FCT Fundação para a Ciência e a Tecnologia

MINISTÉRIO DA CIÊNCIA, TECNOLOGIA E ENSINO SUPERIOR

Final Report

Project: PTDC/MAT/100055/2008

Title/Título

Coupled cell systems: the interplay of network structure and dynamics/
Sistemas de células acopladas: interacção entre estrutura da rede e dinâmica

Concurso * Programa * Área: Concurso 2008 * COMPETE * Matemática

Starting date: 01-01-2011

Ending date: 31-03-2014

Research team:

- Ana Paula da Silva **Dias** (Principal investigator);
- Manuela Alexandrina David de **Aguiar** (Investigator; from the starting date, 01-01-2011);
- Célia Sofia Mota da Cunha **Moreira** (Investigator; joined the team at 10-02-2012);

Contents

1. Details on the research work done during the period of the project

The plan of the project was to focus at the role of the network architecture in the dynamics of the associated coupled cell systems (CCS). In this context, a CCS is formed by interacting individual dynamical systems (the cells, where a cell indicates a system of ordinary differential equations (ODE)); the architecture of the CCS is codified in terms of a directed graph - the coupled cell network (CCN)- indicating which cells are identical, which cells interact and which interactions (couplings) are of the same type.

Robust heteroclinic cycles and networks can occur in the games (or population dynamics) or symmetric settings. The works **[A11]** and **[RLA11a, RLA11b]** analyse the existence of chaotic dynamics in the neigbourhood of a heteroclinic network in the games and symmetric setting, respectively. In **[A11],** we give conditions for switching dynamics in general bimatrix games and show that switching near an edge network can never occur for replicator dynamics. Switching dynamics near a heteroclinic network occurs whenever every (infinite) sequence of connections in the network is shadowed by at least one trajectory in its neighbourhood. In **[RLA11a, RLA11b],** we study the dynamics of a generic vector field in the neighbourhood of a heteroclinic cycle of non-trivial periodic trajectories whose invariant manifolds meet transversely. The main part of the analysis is a proof of chaotic cycling: there are trajectories that follow the cycle making any prescribed number of turns near the periodic solutions, for any given bi-infinite sequence of turns.

The existence of robust heteroclinic cycles is a first step in the understanding of recurrent phenomena forced by the network architecture. In **[AADF11]** we find conditions for coupled cell systems to support robust heteroclinic cycles. The structure of such systems is consistent with networks determining the existence of synchrony subspaces (spaces defined in terms of equalities of certain cell coordinates that are flow-invariant for all coupled cell systems associated with a given network structure) where heteroclinic connections occur in a robust way. It is thus important to know the synchrony subspaces determined by a network structure.

In fact, as the intersection of synchrony subspaces for a CCN is also a synchrony subspace of the network, it follows then that, given a coupled cell network, its set of synchrony subspaces, taking the inclusion partial order relation, forms a lattice. In **[AD14a]** we show how to obtain the lattice of synchrony subspaces for a general network and present an algorithm that generates that lattice. We prove that this problem is reduced to get the lattice of synchrony subspaces for regular networks (in which all cells have the same type and receive the same number of inputs and all arrows have the same type). For a regular network, the lattice of synchrony subspaces is obtained based on the eigenvalue structure of the network adjacency matrix. The two fundamental difficulties that had to be overcomed where how to list the possible polydiagonal subspaces that contain (generalized) eigenvectors of the network adjacency matrix and how to generalize the concept of irreducible (synchrony) subspace aiming to describe the lattice of synchrony subspaces through a (small) set of irreducible synchrony subspaces. The lattice of synchrony subspaces, as a set, is a subset of the lattice of the invariant subspaces under the network adjacency matrix, but it is not in general a sublattice - the meet operation is the same, the intersection of subspaces, but the join of two synchrony subspaces is not given in general by their sum. Concerning these two difficulties, it is defined the concept of minimal synchrony subspace associated to an eigenvector or Jordan chain - the intersection of all synchrony subspaces containing the eigenvector or Jordan chain, and proved then that the set of all minimal synchrony subspaces forms a sum-dense set for the lattice of synchrony subspaces -- that is, any synchrony subspace can be given by a sum of minimal synchrony subspaces. It follows then that to list the possible polydiagonal subspaces containing (generalized) eigenvectors - can be reduced to the listing of the minimal synchrony subspaces. It is then introduced the concept of sum-irreducible synchrony subspace - it cannot be represented as a sum of proper synchrony subspaces. It is shown then that every synchrony subspace associated with a network is a sum of sum-irreducible synchrony subspaces. Joining the two results, as just mentioned, the lattice of synchrony subspaces associated with a regular network is generated by the the set of the minimal synchrony subspaces that are sum-irreducible. We also present an algorithm that outputs the lattice of synchrony subspaces, together with its irreducible sum-dense set, for a regular network.

