Iberian meeting on numerical semigroups
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CMUP
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FACULDADE DE CIÊNCIAS
UNIVERSIDADE DO PORTO
Departamento de Matemática Pura

Organizers:
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P. A. García-Sánchez, Universidad de Granada

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Aguiló, Francesc
Universitat Politècnica de Catalunya

SOME CONTRIBUTIONS TO THE FROBENIUS' PROBLEM OF 3 ELEMENTS

Given a 3 *Frobenius' Set* (F3S) $A = \{a, b, N\} \subset \mathbb{N}$, with $a < b < N$ and $\gcd(a, b, N) = 1$, let us consider $\mathscr{R}(A) = \{\alpha a + \beta b + \gamma N \mid \alpha, \beta, \gamma \in \mathbb{N}\}$ and $\overline{\mathscr{R}}(A) = \mathbb{N} \setminus \mathscr{R}(A)$.

The *Frobenius' Problem* related to A, $\mathrm{FP}(A)$, consists on the study of the set $\overline{\mathscr{R}}(A)$. The *solution* of $\mathrm{FP}(A)$ is the explicit description of $\overline{\mathscr{R}}(A)$, however this is a difficult task. Usually *partial solutions* are given, like the cardinal of $\overline{\mathscr{R}}(A)$ or the *Frobenius' Number* $\mathfrak{f}(A) = \text{máx}\,\overline{\mathscr{R}}(A)$.

An $O(h(N))$ *almost-closed* formula (acf) for $v(N)$ is a closed formula for computing $v(N)$, depending on a fixed number of parameters, which can be computed using an $O(h(N))$ algorithm.

Give any F3S, $A = \{a, b, N\}$, three $O(\log N)$ almost closed formulae are given to compute $|\overline{\mathscr{R}}(A)|$, $\mathfrak{f}(A)$ and $\overline{\mathscr{R}}(A)$ (in this case the acf is a parameterization of the set).

These acf are used to obtain closed formulae of some polynomial sequences of F3S $\{A_t\}_{t \geq t_0}$.

D'Anna, Marco
Università di Catania

NUMERICAL SEMIGROUP RINGS WHOSE ASSOCIATED GRADED RING IS BUCHSBAUM

This is a joint work with M. Mezzasalma and V. Micale.

Let $R = K[[t^{g_1}, \ldots, t^{g-n}]]$ be the semigroup ring associated to a numerical semigroup S. We study when its associated graded ring $G(\mathbf{m})$ is Buchsbaum; in particular, we give a theoretical characterization for $G(\mathbf{m})$ to be Buchsbaum not Cohen-Macaulay; moreover, we introduce some family of invariants for S, connected with the Apery sets of S and of its blow-up S', and we use them in order to give a necessary and a sufficient condition for $G(\mathbf{m})$ to be Buchsbaum.

Araújo, António
CMAF/Universidade Aberta

NUMERICAL SEMIGROUPS ASSOCIATED TO ALGEBRAIC CURVES

This is a joint work with Orlando Neto

It is well known that we can associate to a germ of plane curve a numerical semigroup that completely characterizes its topological type. We will introduce another numerical semigroup that is an analytic invariant of plane curves and will use it to solve problems of analytical classification for certain classes of space curves.

Barucci, Valentina
Università di Roma La Sapienza

ALMOST SYMMETRIC NUMERICAL SEMIGROUPS

Let S be a numerical semigroup of maximal ideal $M = S \setminus \{0\}$. S is called almost symmetric if $M + \Omega = M$, where g is the Frobenius number of S and $\Omega = \{z \in \mathbb{Z} \mid g - z \notin S\}$.

Several characterizations of the canonical ideal and of almost symmetric semigroups will be given and, if time permits, it will be shown how for this class of numerical semigroups some open problems have a positive answer.

Branco, Manuel
Universidade de Évora

PROPORTIONALLY MODULAR DIOPHANTINE INEQUALITIES AND THEIR MULTIPLICITY
This is a joint work with J. C. Rosales and P. Vasco
Let I be an interval of positive rational numbers. Then the set $\mathrm{S}(I)$ of numerators of the rational numbers belonging to I is a numerical semigroup. We study maximal ideals I for which S has fixed multiplicity.

