma_\gamma}{\gamma} \sim 10^{-3}$$



Light Deflection

- The β parameter enters as a second-order term into the light deflection

$$\Delta\alpha = f\left((g_{00})^{-1}, g_{\mu\nu}\right)$$

$$g_{00} = -1 + 2U_{\odot} - 2\beta U_{\odot}^2$$
$$g_{ii} = 1 + 2\gamma U_{\odot}$$

- This is the reason why the light deflection is an intrinsically bad method to estimate the β parameter.

The Nordtvedt Effect

- The Nordtvedt effect refers to the possibility that massive bodies fall with different accelerations depending on their gravitational self-energy, thus violating the weak equivalence principle.

$$\mathrm{a} = \frac{M_{\mathrm{P}}}{M} \nabla U$$

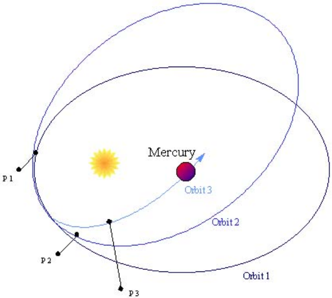
$$\frac{M_{\mathrm{P}}}{M} = 1 - \eta \frac{E_{\mathrm{g}}}{M}$$

$$\eta = 4\beta - \gamma + f\left(\xi, \alpha_1, \alpha_2, \zeta_1, \zeta_2\right)$$

- It is measured with LLR techniques, not with astrometry.

Perihelion Shift Excess

- The effect of the perihelion shift excess depends on a combination of both γ and β (and other PPN parameters).



$$\Delta\omega = \frac{6\pi m}{a\left(1-e^2\right)}\left[\frac{1}{3}\left(2+2\gamma-\beta\right)+f\left(\alpha_1,\alpha_2,\alpha_3,\zeta_2,J_2\right)\right]$$

Integration of NEOs Orbits

- Gaia will not be able to estimate the value of β from Mercury (too close to the Sun).
- It will instead revert to the observation of Near-Earth Objects, which are tipically in less favourable conditions, but their number is large ($\sim 10^5$).
- Determining the relativistic orbital corrections for several objects at different inclinations gives also the opportunity of estimating also the solar J_2 (but beware of $\mathrm{corr}(\beta, J_2) > 0.9$!).
- Numerical simulations have suggested formal accuracies of

$$\sigma_\beta \simeq 1.5 \cdot 10^{-4} \qquad \sigma_{J_2} \simeq 5 \cdot 10^{-9}$$

using the Gaia observations for some 1300 asteroids.

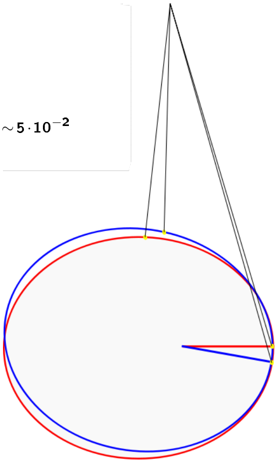
Estimated Accuracy

- The shift excess for Mercury is $\Delta\omega \simeq 0.104''/\mathrm{orbit}$
- At opposition $V_{\mathrm{merc}} \gtrsim -1.5$, $\sigma_\alpha \sim 0.5\,\mathrm{mas}$ (formal) for a single exposure of $0.1\,\mathrm{s}$, i.e.

$$\left.\begin{array}{l}\Delta\alpha \left\{\begin{array}{l}\simeq 0.27\Delta\omega \simeq 28\,\mathrm{mas}\\ = k\Gamma \ (\Gamma \equiv 2+2\gamma-\beta)\end{array}\right.\\ \sigma_{\Delta\alpha} \simeq 0.5\,\mathrm{mas}\end{array}\right\} \Rightarrow \frac{\sigma_\beta}{\beta} \simeq 3\frac{\sigma_\Gamma}{\Gamma} = 3\frac{\sigma_{\Delta\alpha}}{\Delta\alpha} \sim 5\cdot 10^{-2}$$

- Therefore, one day per orbit (about 88 days) with Mercury at opposition means (assuming ~ 3 orbits per year)

Years	# of obs.$\cdot 10^{-7}$	$\left(\sigma_\beta/\beta\right)\cdot 10^6$
1	9.5	5.5
2	19.8	3.9
3	28.4	3.2
4	37.8	2.8
5	47.8	2.5



Astrometric Effect of J_2: the q-effect

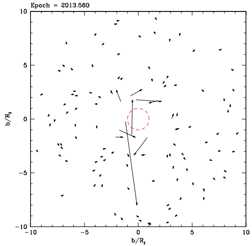
- A non-spherically symmetric distribution of the mass in a perturbing body, according to GR, induces specific patterns in the nearby light deflection, which can be modeled w.r.t. the higher order multipoles of its gravity field (q-effect). General Relativity $\Rightarrow \varepsilon = 1$.