Also, in **[M14a]**, given a regular network, it is defined a special class of Jordan subspaces of the corresponding adjacency matrix which is used to prove that the synchrony subspaces are precisely the polydiagonals that are direct sums of subspaces in that class - the special Jordan subspaces. It is also shown that these subspaces play an important role in the lattice structure of all synchrony subspaces because every join-irreducible element of the lattice is the smallest synchrony subspace containing at least one special Jordan subspace.

Taking into account our aim of using small networks to construct bigger networks presenting heteroclinic behaviour, we consider networks obtained by binary network operations and determine a relation between the lattice of synchrony subspaces for a network obtained by a binary operation and the lattice of synchrony subspaces for its component networks. In **[AR12b]** we consider the join and coalescence of networks and in **[AD14b]** we consider the product of networks.

In fact, most real world networks are evolving networks, that is, their topology evolves with time, either due to a rewiring of a link, the appearance or disappearance of a link or node, or by a merging of small networks into a larger one. The dynamics of network topology reflects frequent changes in the interactions among network components and translates into a rich variety of evolutionary patterns. In **[ADR14],** we analyze the transition of the lattice of synchrony subspaces of a network that is caused by structural changes in the network topology, such as deletion and addition of cells or edges, and rewirings of edges.

The works **[AR12b] , [ADR14]** and **[AD14b]** complete the characterization of the evolution of synchrony for coupled cell networks under the most common unary and binary network (graph) operations.

As one of the most prevailing and studied phenomena in dynamical systems, bifurcation describes the sudden change of properties of systems subject to variation of a parameter. In the case of change of stability of an equilibrium, a bifurcation can be usually foreseen by a critical eigenvalue associated to the linearization at the equilibrium. While the bifurcation analysis for simple critical eigenvalues is straightforward, multiple eigenvalues can lead to complicated bifurcating behavior of the system such as multiple bifurcations and secondary bifurcations. In **[AR12a],** we analyze the impact of interior symmetries (where a group of permutations of a subset of cells partially preserves the network structure) on the multiplicity of the eigenvalues of the Jacobian matrix at a fully synchronous equilibrium for the coupled cell systems associated to homogeneous networks. We consider also the special cases of regular and uniform networks. It follows from our results that the interior symmetries, as well as the reverse interior symmetries and quotient interior symmetries, of the network force the existence of eigenvalues with algebraic multiplicity greater than one. In **[ADP11],** it is also considered interior symmetric coupled cell networks. In this setup, the full analogue of the Equivariant Hopf Theorem for networks with symmetries was obtained by Antoneli, **Dias** and Paiva [SIAM J. Appl. Dyn. Syst. 7 (2008), no. 1, 220–248). In this work we present an alternative proof of this result using center manifold reduction.

Another situation where symmetry and network structure are combined proving that both are essential in implying that from the (local) bifurcation point of view, phenomena can be generic that would not be generic if the network structure was not taken into account, is addressed and **[DP11a, DP11b].** Specifically, coupled cell systems associated with networks with abelian symmetry are considered. Abelian groups are identified in terms of their character tables such that very degenerate codimension-one local Hopf bifurcation occurs – degenerate in the sense that the center subspace associated with the bifurcation is high-dimensional, which would not be expected if only one would consider the abelian symmetry group of the coupled cell systems involved – that is, the vector fields are symmetric but have also a structure that has to be consistent with the network structure. This implies in particular that there can occur a richer dynamics in those degenerate situations.