Bras-Amorós, Maria
Universitat Rovira i Virgili

RESULTS ON NUMERICAL SEMIGROUPS WITH APPLICATIONS TO ALGEBRAIC GEOMETRY CODES
In the theory of algebraic geometry codes, Weierstrass semigroups are crucial for defining bounds on the minimum distance as well as for defining improvements on the dimension of codes. We will talk about these applications as well as some theoretical problems related to classification, characterization and counting of numerical semigroups.

Delgado, Manuel; García-Sánchez, Pedro A.; Morais, José João
Universidade do Porto; Universidad de Granada

SOFTWARE DEMO
We will make an interactive demonstration of the GAP package numericalsgps in the computer room.

Farrán Martín, José Ignacio
Universidad de Valladolid

COMPUTING FENG-RAO DISTANCES AND APPLICATION TO AG CODES
We discuss several methods to compute the so-called Feng-Rao distances in numerical semigroups, and present special results for concrete types of semigroups. This computation is useful to estimate the minimum distance of a special type of Algebraic Geometry error-correcting codes (and the error-correction capacity, in particular), where the Weierstrass semigroup at one rational point is considered.

Fel, Leonid
Technion Institute

WEAK ASYMPTOTICS IN THE 3-DIM FROBENIUS PROBLEM
We consider the Frobenius problem for numerical semigroup $\langle d_1, d_, d_3\rangle$ generated by three positive integers, $\gcd(d_1, d_2, d_3) = 1$, and calculate weak (Cesaro) asymptotics of its conductor $C(d_1, d_2, d_3)$ and fraction $p(d_1, d_2, d_3)$ of a segment $[0, C(d_1, d_2, d_3) - 1]$ occupied by semigroup. Four conjectures proposed by V. Arnold in 1999-2003 and devoted to statistics of numerical semigroups $\langle d_1, \ldots, d_m\rangle$, $m \geq 3$, are discussed for $m = 3$.

Fröberg, Ralf
Stockholms Universitet

ON THE HOMOLOGY OF SEMIGROUP RINGS
Let $k[S] = k[X_1, \ldots, X_n]/I$ be a numerical semigroup ring. Constructing the minimal graded $k[X_1, \ldots, X_n]$-resolution of $k[S]$, we may derive a formula for the Frobenius number of S in some cases. I will also discuss the (infinite) $k[S]$-resolution of k, and of the module of derivations $\mathrm{Der}_k(k[S])$.

García Marco, Ignacio
Univesidad La Laguna

AN ALGORITHM FOR CHECKING WHETHER THE TORIC IDEAL OF AN AFFINE MONOMIAL CURVE IS A COMPLETE INTERSECTION

This is a joint work with Isabel Bermejo and Juan José Salazar-González.

Let K be an arbitrary field and $\{d_1,\ldots,d_n\}$ a set of all-different positive integers. The aim of this talk is to propose and evaluate an algorithm for checking whether or not the toric ideal of the affine monomial curve $\{(t^{d_1},\ldots,t^{d_n}) \mid t \in K\} \subset A^n$ is a complete intersection. The algorithm is based on new results regarding the toric ideal of the curve, and it can be seen as a generalization of the classical result of Herzog for $n = 3$. Computational experiments show that the algorithm is able to solve large-size instances.

Herzinger, Kurt
United States Air Force Academy

UNITARY NUMERICAL SEMIGROUPS AND PERFECT BRICKS

We examine numerical semigroups of embedding dimension 4 that have a non-principal ideal I such that $\mu_S(I)\mu_S(S-I) = \mu_S(I+(S-I))$. We discuss what it means for the pair (S,I) to be balanced, unitary, and a perfect brick. These topics will be connected to the study of torsion in the tensor product of a fractional ideal with its inverse over a 1-dimensional local domain

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Kaplan, Natan
Cambridge University

DELTA SETS IN NUMERICAL MONOIDS

Let M be an additive submonoid of $\mathbb{N}_0$. There exists a minimal set of natural numbers which generates M. Let $S = \langle n_1,\ldots,n_t\rangle$ be any generating set of M. For $m \in M$, if $m = \sum_{i=1}^t x_i n_i$, then $\sum_{i=1}^t x_i$ is called a *factorization length* of m with respect to S. We denote by $\mathcal{L}^S(m) = \{l_1,\ldots,l_k\}$ (where $l_i < l_{i+1}$ for each $1 \le i < k$) the set of all possible factorization lengths of m. The Delta set of m with respect to the generating set S is defined by $\Delta^S(m) = \{l_{i+1} - l_i \mid 1 \le i < k\}$ and the Delta set of M by $\Delta^S(M) = \cup_{m\in M}\Delta^S(m)$.