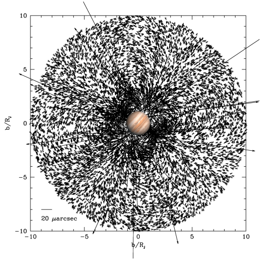
$$\Delta\Phi = \Delta\Phi_1 \mathbf{n} + \Delta\Phi_2 \mathbf{m}$$

$$\Delta\Phi_1 = \frac{2(1+\gamma)M}{b}\left[1+\varepsilon J_2 \frac{R^2}{b^2}\left(1-2(\mathbf{n}\cdot\mathbf{z})^2-(\mathbf{t}\cdot\mathbf{z})^2\right)\right]$$

$$\Delta\Phi_2 = \frac{4(1+\gamma)M\varepsilon J_2 R^2 \gamma}{b^3}(\mathbf{m}\cdot\mathbf{z})(\mathbf{n}\cdot\mathbf{z})$$

- These effects has never been measured $\Rightarrow$ new GR tests (first order at which GW can be modelled).
- Such kind of experiments are foreseen for Gaia (Crosta & Mignard, 2006).
- GAME, as a pointed mission, is able to select the most favourable observing conditions for this measure.





Estimating the q-effect

The Differential Approach

- This method exploits the fact the q-effect has a limited duration as Jupiter moves "away" from its closest approach to suitable stars, comparing the configuration of a stellar field with and without a perturbing body in between.
- This approach requires a favorable configuration and for Gaia it depends on the initial conditions of the Scanning Law (up to 10σ-level configurations with the present mission parameters), while GAME in principle is more flexible.

The Global Approach

- Only in the case of Gaia, another possibility is to introduce the quadrupole contribution to the light deflection into the model for the Gaia abscissa.
- In this way ε, similarly to γ, can be obtained as a by-product of the sphere reduction.

$$\mathbf{n} = \sigma + \delta\sigma_{pN} + \delta\sigma_Q$$

$$\delta\sigma_Q = \varepsilon\sigma \times \Delta_Q \dot{x}_p(t_{\rm obs}) \times \sigma$$

The Mansouri-Sexl Formalism

This is about gravity INDEED!

- **Special Relativity** requires that c is the same for any observer.
- **Lorentz transformations** are the consequence of this hypothesis.
- **Local Lorentz Invariance violation** is foreseen in some quantum gravity/high energy theories.

- The Mansouri-Sexl formalism, similarily to the PPN one, generalizes the Lorentz transformations, introducing a set of arbitrary parameters (a, b, d, ε) (Laemmerzahl, 2006)

$$
\begin{aligned}
T &= \frac{(t - \varepsilon \cdot \mathbf{x})}{a} \\
X &= \frac{\mathbf{x}}{d} - \left(\frac{1}{d} - \frac{1}{b}\right) \frac{\mathbf{w} \cdot \mathbf{x} \mathbf{w}}{\mathbf{w}^2} + \mathbf{w}\, T
\end{aligned}
$$

- From the astrometric point of view, the LLI violation shows itself as an aberration effect that depends on $f = d/b$, and puts to test the same properties of the Michelson-Morley experiment (Klioner, 2008).

The Mansouri-Sexl Formalism

This is about gravity INDEED!

- **Special Relativity** requires that c is the same for any observer.
- **Lorentz transformations** are the consequence of this hypothesis.
- **Local Lorentz Invariance violation** is foreseen in some quantum gravity/high energy theories.

- The Mansouri-Sexl formalism, similarily to the PPN one, generalizes the Lorentz transformations, introducing a set of arbitrary parameters (a, b, d, ε) (Laemmerzahl, 2006)

$$
\begin{aligned}
T &= \frac{(t - \varepsilon \cdot \mathbf{x})}{a} \\
X &= \frac{\mathbf{x}}{d} - \left(\frac{1}{d} - \frac{1}{b}\right) \frac{\mathbf{w} \cdot \mathbf{x} \mathbf{w}}{\mathbf{w}^2} + \mathbf{w} T
\end{aligned}
$$

- From the astrometric point of view, the LLI violation shows itself as an aberration effect that depends on $f = d/b$, and puts to test the same properties of the Michelson-Morley experiment (Klioner, 2008).

For Further Reading I

S. Capozziello and M. Funaro.
Introduzione alla Relatività Generale.
Liguori Editore, 2005.

F. de Felice and D. Bini.
Classical measurements in curved space-times.
Cambridge University Press, 2010.

C. M. Will.
Theory and experiment in gravitational physics.
Cambridge University Press, 1993.

M. Crosta and F. Mignard.
Microarcsecond light bending by Jupiter.
Classical and Quantum Gravity. 23(15):4853–4871, 2006.

For Further Reading II

- M. Gai et al.
  Gamma Astrometric Measurement Experiment (GAME) - Implementation and performance.
  Advances in Space Research. 44(5):588–596, 2009.
- A. Vecchiato et al.
  Gamma astrometric measurement experiment (GAME) - Science case.
  Advances in Space Research. 44(5):579–587, 2009.
- A. Vecchiato et al.
  Testing general relativity by micro-arcsecond global astrometry.
  Astronomy and Astrophysics. 399:337–342, 2003.

For Further Reading III

- C. M. Will.
  The Confrontation between General Relativity and Experiment.
  Living Reviews in Relativity. 9, 2006.
- C. Laemmerzahl.
  Test theories for Lorentz invariance.
  Lect. Notes Phys. 702:349, 2006.
- S. A. Klioner.
  Explicit exact expression for the Thomas precession.
  GAIA-CA-TN-LO-SK-004-2, 2008.