A lift of a given network is a network that admits the first network as quotient. Assuming that a bifurcation occurs for a coupled cell system consistent with the structure of a regular network , it is well-known that surprisingly some lifts exhibit new bifurcating branches of solutions. Using the concept of ODE-equivalence, we are able to prove in **[DM12]** that it is always possible to study a degenerate bifurcation arising in a system whose regular network has multiple arrows as a bifurcation of a bigger system associated with a regular uniform network (that have no loops and no multiple arrows). For that, we show that, given a regular network with loops or multiple arrows, it is always possible to construct a uniform lift whose adjacency matrix has only two possible eigenvalues, namely, 0 and −1, in addition to all eigenvalues of the initial network adjacency matrix. Moreover, this uniform lift has the minimal number of cells over all uniform lifts. We also prove that if a non-vanishing eigenvalue of the initial adjacency matrix is fixed then it is always possible to construct a uniform lift that preserves the number of eigenvalues with the same real part of that eigenvalue. Finally, for the eigenvalue zero we show that such a construction is not always possible proving that there are networks with multiple arrows whose uniform lifts all have the eigenvalue 0, in addition to all eigenvalues of the initial network adjacency matrix.

Considering now the lifting problem restricted to uniform networks, it is shown in **[M14b],** that, from the bifurcation point of view, rings and their lifts are special networks. It is also proved that, generically, it is possible to construct lifts that just exhibit the bifurcating branches determined by the quotient network. Moreover, it is identified all generic situations to construct lifts that may exhibit bifurcating branches that do not appear in the quotient itself.

At the end of the project, as planned, **Aguiar** and **Dias** have organized the "Workshop on Coupled Cell Networks and Dynamics", University of Porto, February 3-5, 2014. http://cmup.fc.up.pt/cmup/coupledcells/ The aim of the workshop was to bring together researchers interested in coupled cell networks in order to share recent progress and advances in theory and applications of coupled cell networks. A priority of the workshop was to promote interactions among researchers, as well as graduate students and postdoctoral fellows, to bring fruitful directions and collaborations. We had around 30 participants. The invited speakers presented very high quality presentations and most of them have stayed till the end of the workshop week so that there were several discussions about what has been done in the area of coupled cell networks and dynamics and what can be possible good future directions.

2. Aims

2.1 Proposed aims

The plan of the project was to focus at the role of the network architecture (the coupled cell network (CCN)) at the dynamics of the associated coupled cell systems (CCS).

The main objectives of the plan were:

(1) To use graph algorithms to establish the transition between ODE-equivalent networks and to generate the minimal networks of each network ODE-class.

(2) To find conditions for the network structure to support robust heteroclinic cycles; starting by developing a strategy for analyzing the dynamics of large networks in terms of small modules where the dynamics is well understood. Two ways of tackling that problem can be:

(2.1) Ways of combining small networks with known dynamics to construct larger networks with specific dynamical properties: for example using product networks (networks where the nodes of one network are replaced by copies of another network) planning to combine small networks with known dynamics to construct larger product networks with specific dynamical properties;

(2.2) A synchrony subspace is a space defined in terms of equalities between certain groups of cell coordinates, that are flow-invariant under any CCS associated with a given CCN. For example, the occurrence of robust heteroclinic behaviour is always associated with the presence of flow-invariant subspaces. For any synchrony subspace determined by a network, the restrictions of the original CCS to that space correspond to CCS associated with a smaller network (the quotient network). As dynamics on a quotient network describe (partially) synchronous dynamics on the whole network, using (quotient) networks with known dynamics is a way of finding networks with pre-defined dynamics.

(3) The concept of `synchrony-breaking (local) bifurcation´ in networks, which is in parallel to the term `symmetry-breaking (local) bifurcation´ in the symmetric context, corresponds to the situation where a synchronous state looses stability and other states appear. In the symmetric context, techniques based at the representations of groups have an important role in the analysis of the local bifurcations in symmetric systems. The natural question that we can pose is if there is an analogous theory for networks and as a start try to identify which are the properties of the network architecture that force the different types of bifurcations, for example, nilpotent.