In this talk we will discuss some general results on $\Delta^S(M)$ and then focus on the work of the 2007 Trinity University REU program on the case where S is a nonminimal generating set of M. For example we will discuss a recent result that given a numerical monoid M and any natural number N there exist two generating sets S' and S'' of M such that $\mid \Delta^{S'}(M) \mid = 1$ but $\mid \Delta^{S''}(M) \mid > N$.

This talk will assume no familiarity with numerical monoids and should be accessible to a broad audience.

Krause, Ulrich
Universität Bremen

FACTORIZATION IN SEMIGROUP RINGS

Whereas for a factorial domain D the semigroup ring $D[\mathbb{N}_0]$ is factorial again this is no longer true if $\mathbb{N}_0$ is replaced by a proper numerical semigroup S. What then can be said about factorization properties of $D[S]$? The talk explores whether $D[S]$ is a Cale domain which means that for each atom some power can be uniquely factored into elements of an essentially unique basis. It will be shown that for a field D and a numerical semigroup $S \neq \mathbb{N}_0$, the numerical semigroup ring $D[S]$ is a Cale domain if and only if D is a finite field. Examples and applications as well as extensions of this result will be discussed.

Llena, David
Universidad de Almería

CATENARY AND TAME DEGREE OF NUMERICAL SEMIGROUPS
This is a joint work with S. T. Chapman and P. A. García-Sánchez
We present an algorithm which computes the catenary and tame degree of a numerical semigroup. As an example we explicitly calculate the catenary and tame degree of numerical semigroups generated by arithmetical sequences in terms of their first element, the number of elements in the sequence and the difference between two consecutive elements of the sequence. We will first review some basic definitions concerning this procedure.

Ramírez Alfonsín, Jorge
Université Pierre et Marie Curie (Paris 6)

ON TILING RECTANGLES AND TORUS
In this talk, we discuss some conditions for a rectangle and a torus to be tiled with a set of bricks. These conditions depends on the so-called Frobenius number.

Robles-Pérez, Aureliano M.
Universidad de Granada

PROPORTIONALLY MODULAR DIOPHANTINE INEQUALITIES AND PROPORTIONALLY MODULAR NUMERICAL SEMIGROUPS
This is a joint work with J. C. Rosales
We study systems of proportionally modular Diophantine inequalities. In particular, we prove that every system of this type is equivalent to a system in which all the inequalities have the same modulus which can be chosen to be prime.

Rosales, José Carlos
Universidad de Granada

MINI COURSE
The set of solutions of a proportionally modular Diophantine inequality,
Quotients of a numerical semigroup by a positive integer,
Frobenius varieties

Vasco, Paulo
Universidade de Trás-os-Montes e Alto Douro

THE SMALLEST POSITIVE INTEGER THAT IS SOLUTION OF A PROPORTIONALLY MODULAR DIOPHANTINE INEQUALITY
This is a joint work with J. C. Rosales
Given two integers m and n with $n \neq 0$, we denote by $m \mod n$ the remainder of the division of m by n. A proportionally modular Diophantine inequality is an expression of the form $ax \mod b \leq cx$, where a, b and c are positive integers. Given the proportionally modular Diophantine inequality $ax \mod b \leq cx$, we denote by $\mathrm{S}(a,b,c)$ the set of integer solutions of this inequality, $\mathrm{S}(a,b,c) = \{x \in \mathbb{N} \mid ax \mod b \leq cx\}$. The set $\mathrm{S}(a,b,c)$ is a numerical semigroup.
We give an algorithm that allows us to calculate the smallest positive integer that is solution of an inequality of this type, or equivalently, the multiplicity (the smallest positive integer that belongs to a numerical semigroup) of $\mathrm{S}(a,b,c)$. This algorithm as we will see has a great similarity with the Euclides algorithm for computing the greatest common divisor of two integers. We note that it is an open problem to give a formula for the multiplicity of $\mathrm{S}(a,b,c)$, from the integers a, b and c. This problem is still open in the case $c = 1$.
As an application of the above mentioned algorithm, we also give an algorithmic method that, given three positive integers n_1, n_2 and n_3, calculates the smallest positive multiple of n_3 that belongs to $\langle n_1, n_2 \rangle$.

List of participants

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