(4) Concerning the reverse problem, that is, given observations of network dynamics, we can ask if we can deduce the network architecture.

(5) To organize a workshop where we aimed to get together several experts in the field as well as young researchers. As a consequence we were expecting to expand our research lines and establish contact with other possible collaborators. The workshop was also an excellent opportunity to spread the coupled cell network area in Portugal and in particular to attract young students to the field.

2.2 Achieved aims

On heteroclinic dynamics:

-Robust heteroclinic cycles and networks can occur in the games (or population dynamics) or symmetric settings. The works **[A11]** and **[RLA11a, RLA11b]** analysed the existence of chaotic dynamics in the neigbourhood of a heteroclinic network in the games and symmetric setting, respectively.

- Conditions were found for coupled cell systems to support robust heteroclinic cycles in **[AADF11].**

On flow-invariant subspaces determined by the networks topology:

-Obtained an algebraic and algorithmic description of the lattice of synchrony subspaces for general networks in **[AD14a].**

-A different approach from the work in [AD14a] was considered in **[M14a]** in obtaining a different class of generators of the lattice of synchrony subspaces of a regular network, using a special class of Jordan subspaces of the corresponding adjacency matrix.

-Aiming to use small networks to construct bigger networks presenting heteroclinic behaviour, we considered networks obtained by binary network operations and determined a relation between the lattice of synchrony subspaces for a network obtained by a binary operation and the lattice of synchrony subspaces for its component networks. In **[AR12b]** we considered the join and coalescence of networks and in **[AD14b]** we considered the product of networks.

- In evolving networks (network topology evolves with time, either due to a rewiring of a link, the appearance or disappearance of a link or node, or by a merging of small networks into a larger one), the dynamics of network topology reflects frequent changes in the interactions among network components and translates into a rich variety of evolutionary patterns. In **[ADR14],** we analyzed the transition of the lattice of synchrony subspaces of a network that is caused by structural changes in the network topology, such as deletion and addition of cells or edges, and rewirings of edges.

The works **[AR12b] , [ADR14]** and **[AD14b]** complete the characterization of the evolution of synchrony for coupled cell networks under the most common unary and binary network (graph) operations.

On bifurcations of coupled cell systems determined by the corresponding coupled cell networks:

-Analysed the impact of interior symmetries on the multiplicity of the eigenvalues of the Jacobian matrix at a fully synchronous equilibrium for the coupled cell systems associated to homogeneous networks in **[AR12a]**.

-In **[ADP11],** it is also obtained for interior symmetric coupled cell networks the full analogue of the Equivariant Hopf Theorem for networks with symmetries, using center manifold reduction, which is an alternative proof of the result that was obtained by

Antoneli, **Dias** and Paiva (SIAM J. Appl. Dyn. Syst. 7 (2008), no. 1, 220–248).

-Combined the symmetry properties of networks symmetric under abelian symmetry groups and the network structure itself to identify coupled cell systems with abelian symmetries that do not exhibit the generic behavior (from the point of view of steady-state and Hopf bifurcations) that would be expected from equivariant bifurcation theory in **[DP11a, DP11b].**

-Shown that any steady-state or Hopf bifurcation problem associated to a regular network can be studied as a bifurcation problem associated to a uniform network in **[DM12].**

- In **[M14b],** it is shown that, from the bifurcation point of view, rings and their lifts are special networks. It is proved that generically it is possible to construct lifts from uniform networks that just exhibit the bifurcating branches determined by the quotient network and, moreover, it is identifed all generic situations to construct lifts that may exhibit bifurcating branches that do not appear in the quotient itself.

Workshop organization

-Organization by Aguiar and Dias of the "Workshop on Coupled Cell Networks and Dynamics", University of Porto, February 3-5, 2014. http://cmup.fc.up.pt/cmup/coupledcells/

Remarks on the aims

From the List 2.1 of Proposed aims, only (1) and (4) where addressed more briefly and we will need more time to be able to make relevant progress. In particular, we are contacting a group of researchers in the area of computer science so that item (1) can be addressed from the algorithmic point of view.

Moreover, it is our intention to have all the algorithms obtained at this project (and at the works that the team members have done previously) implemented and available through a free-access web page, in a near future, so that they can be executed, for any coupled cell network, by any user without having to know how the algorithms work.

3. **Indicators**

Indicators	Achieved
A – Publications	
Papers in international journals	10
Papers in national journals	2
B – Communications	
Communications in international conferences	9
Communications in national meetings	4
D – Conferences and seminars organization	8
E - Ph D and Post doctorate supervisions	
Ph D thesis	1
Other (Post doctorate coordination)	1
L – Others – submitted papers + preprints	2+2

4. Publications

4.1 Papers published at international journals

[A11] Publicado – M.A.D. **Aguiar**, "Is there switching for replicator dynamics and bimatrix games?", *Phys. D* 240 (18) (2011) 1475-1488. DOI 10.1016/j.physd.2011.06.016

[AADF11] Publicado – M. **Aguiar**, P. Ashwin, A. **Dias**, and M. Field, "Dynamics of coupled cell networks: Synchrony, heteroclinic cycles and inflation", *J. Nonlinear Sci.* 21 (2) (2011) 271-323. DOI 10.1007/s00332-010-90839

[AD14a] Aceite (em Junho de 2013) uma vez feitas pequenas alterações – M.A.D. **Aguiar** and A.P.S. **Dias**, "Synchrony in coupled cell networks", *J. Nonlinear Sci.* 2014. http://www.fc.up.pt/cmup/apdias/Aguiar_Dias_Synchrony.pdf

[ADR14] Aceite (condicionalmente em Janeiro de 2014) M.A.D. **Aguiar**, A.P.S. **Dias** and H.Ruan, "Synchrony and Elementary Operations on Coupled Cell Networks", Nonlinearity 2014. http://www.fc.up.pt/cmup/apdias/Elementary_9July_sub.pdf

[AR12a] Publicado – M.A.D. **Aguiar** and H. Ruan, "Interior Symmetries and Multiple Eigenvalues for Homogeneous Networks", *SIAM J. Appl. Dyn. Syst.* 11 (4) (2012) 1231–1269. DOI: 10.1137/110851183

[AR12b] Publicado – M.A.D. **Aguiar** and H. Ruan, "Evolution of synchrony under combination of coupled cell networks", *Nonlinearity* 25 (11) (2012) 3155–3187. DOI:10.1088/0951-7715/25/11/3155

[ADP11] Publicado – F. Antoneli, A.P.S. **Dias** and R.C. **Paiva**, "Coupled Cell Networks: Hopf bifurcation and Interior Symmetry", *Discrete Contin. Dyn. Syst.* 2011, Dynamical systems, differential equations and applications. 8th AIMS Conference. Suppl. Vol. I, 71–78. ISBN: 978-1-60133-007-9; 1-60133-007-3 http://cmup.fc.up.pt/cmup/apdias/Antoneli354.pdf

[DM12] Publicado – A.P.S. **Dias** and C.S. **Moreira**, "Spectrum of the elimination of loops and multiple arrows in coupled cell networks", *Nonlinearity* 25 (11) (2012) 3139–3154. DOI:10.1088/0951-7715/25/11/3139

[M14a] Aceite (condicionalmente em Março de 2014) - C.S. **Moreira**, "Special Jordan subspaces and synchrony subspaces in coupled cell networks", SIAM J. Appl. Dynam. Sys. 2014 (first version at arXiv (http://arxiv.org/pdf/1311.3169.pdf)

[RLA11a] Publicado - A.A.P. Rodrigues, I.S. Labouriau and M.A.D. **Aguiar**, “Chaotic double cycling”, *Dyn. Syst.* 26 (2) (2011) 199–233. DOI:10.1080/14689367.2011.557179

4.2 Papers published at proceedings of national meetings (without referee)

[DP11a] Publicado – A.P.S. **Dias** and R.C. Paiva. Hopf bifurcation in coupled cell networks with abelian symmetry. Em: Actas do Encontro Nacional da SPM Leiria 2010, *Bol. Soc. Port. Mat.* 2010, Special Issue,110-115.
http://cmup.fc.up.pt/cmup/apdias/DiasPaivaSPMversaopublicada.pdf

[RLA11b] Publicado – A.A.P. Rodrigues, I.S. Labouriau and M.A.D. **Aguiar**, “Um carrossel caótico: Dinâmica perto de redes heteroclínicas”. Em: Actas do Encontro Nacional da SPM Leiria 2010, *Bol. Soc. Port. Mat.* 2010, Special Issue, 103–109.
http://www.fc.up.pt/cmup/islabour/Contents/OutrasPub/Carrossel_Caotico.pdf

4.3 Papers submitted to international journals

Submetido – [AD14b] M.A.D. **Aguiar** and A.P.S. **Dias**, “Regular synchrony lattices for product coupled cell networks”, Chaos, 2014.
http://www.fc.up.pt/cmup/apdias/SynchronyProduct_final_version.pdf

Submetido – [M14b] C.S. **Moreira,** “On bifurcations in lifts of regular uniform coupled cell networks”, Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences 2014.
http://www.fc.up.pt/cmup/apdias/RoyalProof.pdf

4.4 Preprints

[DP11b] A.P.S. **Dias** and R.C.**Paiva**. A remark on Hopf bifurcation in o ne-dimensional symmetric coupled cell networks. Preprint (2011) and in reformulation (2014).

[M14c] C.S. **Moreira,** “Cellular splitting in coupled cell networks”, in conclusion (2014).

5. Report on the activities

5.1 Academic visits

[**Dias**] Visit by invitation to the Isaac Newton Institute for Mathematical Sciences, Cambridge, UK, January 7-11, 2013, at the occasion of the “Workshop on Symmetry, Bifurcation and Order Parameters”.

[**Aguiar and Dias**] Visit by invitation to the University of Houston, USA, May 14-16, 2012, at the occasion of the "Workshop on Progress and Problems in Dynamics (in honor of Professor Mike Field)".

5.2 Communications at international conferences

[**Moreira**] "Cellular splitting in coupled cell networks", Workshop on Coupled Cell Networks and Dynamics, University of Porto, February 3-5, 2014 (invited speaker).

[**Moreira**] "Coupled cell networks and matrix theory", at the conference "MATTRIAD 2013", Herceg-Novi, Montenegro, September 16-20, 2013 (speaker).

[**Aguiar**] "Synchrony and Graph Operations on Coupled Cell Networks", International Conference Dynamics, Games and Science II, Mathematics of Planet Earth 2013, Lisbon, Portugal, 2-4 September, 2013 (session invited speaker).

[**Dias**] "Coupled Cell Networks and the Synchrony Lattice", International Conference Dynamics, Games and Science II, Mathematics of Planet Earth 2013, Lisbon, Portugal, 2-4 September, 2013 (session invited speaker).

[**Moreira**] "Exploring coupled cell networks", International Conference Dynamics, Games and Science II, Mathematics of Planet Earth 2013, Lisbon, Portugal, 2-4 September, 2013 (session organizer and speaker).

[**Dias**] "Synchrony in Coupled Cell Networks", SIAM Conference on Applications of Dynamical Systems (DS13)", Snowbird, Utah, USA, May 19-23, 2013 (session organizer and speaker).

[**Moreira**] "Spectrum of the elimination of loops and multiple arrows in coupled cell networks", poster, "Workshop on critical transitions in complex systems", Imperial College of London, United Kingdom, March 21, 2012.
http://www2.imperial.ac.uk/~mrasmuss/criticaltransitions/

[**Moreira**] "Spectrum of the elimination of loops and multiple arrows in coupled cell networks", "XXXII Dynamics Days Europe", University of Gothenburg, Sweden, September 5, 2012 (session invited speaker)
http://dynamics-days-europe-2012.org/

[**Dias**] "Network structure and dynamics of coupled cell systems" at the Mathematical Neuroscience workshop, Universidade do Porto, April 27, 2012 (invited speaker).

5.3 Other seminars

[**Dias**] "Regras de boa vizinhança", Seminar integrated in "Jogos da Matemática", Escola de Comércio do Porto, Porto, May 8, 2013(invited speaker).

[**Aguiar**] "The lattice of synchrony subspaces of a coupled cell network", PDMA Seminars, University of Porto, November 30, 2012 (invited speaker).

[**Aguiar**] "The patterns of synchrony of a coupled cell network", in "CMUP Dynamical Systems Seminars", Universidade do Porto, November 16, 2012 (invited speaker).

[**Dias**] "Coupled cell networks", PhD in Mathematics Programme Coimbra/Porto Seminars, University of Porto, October 28, 2011(invited speaker).

5.4 Visitors

-To work and participate as invited speakers at the "Workshop on Coupled Cell Networks and Dynamics", University of Porto, February 3-5, 2014, organized by **Aguiar and Dias,** http://cmup.fc.up.pt/cmup/coupledcells/:

Peter Ashwin ,University of Exeter, UK;
Fatihcan Atay, Max Planck Inst. for Mathematics in the Sciences, Germany;
Mike Field, Rice University, USA;
Raoul-Martin Memmesheimer, Radboud University Nijmegen, Netherlands;
José Fernando Mendes, University of Aveiro, Portugal;
Bob Rink, VU University Amsterdam, The Netherlands;
Francisco C. Santos, IST, University of Lisbon, Portugal;
Yunjiao Wang, University of Houston, USA;

-Mike **Field**, University of Rice, Houston, USA, 27 September - 6 October, 2012;

-Haibo **Ruan**, University of Hamburg, Germany, 17-24 July, 2012;

-Nikita **Agarwal**, Indian Institute of Science Education and Research Bhopal, India, 30 June - 7 July, 2012;

-Haibo **Ruan**, University of Hamburg, Germany, 18-21 January, 2011.

5.5 Supervisions

-Ph D supervision (concluded)

[Co-orientation of **Aguiar** with Isabel Salgado Labouriau (Porto)] Alexandre Artur Pinho Rodrigues, "Heteroclinic Phenomena", PhD in Mathematics, Faculty of Sciences, University of Porto, January, 2012.

-Post-doctorate coordination (on going)

[**Dias**] Supervision of the post-doctorate work of Célia Sofia Mota da Cunha **Moreira,** since February of 2010. Title of the project: Robust Bifurcations in Coupled Cell Networks. Ongoing.

5.6 Sabbatical leave

[Dias] During the period September of 2012 till September of 2013.

5.7 Conferences and Seminars Organization

-International conferences organization

(Organizers: **Aguiar and Dias**) "Workshop on Coupled Cell Networks and Dynamics", University of Porto, February 3-5, 2014. http://www.fc.up.pt/cmup/coupledcells/

(One of the organizers: **Aguiar**) "International Conference on Geometry and Dynamics of Holomorphic Foliations", University of Porto, January 18-21, 2011.
http://www.fc.up.pt/cmup/foliations/

-Minisymposia organization at international conferences

(Organizer: **Moreira**) Minisymposium "Coupled Cell Networks", at the "International Conference Dynamics, Games and Science II, Mathematics of Planet Earth 2013", Lisbon, Portugal, 2-4 September, 2013.
http://mpe2013.org/workshop/dgs-2013-international-conference-and-advanced-school-planet-earth-dynamics-games-and-science-portugal-26-august-to-7-september-2013/

(Co-organization **Dias** and Bob Rink (Free University Amsterdam, Netherlands)) at the "SIAM Conference on Dynamical Systems", Snowbird, USA, May, 2013:

Minisymposium MS8 "Bifurcations in Coupled Cell Networks - Part I";

http://meetings.siam.org/sess/dsp_programsess.cfm?SESSIONCODE=16115

Minisymposium MS23 "Bifurcations in Coupled Cell Networks – Part II".
http://meetings.siam.org/sess/dsp_programsess.cfm?SESSIONCODE=16116

(Organizers: **Aguiar and Dias**) at the "XXXII Dynamics Days Europe", 2-7 September 2012, Gothenburg, Sweden:

Minisymposium "Dynamics of coupled cell systems inspired by real-world networks";
http://dynamics-days-europe-2012.org/index.php?id=5

Minisymposium "Understanding dynamics of real networks using mathematics.
http://dynamics-days-europe-2012.org/index.php?id=5

-National meetings organization

(One of the organizers: **Aguiar**) "IJUP 2012 - 5º Encontro de Jovens Investigadores da U.Porto", Reitoria da Universidade do Porto, February, 22-14, 2012.
http://www.ijup.up.pt/2012/

(One of the organizers: **Aguiar**) "IJUP 2011 - 4º Encontro de Jovens Investigadores da U.Porto", Reitoria da Universidade do Porto, February, 17-19, 2011.
http://www.ijup.up.pt/2011/

-National seminar series organization

[**Moreira**] Organizer of "CMUP Dynamical Systems Seminars", University of Porto, 2012 until July of 2013.
http://cmup.fc.up.pt/cmup/v2/frames/seminars.htm

-Ongoing workshop organization

(Organization of Sofia Castro (Porto) and **Dias)** "Birthday Party Workshop – Isabel Labouriau´s 60th", University of Porto, June 13, 2014. http://cmup.fc.up.pt/cmup/isabel60/

5.8 Summary list of the activities that have benefited from the project funding

Remarks

-Due to the twin pregnancy of the principal investigator of the project, **Dias** was out of work during the period from September of 2010 till September of 2011, first with sick leave due to the pregnancy risk, and then to the maternity leave. FCT was informed by that fact. During the year 2011, the other investigator of the team, **Aguiar,** was active and executing her part of the project as planned, and was able to do that without using the 2011 project funding. There was a transition of the funding of 2011 to the years 2012 and 2013, which from the missions and consultants point of views have benefited the execution of the project, as in that way it was possible then to have more funding during those two years for both categories and taking advantage of the fact that all the team members were active during 2012 and 2013.

-At February of 2012, **Moreira** have joined the team since her post doctorate work (under coordination of **Dias)** was directly connected with the project plan.

Missions

-Participations of **Aguiar, Dias** and **Moreira** at the International Conference Dynamics, Games and Science II, Mathematics of Planet Earth 2013, Lisbon, Portugal, 2-4 September, 2013; **Aguiar** and **Dias**, each, gave a talk at a session; **Moreira** was the organizer of the session and a speaker.

-Participation of **Dias** at the SIAM Conference on Applications of Dynamical Systems (DS13)", Snowbird, Utah, USA, May 19-23, 2013, where has co-organized two sessions and gave a talk in one of the sessions.

-Partial support to the academic visit done by **Dias** to the Isaac Newton Institute for Mathematical Sciences, Cambridge, UK, January 7-11, 2013, at the occasion of the "Workshop on Symmetry, Bifurcation and Order Parameters". This visit was done by invitation.

-Partial support to the participation of **Aguiar** to the "XXXII Dynamics Days Europe", 2-7 September 2012, Gothenburg, Sweden, where **Aguiar** and **Dias** have organized two sessions.

-Partial support to the academic visits done by **Aguiar and Dias** to the University of Houston, USA, May 14-16, 2012, at the occasion of the "Workshop on Progress and Problems in Dynamics (in honor of Professor Mike Field)". These visits were done by invitation.

-Participation (presenting a poster) of **Moreira** at the "Workshop on critical transitions in complex systems", Imperial College of London, United Kingdom, March 21, 2012.

Consultants

-Funding support of the consultants that were also invited speakers of the workshop organized by **Aguiar** and **Dias**, "Workshop on Coupled Cell Networks and Dynamics", University of Porto, February 3-5, 2014. http://cmup.fc.up.pt/cmup/coupledcells/

-Funding support of the visit to the University of Porto of Professor Mike **Field**, University of Rice, Houston, USA, 27 September - 6 October, 2012.

-Support of the visit to the University of Porto of Dr Haibo **Ruan**, University of Hamburg, Germany, 17-24 July, 2012.

- Support of the visit to the University of Porto of Dr Nikita **Agarwal**, Indian Institute of Science Education and Research Bhopal, India, 30 June - 7 July, 2012.